Illustrative Mathematics

F-TF Foxes and Rabbits 2

Alignments to Content Standards

- Alignment: F-TF.B.5

Tags

- *This task is not yet tagged.*

Given below is a table that gives the populations of foxes and rabbits in a national park over a 12 month period. Note that each value of t corresponds to the beginning of the month and $t = 0$ corresponds to the beginning of January.

t, month	0	1	2	3	4	5	6	7	8	9	10	11
r, number of rabbits	1000	750	567	500	567	750	1000	1250	1433	1500	1433	1250
f, number of foxes	150	143	125	100	75	57	50	57	75	100	125	143

Note that the number of rabbits and the number of foxes are both functions of time.

a. Explain why it is appropriate to model the number of rabbits and foxes as trigonometric functions of time.

b. Find an appropriate trigonometric function that models the number of rabbits, $r(t)$, as a function of time, t, in months.

c. Find an appropriate trigonometric function that models the number of foxes, $f(t)$, as a function of time, t, in months.

d. Graph both functions and give one possible explanation why one function seems to “chase” the other function.

Commentary

The example of rabbits and foxes was introduced in the task (8-F Foxes and Rabbits) to illustrate two functions of time given in a table. We are now in a position to actually model the data given previously with trigonometric functions and investigate the behavior of this predator-prey situation.

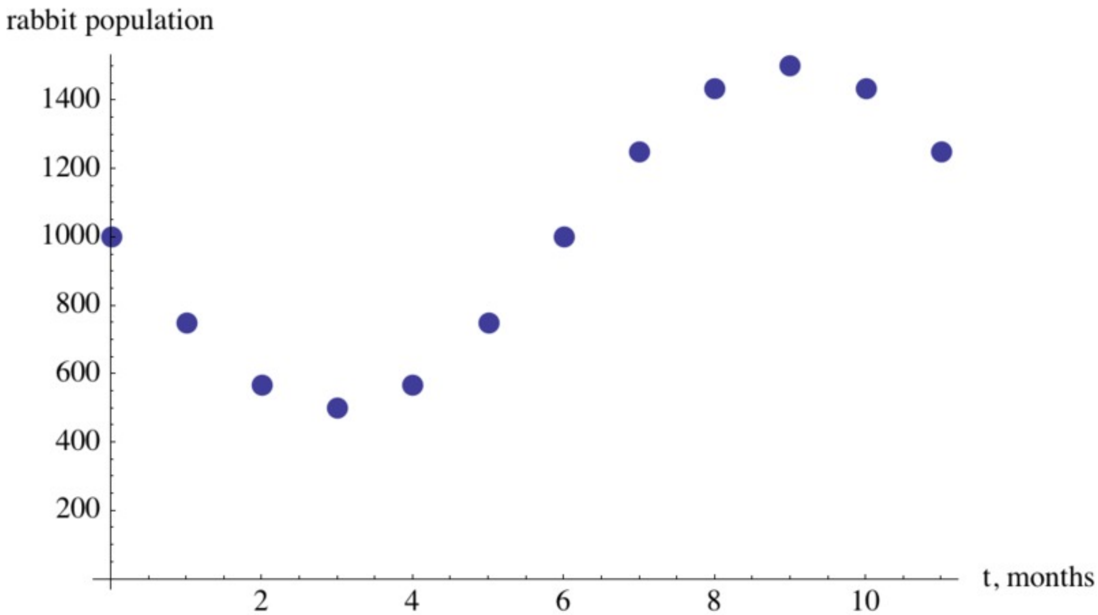
Note that the verb "model" in the task means to find a function which well approximates the data presented in the table. The data in the current task has been simplified (it is much too regular to be completely realistic) so that students can come up with a clear workable model. Indeed, for the given data a sine function can be found which perfectly fits the data, a highly unusual state of affairs. A more realistic situation, and a good follow-up task to the current one, would be to present students with data points that do not perfectly fit on a trigonometric graph. Such a scenario is implemented in the task F-TF Foxes and Rabbits 3. In that case, students have to decide which data values to use for their model and the modeling function will not be a perfect fit for the data.

This task is best used for instruction. It lends itself well for students working together in groups and comparing their modeling functions. Different groups might come up with different function formulas, since it is possible to use positive or negative sine or cosine functions with different horizontal shifts for both populations.

