Illustrative Mathematics

F-IF.6 Mathemafish Population

Alignments to Content Standards

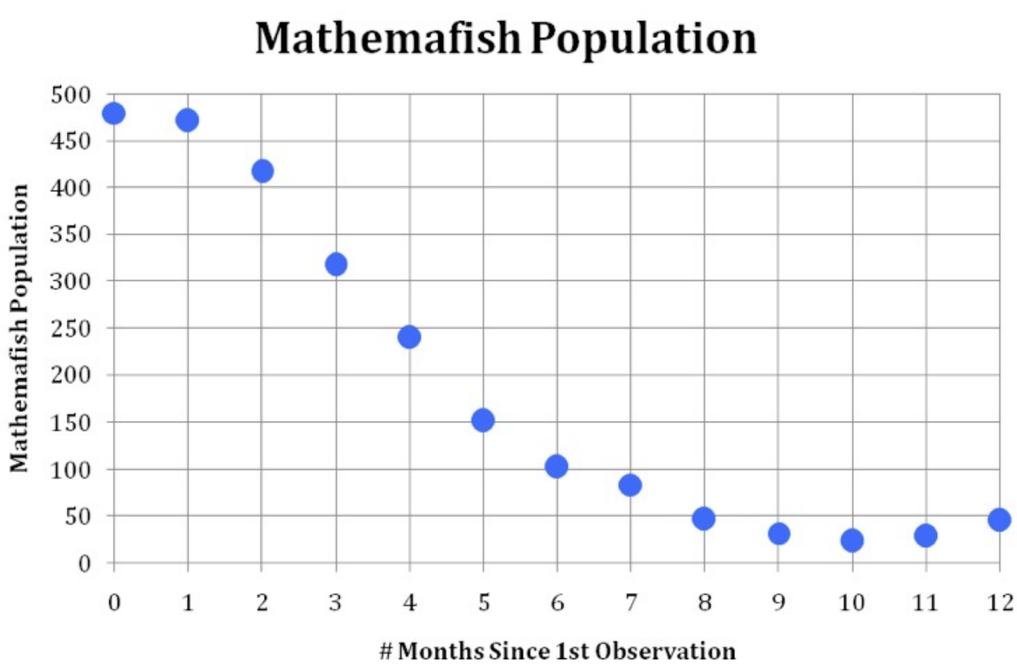
- Alignment: F-IF.B.6

Tags

- *This task is not yet tagged.*

You are a marine biologist working for the Environmental Protection Agency (EPA). You are concerned that the rare coral mathemafish population is being threatened by an invasive species known as the fluted dropout shark. The fluted dropout shark is known for decimating whole schools of fish. Using a catch-tag-release method, you collected the following population data over the last year.

# months since 1st measurement	0	1	2	3	4	5	6	7	8	9	10	11	12
Mathemafish population	480	472	417	318	240	152	103	84	47	32	24	29	46



Through intervention, the EPA was able to reduce the dropout population and slow the decimation of the mathemafish population. Your boss asks you to summarize the effects of the EPA's intervention plan in order to validate funding for your project.

What to include in your summary report:

- Calculate the average rate of change of the mathemafish population over specific intervals. Indicate how and why you chose the intervals you chose.
- When was the population decreasing the fastest?
- During what month did you notice the largest effects of the EPA intervention?
- Explain the overall effects of the intervention.
- Remember to justify all your conclusions using supporting evidence.

