Graphs of Polynomial Functions

Use your graphing calculator to graph the functions below. What are the real roots of the functions?

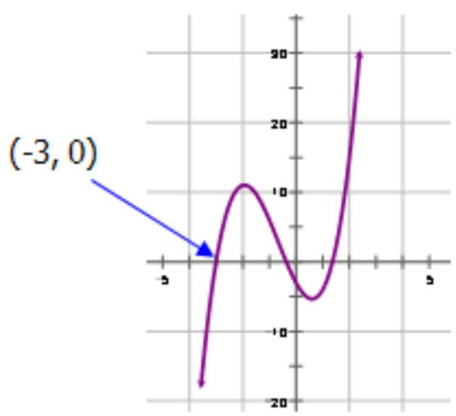
1. $f(x) = x^3 - 6x^2 + 11x - 6$
2. $g(x) = 2x^4 - 4x^3 - 3x^2 + 12x - 8$

Watch This

James Sousa: Determing the Zeros or Roots of a Polynomial Function on the TI83/84

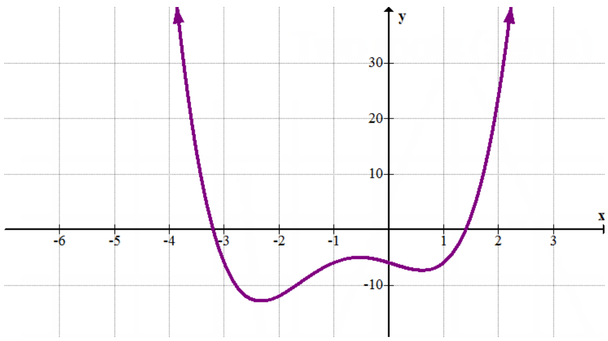
Guidance

Recall that $x - a$ is a factor of polynomial $p(x)$ if $p(a) = 0$. This means that on a graph, factors will appears as x-intercepts of a polynomial because they will occur at points with a y-coordinate equal to zero. For the function graphed below, you can see one of the x-intercepts is the point (–3, 0).

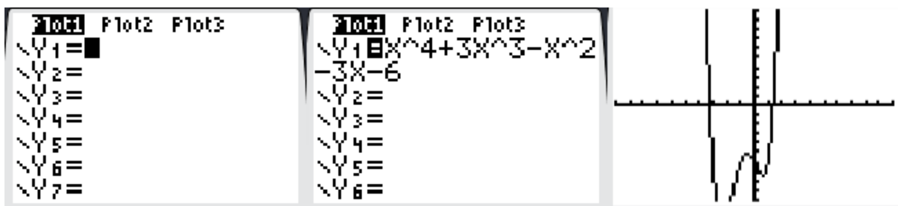


This means that one of the factors of the polynomial must be $(x + 3)$. For the polynomial above,-3 is not only known as an x-intercept. It is also known as a **real root** of the polynomial.*Whenever a root (x-intercept) of a polynomial is an integer, it corresponds to a factor of the function.*

Cubic polynomials are degree three and are of the form $y = ax^3 + bx^2 + cx + d$. Graphs of cubics are like the graph above where overall one end of the graph points up and one end of the graph points down. Quartic polynomials are degree four and are of the form$y = ax^4 + bx^3 + cx^2 + dx + e$. Graphs of quartics are like the graph below where overall both ends of the graph point either up or down.



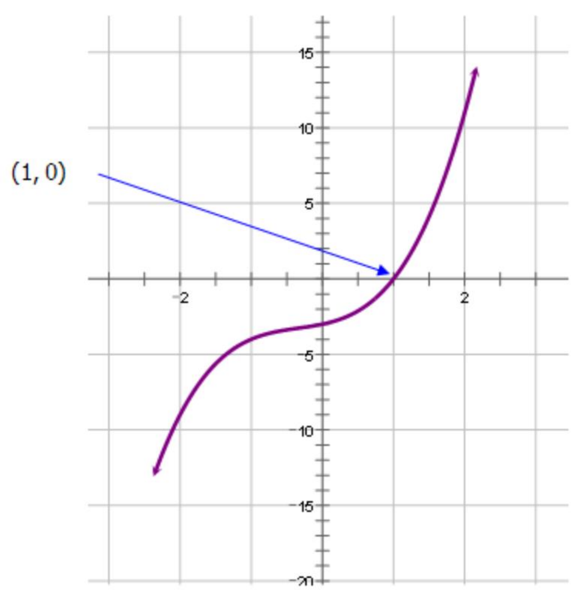
You can use a graphing calculator to graph cubics and quartics. To graph with your graphing calculator, push [Y=], enter your polynomial, push [GRAPH] to see the graph. Then, look at the graph for information about the factors of the polynomial. You can push [TABLE] ([2nd], [GRAPH]) to see the points on the graph more clearly.



