Exponential Functions

Have you ever been to a laboratory or conducted an experiment? Take a look at this dilemma.

“We have been given a dilemma by my friend Professor Smith,” Mr. Travis said upon the class’ return to the classroom.

“What is it?” Janet asked.

“Here we go, see what you can do with this,” Mr. Travis wrote the following problem on the board.

In a laboratory, one strain of bacteria can double in number every 15 minutes. Suppose a culture starts with 60 cells, use your graphing calculator or a table of values to show the sample’s growth after 2 hours. Use the function $b = 60 \cdot 2^q$ where b is the number of cells there are after q quarter hours.

To work on this problem, you have to understand exponential functions. Pay close attention during this Concept and you will know how to solve it by the end of it.

Guidance

Let's think about exponential functions by looking at the following situation.

Two girls in a small town once shared a secret, just between the two of them. They couldn’t stand it though, and each of them told three friends. Of course, their friends couldn’t keep secrets, either, and each of them told three of their friends. Those friends told three friends, and those friends told three friends, and son on... and pretty soon the whole town knew the secret. There was nobody else to tell!

These girls experienced the startling effects of an exponential function.

If you start with the two girls who each told three friends, you can see that they told six people or $2 \cdot 3$.

Those six people each told three others, so that $6 \cdot 3$ or $2 \cdot 3 \cdot 3$ —they told 18 people.

Those 18 people each told 3, so that now is $18 \cdot 3$ or $2 \cdot 3 \cdot 3 \cdot 3$ or 54 people.

You can see how this is growing and you could show the number of people told in each round of gossip with a function: $y = ab^x$ where y is the number of people told, a is the two girls who started the gossip, b is the number of friends that they each told, and x is the number of rounds of gossip that occurred.

This is called an *exponential function* —any function that can be written in the form $y = ab^x$, where a and b are constants, $a \neq 0, b > 0$, and $b \neq 1$.

As we did with linear and quadratic functions, we could make a table of values and calculate the number of people told after each round of gossip. Use the function $y = 2 \cdot 3^x$ where y is the number of people told and x is the number of rounds of gossip that occurred.

x rounds of gossip	0	1	2	3	4	5
y people told	2	6	18	54	162	486

How can you tell if a function is an exponential function?

If your function can be written in the form $y = ab^x$, where a and b are constants, $a \neq 0, b > 0$,and $b \neq 1$, then it must be exponential. In quadratic equations, your functions were always to the 2^{nd} power. In exponential functions, the exponent is a variable. Their graphs will have a characteristic curve either upward or downward.

Exponential Functions

1. $y = 2^x$
2. $c = 4 \cdot 10^d$
3. $y = 2 \cdot \left(\frac{2}{3}\right)^x$
4. $t = 4 \cdot 10^u$

Not Exponential Functions

$$1.\ y = 3 \cdot 1^x \qquad 2.\ n = 0 \cdot 3^p \qquad 3.\ y = (-4)^x \qquad 4.\ y = -6 \cdot 0^x$$

$$\text{because} \quad b = 1 \qquad a = 0 \qquad b < 0 \qquad b \leq 1$$

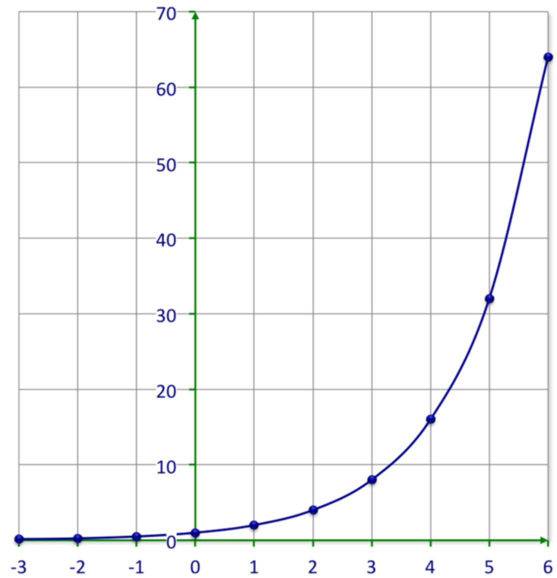
Exponential functions can be graphed by using a table of values like we did for quadratic functions. Substitute values for x and calculate the corresponding values for y .

Take a look at this one.

Graph $y = 2^x$.

Here is the table.

x	$y = 2^x$	y
-3	$y = 2^{-3}$	$\frac{1}{8}$
-2	$y = 2^{-2}$	$\frac{1}{4}$
-1	$y = 2^{-1}$	$\frac{1}{2}$
0	$y = 2^0$	0
1	$y = 2^1$	2
2	$y = 2^2$	4
3	$y = 2^3$	8
4	$y = 2^4$	16
5	$y = 2^5$	32
6	$y = 2^6$	64



Notice that the shapes of the graphs are not parabolic like the graphs of quadratic functions. Also, as the x value gets lower and lower, the y value approaches zero but never reaches it. As the $x$value gets even smaller, the y value may get infinitely close to zero but will never cross the x -axis.

