Illustrative Mathematics

F-IF Influenza epidemic

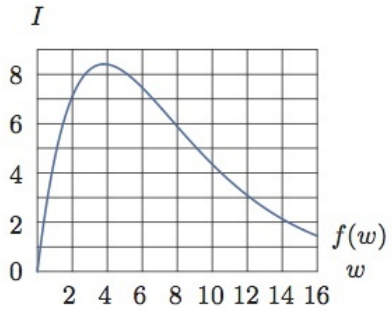
Alignments to Content Standards

- Alignment: F-IF.B.4

Tags

• *This task is not yet tagged.*

An epidemic of influenza spreads through a city. The figure below is the graph of $I = f(w)$ where I is the number of individuals (in thousands) infected w weeks after the epidemic begins.



a. Estimate $f(2)$ and explain its meaning in terms of the epidemic.

b. Approximately how many people were infected at the height of the epidemic? When did that occur? Write your answer in the form $f(a) = b$.

c. For approximately which w is $f(w) = 4.5$; explain what the estimates mean in terms of the epidemic.

d. An equation for the function used to plot the image above is $f(w) = 6w(1.3)^{-w}$. Use the graph to estimate the solution of the inequality $6w(1.3)^{-w} \geq 6$. Explain what the solution means in terms of the epidemic.

(Task from *Functions Modeling Change: A Preparation for Calculus, Connally et al., Wiley 2010.*)

Commentary

The principal purpose of this task is to probe students' ability to correlate symbolic statements about a function using function notation with a graph of the function, and to interpret their answers in terms of the quantities between which the function describes a relationship. It can be used in assessment, or in instruction to bring out some common frailties of student understanding, such as not really understanding what it means for a point to lie on the graph of a function, and, in part (d), not being comfortable with interchanging a function value expressed in function notation and an expression for the function. As it is impossible to read exact numerical data from the graph, students will have to approximate coordinates of data points on the graph, providing a good opportunity for an instructor to address MP6 (attend to precision).

Solutions

Solution: Influenza epidemic

a. To evaluate $f(2)$, we determine which value of I corresponds to $w = 2$. Looking at the graph, we see that $I \approx 7$ when $w = 2$. This means that approximately 7000 people were infected two weeks after the epidemic began.

b. The height of the epidemic occurred when the largest number of people were infected. To find this, we look on the graph to find the largest value of I, which seems to be approximately 8.5, or 8500 people. This seems to have occurred when $w = 4$, or four weeks after the epidemic began. We can say that the height of the epidemic corresponds to the evaluation $f(4) = 8.5$.

c. To find a solution to $f(w) = 4.5$, we must find the value of w for which $I = 4.5$, or 4500 people were infected. We see from the graph that there are actually two values of w at which $I = 4.5$, namely $w \approx 1$ and $w \approx 10$. This means that 4500 people were infected after the first week when the epidemic was on the rise, and that after the tenth week, when the epidemic was slowing, 4500 people remained infected.

d. We are looking for all the values of w for which $f(w) \geq 6$. Looking at the graph, this seems to happen for all values of $w \geq 1.5$ and $w \leq 8$. This means that more than 6000 people were infected starting in the middle of the second week and lasting until the end of the eighth week, after which time the number of infected people fell below 6000.