Example A

Graph the function $f(x) = x^3 + x^2 + x - 3$ to determine the number of real roots (x-intercepts).

Solution: Once you graph the function, this is what you should see:

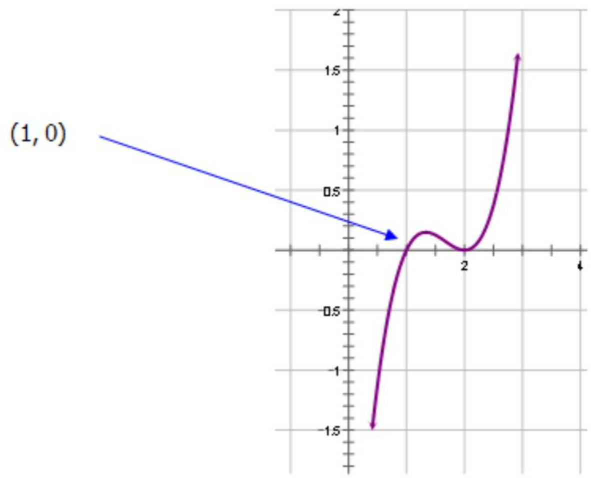


The polynomial $f(x) = x^3 + x^2 + x - 3$ has only one real root (x-intercept) at (1, 0).

Example B

Graph the function $g(x) = x^3 - 5x^2 + 8x - 4$ to determine if $x - 1$ is a factor of the polynomial.

Solution: Once you graph the function, this is what you should see:

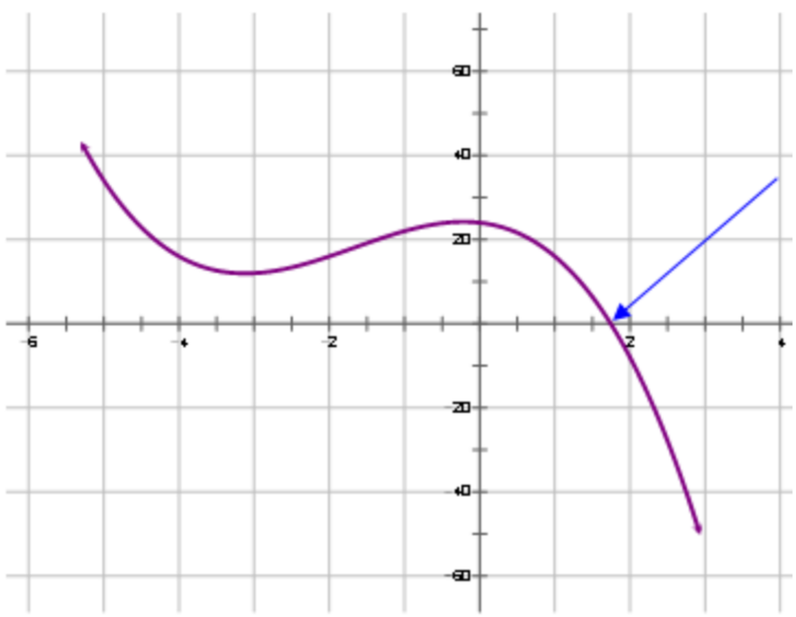


Since (1, 0) is an x-intercept of the polynomial $g(x) = x^3 - 5x^2 + 8x - 4$, $(x - 1)$ is a factor of this cubic.