Identify each function.

Example A

$y = 4^x$

Solution: Exponential function

Example B

$f(x) = 2x - 1$

Solution: Linear function

Example C

$y = ax^2 - bx + c$

Solution: Quadratic function

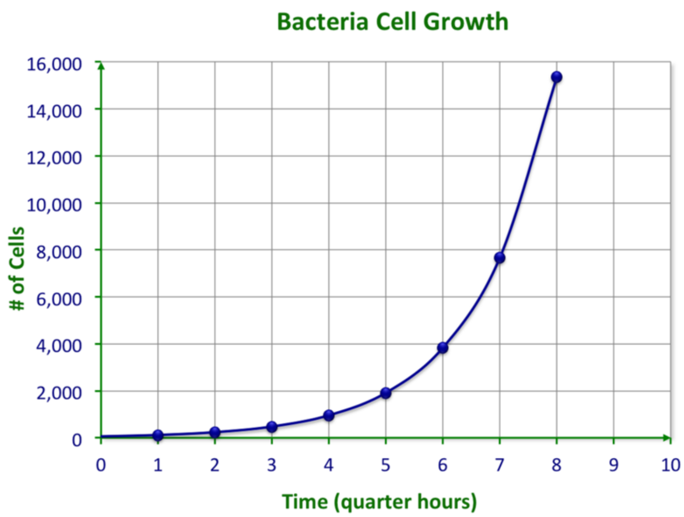
Now let's go back to the dilemma from the beginning of the Concept.

First, we can create a t-table to go with the equation of the function. Here are the values in that table.

q	b
0	60

q	b
1	120
2	240
3	480
4	960
5	1920
6	3840
7	7680
8	15360

Now here is our graph.



Vocabulary

Exponential Functions

results that expand exponentially. The graph curves upward or downward.

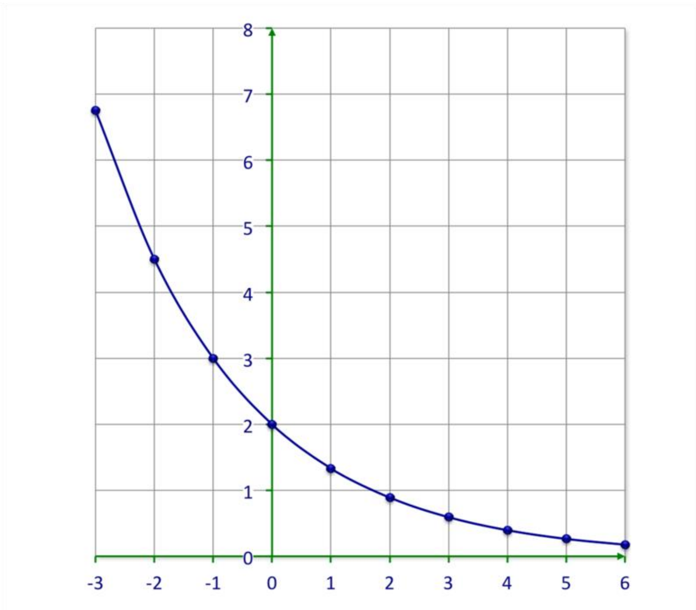
Guided Practice

Here is one for you to try on your own.

Graph $y = 2 \cdot \left(\frac{2}{3}\right)^x$

x	y
-3	$\frac{27}{4}$
-2	$\frac{9}{2}$
-1	3
0	2
1	$\frac{4}{3}$
2	$\frac{8}{9}$
3	$\frac{16}{27}$
4	$\frac{32}{81}$
5	$\frac{64}{243}$
6	$\frac{128}{729}$

Solution



Video Review

Graphing Exponential Functions

Practice

Directions: Classify the following functions as exponential or not exponential. If it is not exponential, state the reason why.

1. $y=7^x$
2. $c=-2\cdot 10^d$
3. $y=1^x$
4. $y=4^x$
5. $n=0\cdot\left(\frac{1}{2}\right)^x$
6. $y=5\cdot\left(\frac{4}{3}\right)^x$
7. $y=(-7)^x$
8. Use a table of values to graph the function $y=3^x$.
9. Use a table of values to graph the function $y=\left(\frac{1}{3}\right)^x$.
10. What type of graph did you make in number 7?
11. What type of graph did you make in number 8?
12. Use a table of values to graph the function $y=-2^x$.
13. Use a table of values to graph the function $y=5^x$.
14. Use a table of values to graph the function $y=-5^x$.
15. Use a table of values to graph the function $y=6^x$.