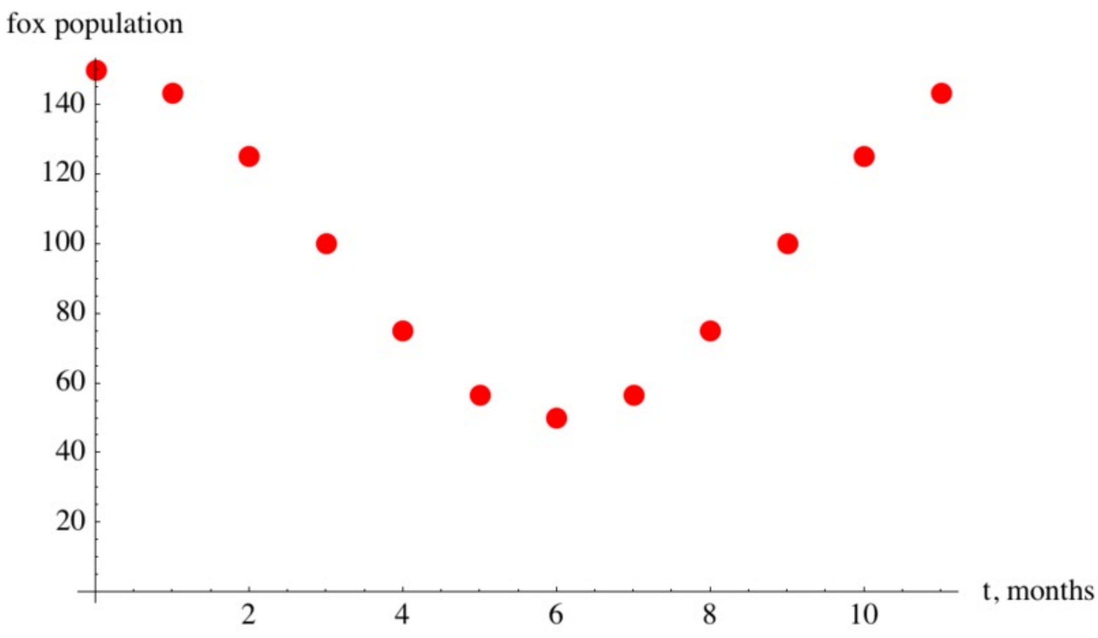
Solutions

Solution: Solution

a. Looking at the table, we notice a pattern in the number of rabbits and foxes. The number of rabbits starts at 1000, decreases to a minimum of 500 after 3 months, increases back to 1000 in the next 3 months, reaches a maximum of 1500 after another 3 months and almost decreases to its starting value of 1000 at the end of the year. Graphing the available points, we see the general shape of a sine or cosine function. Even though we don’t have any additional data, it is reasonable to assume that this pattern will repeat itself in the next year, and we are looking at a periodic function with amplitude 500 rabbits, midline 1000 rabbits and period 12 months.



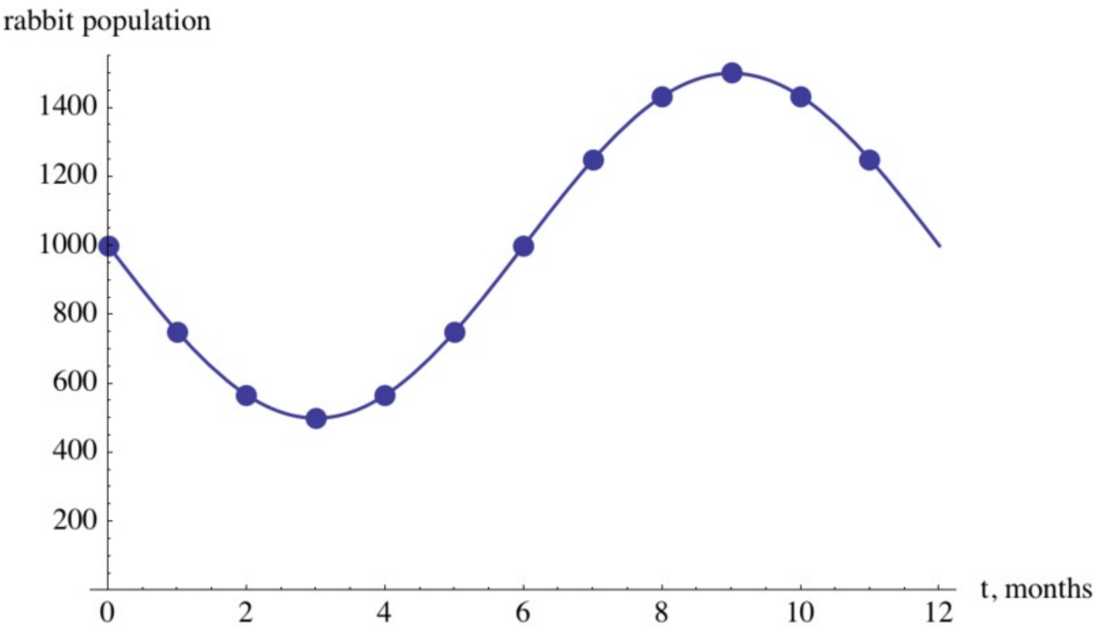
The number of foxes shows a similar pattern starting at a maximum of 150, reaching a minimum of 50 after 6 months and returning close to the maximum at the end of the year. Looking at the graph of the given points, we again see the general shape of a sine or cosine function with amplitude 50 foxes, midline 100 foxes and period 12 months.