Example C

How many real roots (x-intercepts) are there for the polynomial $h(x) = -x^3 - 5x^2 - 2x + 24$?

Solution: Once you graph the function, this is what you should see:



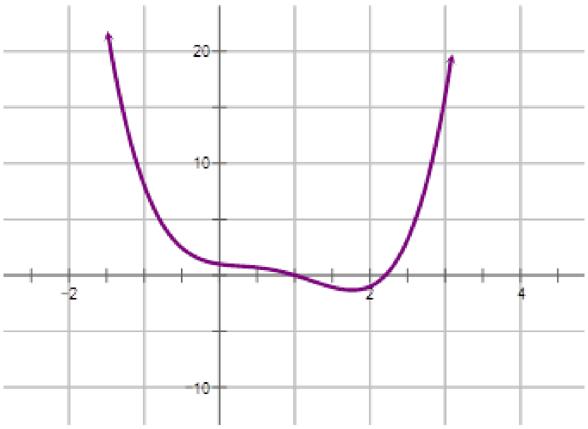
There is one x-intercept so there is one real root.

Example D

Find the real root(s) for the following quartic.

$k(x) = x^4 - 3x^3 + 2x^2 - x + 1$

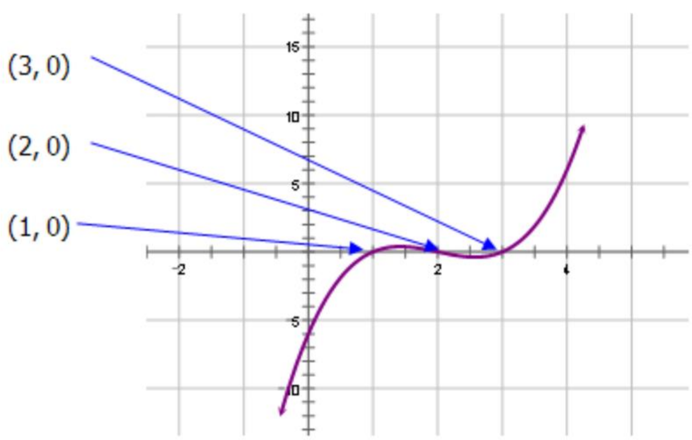
Solution: This is the graph of the quartic:



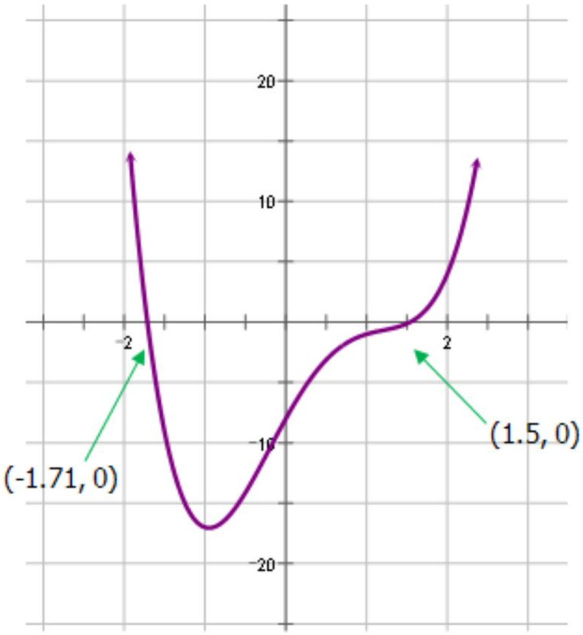
There are two real roots for this quartic. One is (1, 0) and the other occurs around (2.25,0).

Concept Problem Revisited

Here is the graph of the function $f(x) = x^3 - 6x^2 + 11x - 6$:



Here is the graph of the function $g(x) = 2x^4 - 4x^3 - 3x^2 + 12x - 8$. It has two real roots as indicated.



Vocabulary

Cubic Polynomial

A ***cubic polynomial*** is a polynomial where the largest degree is 3. So, for example, $2x^3 + 13x^2 - 8x + 5$ is a cubic polynomial.

Real Root

A ***real root*** is a point where the graph of a function crosses the x -axis.