Commentary

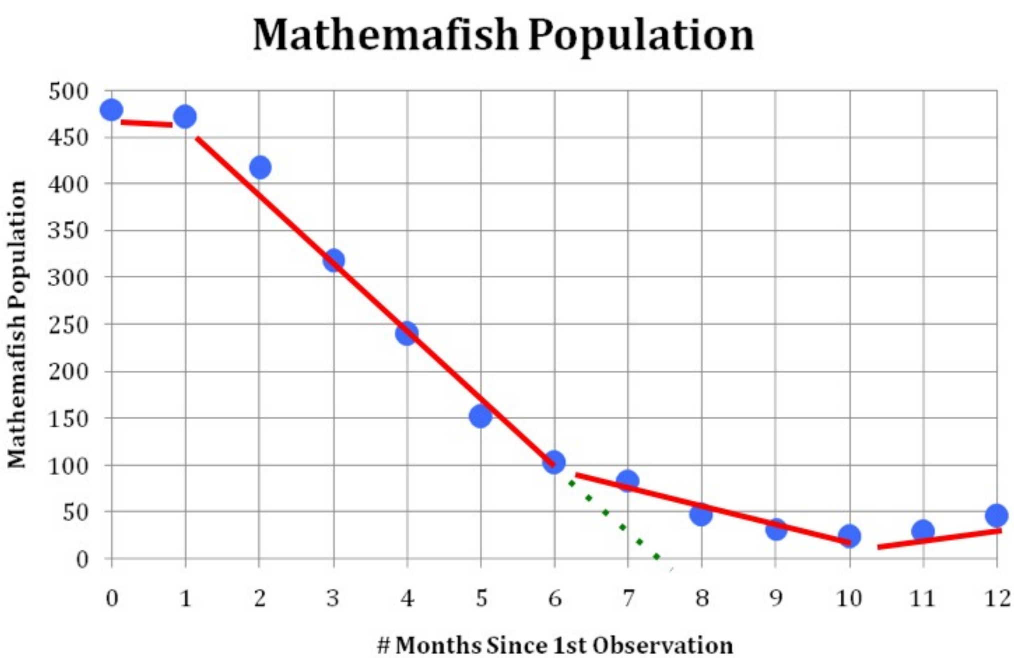
In this problem, students use given data points to calculate the average rate of change of a function over a specific interval, foreshadowing the idea of limits and derivatives to students. Indeed, teachers of advanced classes could closely mirror this idea by starting with the average rate of change for the year and then shrinking the interval to approximate the line tangent to the curve at a specific point. Exposing students to these ideas early on (even without the terminology of calculus) will make it easier for them to conceptualize these concepts when they explore them further in calculus.

Solutions

Solution: Solution

Solutions are asked to be put in the format of a report. We include below a sample collection of the most relevant pieces of information -- alternatives abound.

Looking at the graph, it seems that there were four intervals in which the rate of population change was significantly different; that is, where the slope of the line appeared to change. These intervals are: (0,1), (1,6), (6,10), and (10,12).



$$m_{(0,1)} = \frac{\Delta y}{\Delta x} = \frac{472-480}{1-0} = \frac{-8 \text{ fish}}{1 \text{ month}} = -8\,\frac{\text{fish}}{\text{month}}$$
$$m_{(1,6)} = \frac{\Delta y}{\Delta x} = \frac{103-472}{6-1} = \frac{-369 \text{ fish}}{5 \text{ month}} = -73.8\,\frac{\text{fish}}{\text{month}}$$
$$m_{(6,10)} = \frac{\Delta y}{\Delta x} = \frac{24-103}{10-6} = \frac{-79 \text{ fish}}{4 \text{ month}} = -19.75\,\frac{\text{fish}}{\text{month}}$$
$$m_{(10,12)} = \frac{\Delta y}{\Delta x} = \frac{46-24}{12-10} = \frac{22 \text{ fish}}{2 \text{ month}} = +11\,\frac{\text{fish}}{\text{month}}$$

The mathemafish population decreased along three of the intervals, but decreased the fastest between 1 and 6 months with a rate of change of –73.8 fish each month. We begin to see the effects of the EPA intervention around the 6th month when the rate of *decrease* drops from approximately 74 fish per month to 20 fish per month. If not for the intervention, the fluted dropout shark would have decimated the mathemafish population before month 8.

Population in 6th month: 103 fish

Current rate of change: –73.8 fish/month

$$y = 103 - 73.8x$$

$$0 = 103 \text{ fish} - \left(73.8\,\frac{\text{fish}}{\text{month}}\right)(x \text{ months})$$

$$\left(73.8\,\frac{\text{fish}}{\text{month}}\right)(x \text{ months}) = 103 \text{ fish}$$

$$x \approx 1.39 \text{ months}$$

$$6 + 1.39 = 7.39 \text{ months}$$

Additionally, because of the intervention, by month 10 we even begin to see the population *increase* with a rate of change of 11 fish/month. If the EPA intervention remains effective, we can expect to see continued growth in the mathemafish population.