Illustrative Mathematics

F-IF The Random Walk

Alignments to Content Standards

- Alignment: F-IF.A.2

Tags

- *This task is not yet tagged.*

Imagine Scott stood at zero on a life-sized number line. His friend flipped a coin 50 times. When the coin came up heads, he moved one unit to the right. When the coin came up tails, he moved one unit to the left. After each flip of the coin, Scott's friend recorded his position on the number line.

a. Let f assign to the whole number n, when $1 \leq n \leq 50$, Scott's position on the number line after the n^{th} coin flip. Explain why f is a function.

b. Write a sentence explaining what $f(5) = 5$ means in everyday language.

c. Before Scott began the random walk, he asked his friend to calculate the probability that $f(3) = 0$. What should his friend respond?

Commentary

This task requires interpreting a function in a non-standard context. While the domain and range of this function are both numbers, the way in which the function is determined is not via a formula but by a (pre-determined) sequence of coin flips.

In addition, the task provides an opportunity to compute some probabilities in a discrete situation. The task could be used to segue the discussion from functions to probability, in particular the early standards in the S-CP domain.

Solutions

Solution: 1

a. We know that f is a function because for each whole number $1 \leq n \leq 50$ Scott is at exactly one point on the number line. It is important here that the coin flipping and Scott's movement have already taken place so that f assigns a genuine spot on the number line to each whole number $1 \leq n \leq 50$, not some hypothetical spot to be determined in the future.

b. The statement $f(5) = 5$, in everyday language, means that after 5 coin flips, Scott is standing on the number 5 of the number line.

c. To find the probability that $f(3) = 0$ we can list all possible outcomes of the three coin tosses and Scott's location after those three tosses. In the first column of the table we abbreviate heads by `H' and tails by `T' so, for example HTH means that the first coin toss came out heads, the second was tails, and the third was heads.

Coin tosses	**Final Position**
HHH	3
HHT	1
HTH	1
HHT	1
HTT	-1
HTH	-1
TTH	-1
TTT	-3

There are eight possible outcomes and none of them is at the origin so this means that the probability that Scott is at the origin after three coin tosses is $\frac{0}{8}$ or 0.

Solution: Alternate solution for part (c)

The third part of this problem can be solved by studying the walk in terms of even and odd numbers. In order to finish at 0, Scott would have to move the same number of steps to the left and to the right: in other words, he would have to move an even number of times. Three is not an even number so after three moves it is impossible for Scott to be at 0.

Another way to present this argument is as follows. On the first move, Scott must go to an odd number, either $+1$ or -1. On the next move, he must go to an even number, either 0 or 2 if the first step was to go to $+1$ or -2 or 0 if the first step was to go to -1. This succession of even and odd will continue because an odd number plus or minus one is an even number and an even number plus or minus one is an odd number. So after an odd number of steps Scott must be on an odd number and cannot be on 0 or any other even number.