

Average Rate of Change

Suppose you and your family are planning to drive to DisneyWorld. You know that the trip includes the following approximate sections:

200 Miles East

100 Miles South

350 Miles East

50 Miles North

150 Miles East

450 Miles South

How could you generalize the description of the journey?

Watch This

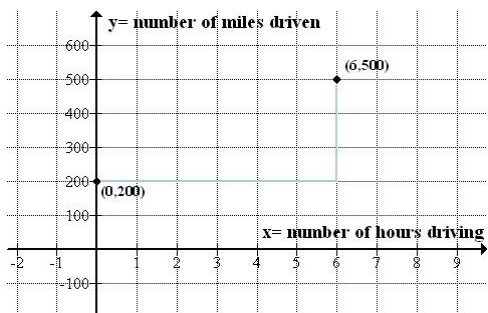
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Guidance

Average Rate of Change

Consider the following situation: you are on a week long road trip with your friend. When you begin to drive on the second day, you have already driven a total of 200 miles. After 6 hours of driving on the second day, you have driven a total of 500 miles. On average, how many miles did you drive per hour on the second day of the trip?

The graph below shows this situation, with the x axis representing the number of hours driving (on the second day), and the y axis representing the number of miles driven. The first point on the graph, $(0, 200)$, says that at the beginning of the second day you have already driven 200 miles. The second point on the graph, $(6, 500)$, says that after 6 hours of driving on the second day you have driven 500 miles total.



Notice that in total, during your 6 hours of driving, you have driven 300 miles. The rate at which you drove is 300 miles in 6 hours, or 50 miles per hour. We refer to this rate as the **average rate of change** because it is an average across the 6 hours. That is, you did not necessarily drive 50 miles every hour. There could have been one hour where you drove 70 miles and another hour where you drove only 30 miles.

We can represent the average rate of change on the graph by indicating how much each quantity has changed: The y values increased by 300, and the x values increased by 6. The average rate of change is the ratio of these changes in each variable. This is how we can define average rate of change in general:

$$\text{Average rate of change} = \frac{\text{change in } y}{\text{change in } x}$$

We can examine the average rate of change of a function, whether it is represented as data, as in the previous example, or by an equation.

Notice that the average rate of change of the function $f(x) = 4x$ is the slope of the line, 4. While a linear function has a constant slope, other functions, such as $f(x) = x^2$, will not. You will explore this idea in greater detail in your study of calculus.

Example A:

Find the average rate of change on the given interval:

$$f(x) = x^2 \text{ on } [0, 2]$$

Solution:

$$f(x) = x^2$$

The endpoints of the interval are (0, 0) and (2, 4). Therefore the change in y is 4 and the change in x is 2. The average rate of change is $4/2 = 2$.

Example B:

Find the average rate of change on the given interval:

$$f(x) = 4x \text{ on } [1, 7]$$

Solution:

$$f(x) = 4x$$

The endpoints of the interval are (1, 4) and (7, 28). Therefore the change in y is $28 - 4 = 24$ and the change in x is $7 - 1 = 6$. The average rate of change is $24/6 = 4$.

Example C:

What is the average rate of change of the function $f(x) = 3x^2$ on the interval [2, 5]?

Solution:

The two points are (2, 12) and (5, 75). The average rate of change is $63/3 = 21$.

Concept question wrap-up

One way to generalize the information about your trip is to identify your average rate of change in position from home to DisneyWorld.

Average Rate of Change = (Change in y) / (Change in x)

The trip segments were:

200 Miles East

100 Miles South

350 Miles East

50 Miles North

150 Miles East

450 Miles South

If we assume that travelling South is $-y$, and traveling East is $+x$:

The total change in $y = -100 + 50 - 450 = -500$

The total change in $x = +200 + 350 + 150 = 700$

Your trip includes a total of 1200 miles of travel, and you will average 5 miles South for every 7 miles East. Your average rate of change is $-5/7$.

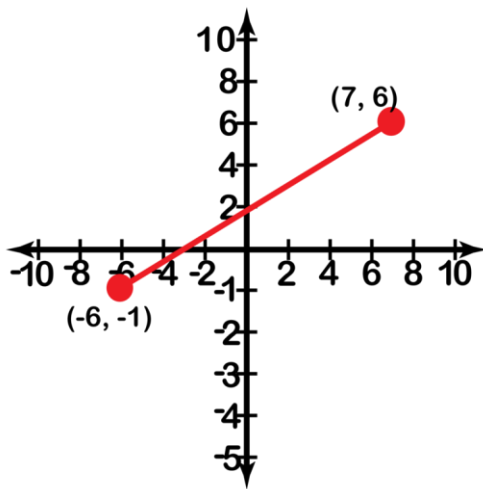
Vocabulary

Average rate of change is the change in y co-ordinates of a function, divided by the change in x co-ordinates.

