Illustrative Mathematics

F-BF Building a quadratic function from $f(x) = x^2$

Alignments to Content Standards

- Alignment: F-BF.B.3

Tags

Tags: GeoGebra

Suppose $f(x) = x^2$ where x can be any real number.

a. Sketch a graph of the function f.

b. Sketch a graph of the function g given by

$$g(x) = f(x) + 2.$$

How do the graphs of f and g compare? Why?

c. Sketch a graph of the function h given by

$$h(x) = -2 \cdot f(x).$$

How do the graphs of f and h compare? Why?

d. Sketch a graph of the function p given by

$$p(x) = f(x + 2).$$

How do the graphs of f and p compare? Why?

Commentary

This is the first of a series of task aiming at understanding the quadratic formula in a geometric way in terms of the graph of a quadratic function. Here the student works with an explicit function and studies the impact of

- An additive scaling of f
- A multiplicative scaling of f
- A linear change of variables applied to f

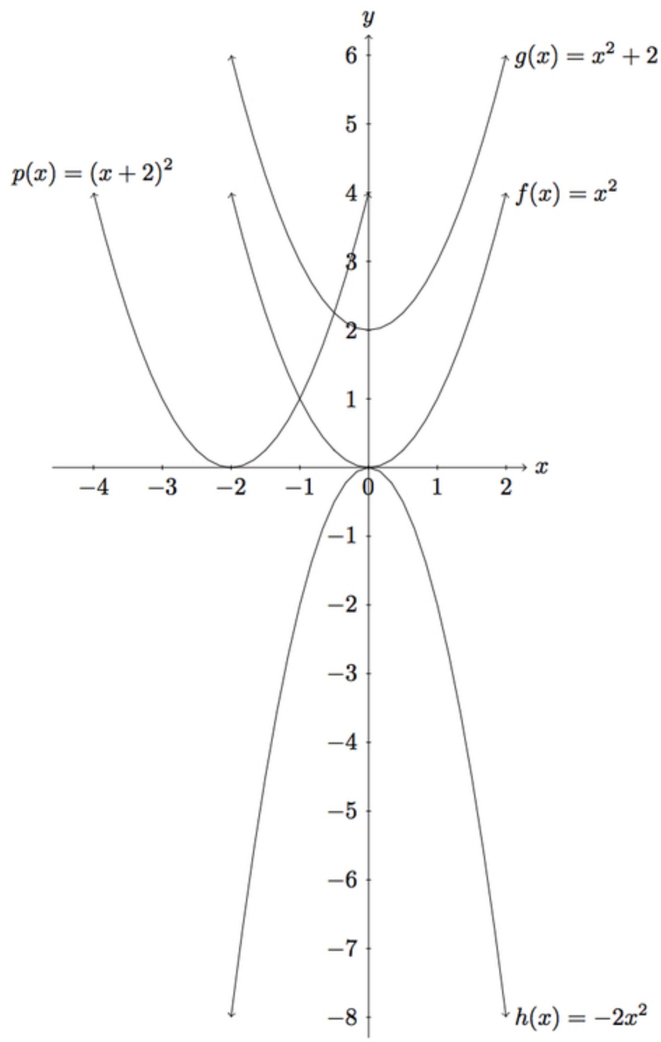
The students can either sketch the graphs by hand or use graphing calculators. The important part of this task is recognizing the impact of the different transformations on the graphs. An alternative way to proceed with this task would be to give the students the graphs and the list of functions f, g, h, p and ask them to identify the functions with their graphs and explain. This task is appropriate for either instruction or assessment.
This task includes an experimental GeoGebra worksheet, with the intent that instructors might use it to more interactively demonstrate the relevant content material. The file should be considered a draft version, and feedback on it in the comment section is highly encouraged, both in terms of suggestions for improvement and for ideas on using it effectively. The file can be run via the free online application GeoGebra , or run locally if GeoGebra has been installed on a computer.

This applet has multiple tasks in one file. There are buttons to select which task is needed and an arrow indicates which task is selected. This task was designed to interactively show how functions vary based on having various operations applied to them. The initial state of this applet shows a given function f(x) in red. On the left side of the screen there is a different window that has various buttons with different colored functions next to them. By clicking these green or red buttons you can toggle the different functions on or off to show how the operations affect the graph of f(x). You are able to change the function to something else by typing in the new function in the top input box. Also you can change the various translated functions using the three other input boxes that are labeled a, b, and c. If you need to reposition the screen you can use the tool at the top of the screen that looks like four arrows to drag the screen to a different position. Also you can use the pointer button at the top of the screen to drag the function f(x) to show how the other functions change.

Solutions

Solution: 1

The graphs of f, g, h, and p are shown (within the range of $-2 \leq x \leq 2$) below together to reveal their geometric relationship:



a. Since $f(x) = x^2$ the graph of f is a parabola. It opens upward and its vertex is at $(0, 0)$.

b. Looking at the two points $(x, f(x))$ and $(x, g(x))$ we have

$$(x, g(x)) = (x, f(x) + 2)$$

for every value of x. So for each point $(x, f(x))$ on the graph of f, there is a corresponding point $(x, f(x) + 2)$ on the graph of g, located two units above $(x, f(x))$ This means that the graph of g is the same as the graph of f, displaced 2 units upward.

c. The point $(x, h(x))$ is the same as $(x, -2f(x))$ so the graph of h is the same as the reflection of the graph of $2f$ about the x-axis. So the values of f are first doubled, exaggerating the slope of the graph, and then the graph is reflected about the x-axis.

d. The point $(x, p(x))$ on the graph of p is the same as $(x, f(x+2))$ which is two units to the right of the point $(x, f(x))$ on the graph of f. So the graph of p looks like the graph of f, translated two units to the left: put in other words, $p(x-2) = f(x)$ so the point $(x-2, f(x))$ is on the graph of p while $(x, f(x))$ is the corresponding point of the graph of f and so the graph of p is the graph of f shifted two units to the left.