Illustrative Mathematics

F-TF Special Triangles 2

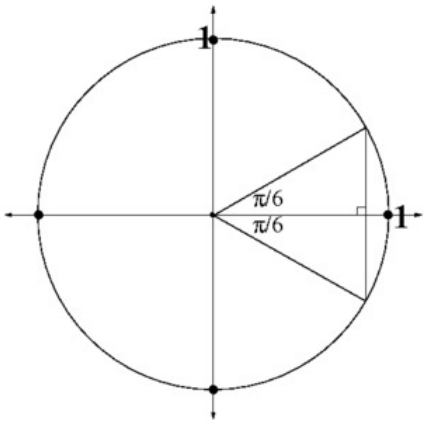
Alignments to Content Standards

- Alignment: HSF-TF.A.3

Tags

• *This task is not yet tagged.*

Use the unit circle and indicated triangle below to find the exact value of the sine and cosine of the special angle $\pi/6$.



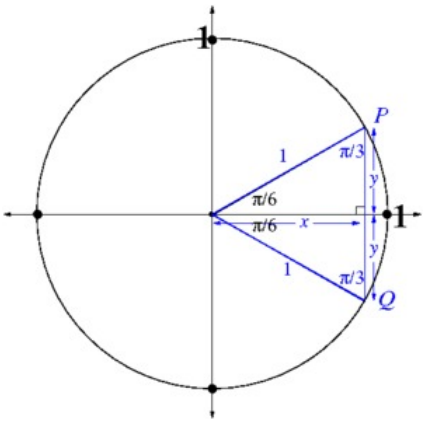
Commentary

Using known facts about the unit circle and isosceles triangles together with the Pythagorean Theorem, we can derive the sine and cosine of special angels, in this case of $\pi/6$. This task can be done as a mini lecture soliciting responses from the students, or as a challenge problem for students to ponder and discuss. It is a very nice connection between geometry and algebra that uses quite simply the symmetry of the triangle. A natural mathematical practice to focus on in this task is SMP 3 - Make a viable argument and critique the reasoning of others. Similar tasks derive the exact values of sine and cosine of $\pi/4$ and $\pi/3$. A variant of this task would be to write down the steps in the proof and to ask students to supply the justification.

Solutions

Solution: 1

First we label the vertices of the triangle that lie on the unit circle as points P and Q . We know that P has coordinates $(\cos(\pi/6), \sin(\pi/6))$.



Note that the blue triangle is isosceles since two of its sides are radii of the circle and therefore both have length 1. Therefore, the angles opposite these sides have the same measure, and therefore must both measure $\pi/3$ in order for the angles in the blue triangle to add up to π. This means that the blue triangle is equilateral, and so $\overline{PQ} = 1$. But we can also observe that the two smaller triangles that make up the blue triangle are congruent by SAS (or since Q is the reflection of P across the x-axis); therefore, $y = 1/2$, and by using the Pythagorean Theorem, we have $x^2 + (1/2)^2 = 1^2$, which implies $x^2 = 3/4$ and $x = \sqrt{3}/2$. Thus, $P = (\sqrt{3}/2,\ 1/2)$, and we conclude that

$$\cos\ \pi/6 = \frac{\sqrt{3}}{2} \text{ and } \sin \pi/6\ = \frac{1}{2}.$$