Illustrative Mathematics

A-SSE Graphs of Quadratic Functions

Alignments to Content Standards

- Alignment: A-SSE.B.3
- Alignment: F-IF.C.7

Tags

• *This task is not yet tagged.*

a. Graph these equations on your graphing calculator at the same time. What happens? Why?

$$\begin{aligned} y_1 &= (x-3)(x+1) \\ y_2 &= x^2-2x-3 \\ y_3 &= (x-1)^2-4 \\ y_4 &= x^2-2x+1 \end{aligned}$$

b. Below are the first three equations from the previous problem.

$$\begin{aligned} y_1 &= (x-3)(x+1) \\ y_2 &= x^2-2x-3 \\ y_3 &= (x-1)^2-4 \end{aligned}$$

These three equations all describe the same function. What are the coordinates of the following points on the graph of the function? From which equation is each point most easily determined? Explain.

i. vertex: _____

ii. y-intercept: _____

iii. x-intercept(s): _____

c. Make up an equation for a quadratic function whose graph satisfies the given condition. Use whatever form is most convenient.

i. Has a vertex at $(-2,-5)$.

ii. Has a y-intercept at $(0,6)$

iii. Has x-intercepts at $(3,0)$ and $(5,0)$

iv. Has x-intercepts at the origin and $(4,0)$

v. Goes through the points $(4,2)$ and $(1,2)$

Commentary

This exploration can be done in class near the beginning of a unit on graphing parabolas. Students need to be familiar with intercepts, and need to know what the vertex is. It is effective after students have graphed parabolas in vertex form ($y = a(x–h)^2 + k$), but have not yet explored graphing other forms. Part (a) is not obvious to them; they are excited to realize that equivalent expressions produce the same graph. Parts (b) and (c) lead to important discussions about the value of different forms of equations, culminating in a discussion of how we can convert between forms and when we might want to do so.

A natural extension of this task is to have the students share some of the different equations that they found for a given condition and have them graph two or more simultaneously. For example, students could graph three different equations that all have the same x-intercepts and discuss the effect that the different constant factors have on the graph.

Solutions

Solution: Solutions

a. When you graph these four equations, only two different parabolas are shown. This is because the first three equations are equivalent, and so all produce the same graph. We can see the equivalence as follows:

   - If we multiply the factors given in the first equation, we'll get the second equation:

$$\begin{aligned}&(x-3)(x+1) = \\ &x^2 - 3x + 1x - 3 = \\ &x^2 - 2x - 3\end{aligned}$$

   - Similarly, if we multiply out the perfect square and combine like terms in the third equation, we also get the second one:

$$\begin{aligned}&(x-1)^2 - 4 = \\ &x^2 - 2x + 1 - 4 = \\ &x^2 - 2x - 3\end{aligned}$$

   The fourth function produces a different graph. We can see that the difference between it and y_2 is just 4, so that graph is 4 units below the other one.

b. i. The vertex is $(1, -4)$ which is most visible in y_3 since the vertex occurs at the point where the squared portion is zero.

   ii. The y-intercept is $(0, -3)$, which is visible as the constant in y_2 since the other terms are 0 when you plug in $x = 0$.

   iii. The x-intercepts are $(3, 0)$ and $(-1, 0)$, which are most visible in y_1 since you can find the roots of the polynomial using the zerofactor property and thus the intercepts correspond to the zeros of each factor.

c. Note: each of these problems has many possible answers. We're including three possible answers for each one, to demonstrate the type of variability you might expect to see in a class. Asking students for three possible answers is a great extension for students - it gets them thinking about the effects of the different parts of the equation.

   i. The following have a vertex at $(-2, -5)$:

$$y = (x+2)^2 - 5$$
$$y = -(x+2)^2 - 5$$
$$y = 3(x+2)^2 - 5$$

   ii. The following have a y-intercept of $(0, -6)$:

$$y = x^2 - 6$$
$$y = x^2 + 13x - 6$$
$$y = 2x^2 - 6$$

   iii. The following have x-intercepts of $(3, 0)$ and $(5, 0)$:

$$y = (x-3)(x-5)$$
$$y = 2(x-3)(x-5)$$
$$y = -7(x-3)(x-5)$$

   iv. The following have x-intercepts at the origin and $(-4, 0)$:

$$y = x(x+4)$$
$$y = 12x(x+4)$$
$$y = -x(x+4)$$

   v. The following go through the points $(-4, 2)$ and $(1, 2)$:

$$y = (x+4)(x-1) + 2$$
$$y = 2(x+4)(x-1) + 2$$
$$y = -\frac{1}{2}(x+4)(x-1) + 2$$

   (Note: students will likely need to experiment quite a bit to find an equation that satisfies these constraints.)