Quartic Polynomials

Quartic polynomials have a degree of 4. So for example $x^4 - 2x^3 - 13x^2 - 14x + 24$ is a quartic because it has a degree of 4.

Guided Practice

1. Find the real roots for the cubic $y = x^3 + 3x - 4$ using a graph.

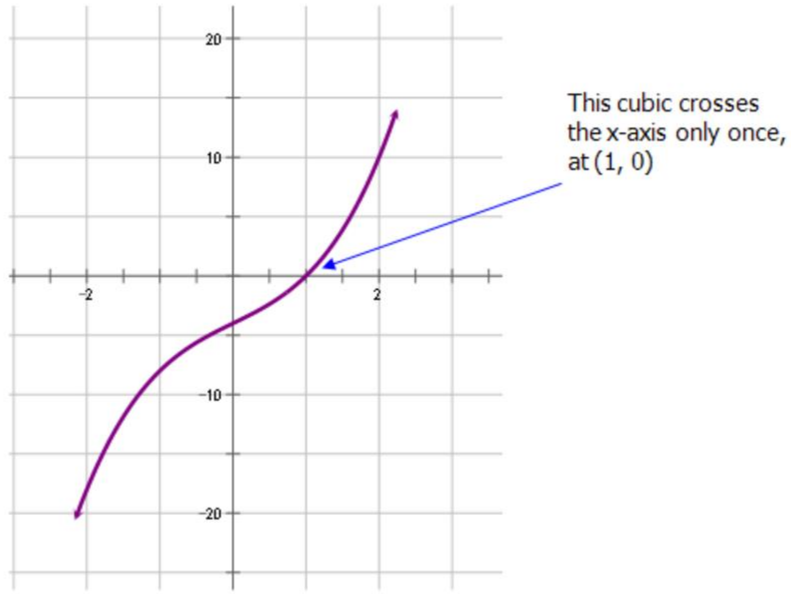
2. Graph the function $g(x) = 3x^3 + 8x^2 + 3x - 2$ and determine the number of real roots. Is $(x - 2)$ one of the factors of this polynomial?

3. Graph the function $m(x) = -2x^3 + 10x^2 + 8x - 1$ to determine if $x - 1$ is a factor of the polynomial.

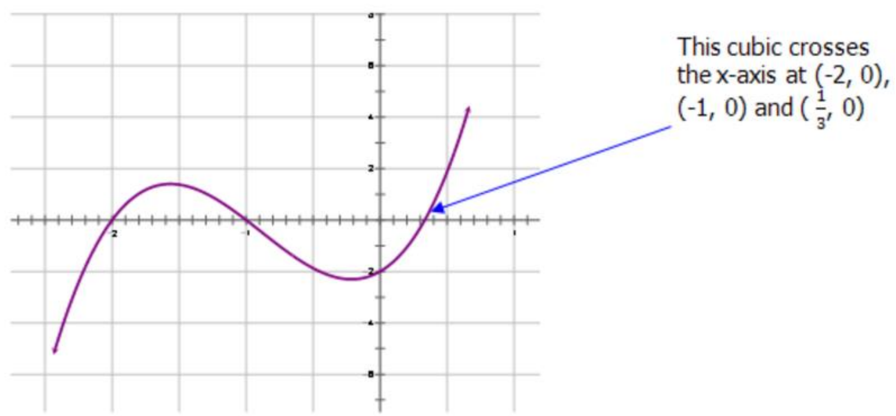
4. Describe the graph of the following quartic: $j(x) = -x^4 - 3x^3 + 2x^2 + x - 6$.

Answers

1.

