Illustrative Mathematics

F-IF Which Function?

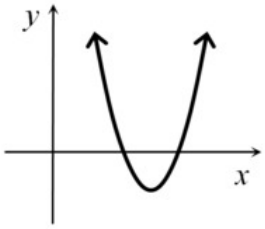
Alignments to Content Standards

- Alignment: F-IF.C.8.a

Tags

• *This task is not yet tagged.*

Which of the following could be the function of a real variable x whose graph is shown below? Explain.



$f_1(x) = (x+12)^2 + 4$

$f_2(x) = -(x-2)^2 - 1$

$f_3(x) = (x+18)^2 - 40$

$f_4(x) = (x-12)^2 - 9$

$f_5(x) = -4(x+2)(x+3)$

$f_6(x) = (x+4)(x-6)$

$f_7(x) = (x-12)(-x+18)$

$f_8(x) = (24-x)(40-x)$

Commentary

The task addresses knowledge related to interpreting forms of functions derived by factoring or completing the square. It requires students to pay special attention to the information provided by the way the equation is represented as well as the sign of the leading coefficient, which is not written out explicitly, and then to connect this information to the important features of the graph. Students who have had plenty of experience re-writing quadratic expressions will be able to determine the sign of the leading coefficient without actually writing the quadratic expressions in $ax^2 + bx + c$ form.

Solutions

Solution: Answers

All of these are expressions for quadratic functions. Since quadratic functions have graphs that are parabolas and the given graph appears to be a parabola, the given expression meet a minimum criteria for consideration.

The graph of $f_1(x) = (x + 12)^2 + 4$ has a vertex of $(-12, 4)$ which is in the second quadrant, so it does not match the graph.

The graph of $f_2(x) = -(x - 2)^2 - 1$ has maximum rather than a minimum value at $x = 2$ since the leading coefficient is negative (in other words, the graph opens downward), so it does not match the graph.

The graph of $f_3(x) = (x + 18)^2 - 40$ has a vertex of $(-18, -40)$ which is in the third quadrant, so it does not match the graph.

The graph of $f_4(x) = (x - 12)^2 - 9$ has a vertex of $(12, -9)$ which is in the fourth quadrant, and the leading coefficient is positive (so the graph would open upward) so this could describe the function whose graph is given.

The graph of $f_5(x) = -4(x + 2)(x + 3)$ has x-intercepts of $(-2, 0)$ and $(-3, 0)$. Since the x-intercepts are both positive for the given graph, they do not match.

The graph of $f_6(x) = (x + 4)(x - 6)$ has x-intercepts of $(-4, 0)$ and $(6, 0)$. The x-intercepts are both positive for the give graph, so they do not match.

The graph of $f_7(x) = (x - 12)(-x + 18)$ has a leading coefficient that is negative and so has a maximum rather than a minimum value (at $x = 15$) and thus cannot match the graph.

The graph of $f_8(x) = (x - 24)(x - 40)$ has x-intercepts of $(24, 0)$ and $(40, 0)$. Since the x-intercepts are both positive for the graph and the leading coefficient is positive (so the graph would open upward), this could possibly be the equation for this graph.

The functions f_4 and f_8 both have graphs of the approximate shape given, though we note that they certainly would not appear identical if plotted simultaneously. For example, the vertex of the graph of f_4 occurs at $x = 12$ whereas that of f_8 occurs at $x = 32$.