Guided Practice

Questions

- 1) Find the average rate of change of the function $f(x) = x^2$ as x varies from 1 to 3.
- 2) If the temperature at 1pm was 82deg, and at 9pm was 50deg, what was the average rate of change of temperature over the given time interval?
- 3) Brian drove to town to get some milk, he left at 9am, drove 8mi north, 5mi west, 3mi north, and then 2mi west. He arrived at the store at 9:45am. What was his average rate of change of location?
- 4) If Kelli spends \$5 at 1pm, \$7 at 2:30pm, \$12 at 4pm, and \$2 at 4:30pm, what is her average rate of spending?
- 5) What is the average rate of change shown in the image below?



Answers

- 1) The average rate of change is the [slope](#) of the line that passes through the two points $(3, 9)$ and $(1, 1)$ on the graph.

$$\frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

$$\frac{f(3) - f(1)}{3 - 1}$$

$$\frac{9 - 1}{3 - 1}$$

$$4$$

- 2) The temperature dropped by 32deg over a period of 8 hours:

$$\frac{32}{8} = 4$$

\therefore 4deg per hour

3) Brian drove a total of 18miles, in a total of 45mins:

$$\frac{18mi}{45min} = \frac{2}{5} \text{ miles per minute}$$

4) Kelli spent a total of \$26 over a total of 3 1/2 hours:

$$\frac{\$26}{7/2hr}$$

$\frac{\$52}{7}$ Multiply top and bottom by 2 to remove the fraction in denominator.

$$\approx \frac{\$7.43}{hr}$$

5) The two points in the image are: $(-6, -1) : (7, 6)$

The average rate of change is the same as the slope of the line.

Recall that $m = \frac{\Delta y}{\Delta x} \therefore \frac{y_2 - y_1}{x_2 - x_1}$

$$\frac{(6) - (-1)}{(7) - (-6)}$$

$$\frac{7}{13}$$

Practice

For $y = x^3$, find the average rate of change as:

- x increases from 1 to 3
- x increases from -4 to -1
- Suppose $f(1) = 2$ and the average rate of change of f between 1 and 5 is 3. Find $f(5)$
- Jamie went on a bicycle trip and stopped regularly at half-hour intervals. At each break he recorded his total distance since leaving home. What was his average speed in km/h, during the first half of the trip? During the last half? Jamie hoped to average at least 11.5km/h over the course of the trip. Did he? Explain.

Stops	Time (h)	Distance (km)
1	0.5	7

Stops	Time (h)	Distance (km)
2	1.0	15
3	1.5	21
4	2.0	24
5	2.5	28
6	3.0	36

- On Monday, the price of a gallon of gas was \$3.74, and on Saturday, the price had risen to \$4.09. What is the average rate of change of the price of a gallon of gas?
- According to census figures, the population of Clovis was 31,194 in 1980 and 32,511 in 2001. What was the average rate of change of the population over that time interval?
- Griego and Sons will deliver 12 yd³ of gravel for \$240 and 30 yd³ for \$575. What is the average rate of change of the cost as the number of cubic yards varies from 12 to 30?
- When a load of 5 pounds is placed on a spring, its length is 6 inches, and when a load of 9 pounds is placed on the spring, its length is 8 inches. What is the average rate of change of the length of the spring as the load varies from 5 pounds to 9 pounds?
- Driving at a speed of 75 mph, a Mini Cooper's fuel efficiency is 29 miles per gallon. If the driver slows to a speed of 60 mph, he will have a fuel efficiency of 34 miles per gallon. What is the average rate of change of the fuel efficiency as the speed drops from 75 mph to 60 mph?
- Let $y = f(x) = x^2 + x + 2$ a) Find the average rate of change of y with respect to x between $x = -1$ and $x = 2$ b) Draw the graph of f and the graph of the secant line through $(-1, -2)$ and $(2, 4)$
- Amy takes a trip from Chicago to Milwaukee. Due to road construction, she drives the first 10 miles at a constant speed of 20 mph. For the next 30 miles she maintains a constant speed of 60 mph and then stops at McDonald's for 10 minutes for a snack. She drives the next 45 miles at a constant speed of 45 mph. a) What was Amy's average **driving** speed for the trip? b) What is her average speed for the entire trip (including the stop at McDonald's)?
- The weight $w(t)$ (in grams) of a tumor t weeks after it forms is given by $w(t) = \frac{t^2}{15}$. Find the average rate at which the tumor is growing during the fifth week after it was formed.