Illustrative Mathematics

F-TF Special Triangles

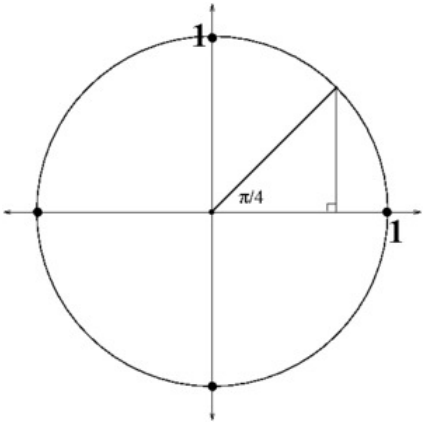
Alignments to Content Standards

- Alignment: HSF-TF.A.3

Tags

• *This task is not yet tagged.*

Use the unit circle and indicated triangle below to find the exact value of the sine and cosine of the special angle $\pi/4$.



Commentary

Using known facts about the unit circle and isosceles triangles together with the Pythagorean Theorem, we can derive the sine and cosine of special angels, in this case of $\pi/4$.

This task can be done as a mini lecture soliciting responses from the students, or as a challenge problem for students to ponder and discuss. It is a very nice connection between geometry and algebra that uses quite simply the symmetry of the triangle.

A natural mathematical practice to focus on in this task is SMP 3 - Make a viable argument and critique the reasoning of others. Similar tasks, to derive the exact values of sine and cosine of $\pi/3$ and $\pi/6$, are in progress.

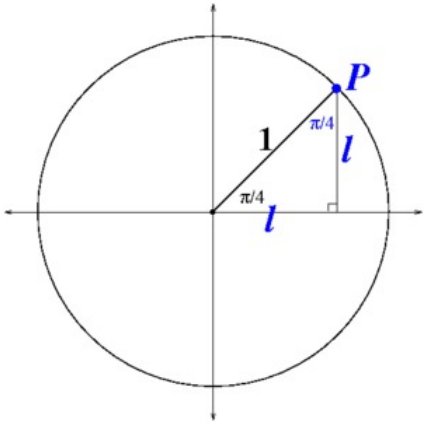
A variant of this task would be to write down the steps in the proof and to ask students to supply the justification.

Solutions

Solution: 1

First we label the vertex of the triangle that lies on the unit circle as point P. We know that P has coordinates $(\cos(\pi/4), \sin(\pi/4))$.

Note that the given triangle is an isosceles right triangle with hypotenuse 1, so we begin by letting l represent the length of each side of the triangle.



Then we have

$$l^2 + l^2 = 1^2 \Longrightarrow 2l^2 = 1 \Longrightarrow l^2 = 1/2,$$

which means that $l = \sqrt{1/2}$, which can also be written as $l = \sqrt{2}/2$. It follows that $P = (\sqrt{2}/2,\ \sqrt{2}/2)$, and we conclude that

$$\cos(\pi/4) = \frac{\sqrt{2}}{2} \text{ and } \sin(\pi/4) = \frac{\sqrt{2}}{2}.$$

(Note that the last step of rationalizing the denominator is not really necessary, it is just an equivalent way of writing the solution.)