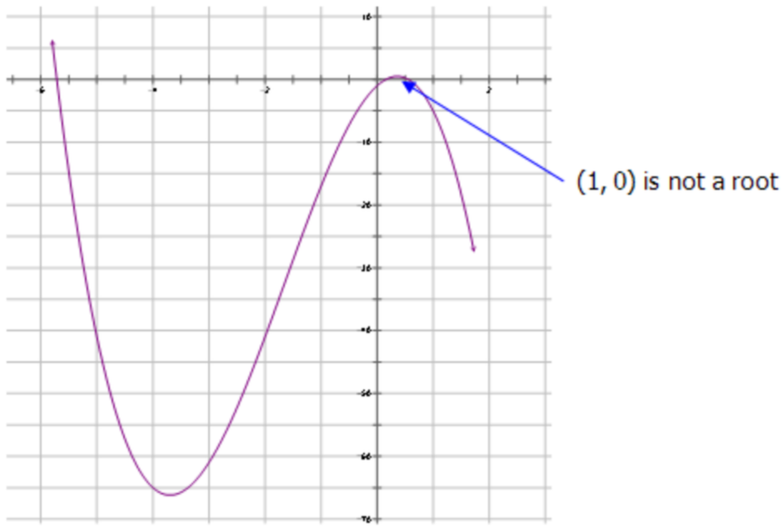


2.

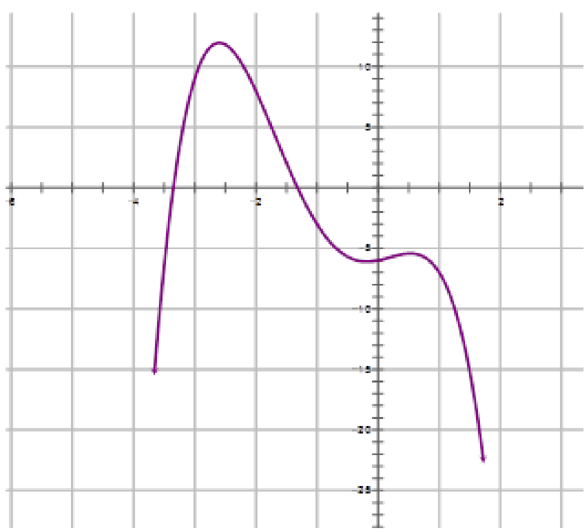


Since one of these root values is (–2, 0), the factor for the polynomial would be $(x+2)$ and not $(x-2)$.

3. If $(x-1)$ was one of the factors then one of the roots would have to be (1, 0). This is not the case.



4. The graph has an M shape. It looks like an M because of the –1 coefficient before x^4 . There are two real roots located at (–3.35, 0) and (–1.32, 0).



Practice

Find the real roots for the following cubic polynomials using a graph.

1. $y = x^3 - 2x^2 - 9x + 18$
2. $y = x^3 + 5x^2 - 4x - 20$
3. $y = 3x^3 - 6x^2 + 12x - 5$
4. $y = 2x^3 - 8x^2 + 3x - 12$
5. $y = -2x^3 - 3x^2 - 5x + 10$

Graph the functions below and determine the number of real roots. Give at least one factor of each polynomial from the graphed solution.

6. $y = x^3 - 3x^2 - 2x + 6$
7. $y = x^3 + x^2 - 3x - 3$
8. $y = x^3 + 2x^2 - 16x - 32$
9. $y = 2x^3 + 13x^2 + 9x + 6$
10. $y = 2x^3 + 15x^2 + 4x - 21$

Graph the functions below to determine if $x - 1$ is a factor of the polynomial.

11. $y = x^3 - 2x^2 + 3x - 6$
12. $y = x^3 + 3x^2 - 2x - 2$
13. $y = 3x^3 + 8x^2 - 5x - 6$
14. $y = x^3 + x^2 - 10x + 8$
15. $y = 2x^3 - x^2 - 3x + 2$

Indicate the real root(s) on the following quartic graphs:

16. $y = x^4 - 3x^3 - 6x^2 - 3$
17. $y = x^4 - 8x^2 - 8$
18. $y = 2x^4 + 2x^3 + x^2 - x - 8$
19. $y = x^4 - 6x^2 - x + 3$
20. $y = x^4 + x^3 - 7x^2 - x + 6$

Describe the following graphs:

21. $y = x^4 - 5x^2 + 2x + 2$
22. $y = x^4 + 3x^3 - x - 3$
23. $y = -x^4 + x^3 + 4x^2 - x + 6$
24. $y = -x^4 - 5x^3 - 5x^2 + 5x + 6$
25. $y = -2x^4 - 4x^3 - 5x^2 - 4x - 4$