b. We already found the amplitude, midline and period for $r(t)$ in part (a). Now we just have to decide if we want to use sine or cosine to model the function. Looking at the graph, we observe that the rabbit population has a vertical intercept at its midline and then decreases. This suggests the use of a negative sine function $r(t) = -A\sin(B(t - D)) + C$ where A is the amplitude, $B = \frac{2\pi}{\text{period}}$, C is the midline, and D is the horizontal shift. Because of the choice of using a a negative sine function, the horizontal shift is zero.

Since $A = 500$, $C = 1000$ and $B = \frac{2\pi}{\text{period}} = \frac{2\pi}{12}$, we have $r(t) = -500\sin(\frac{\pi}{6}t) + 1000$.

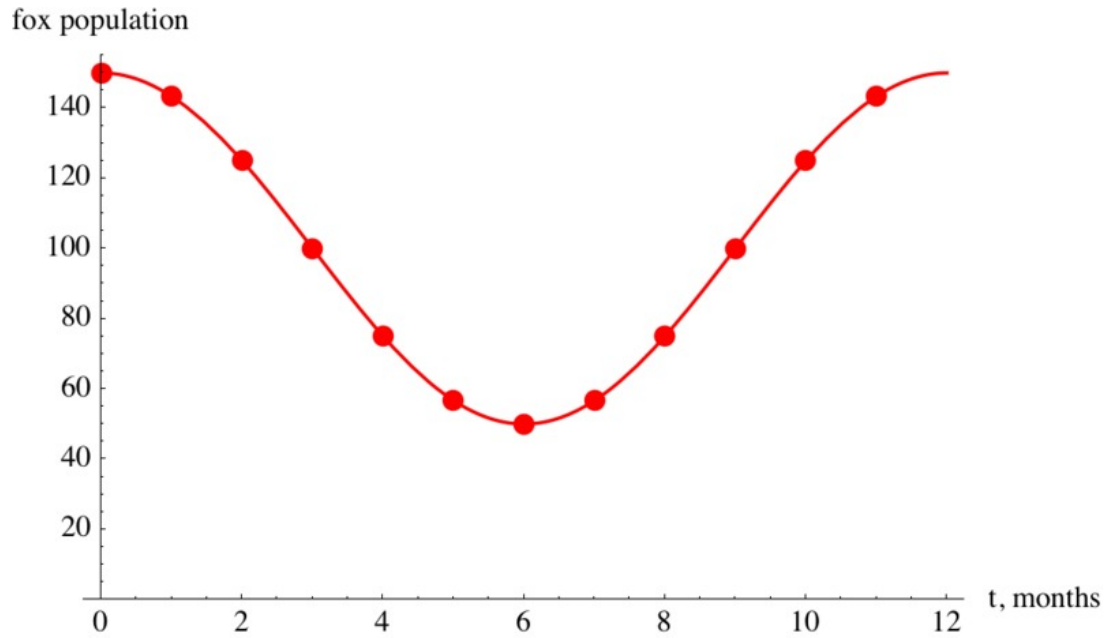
Graphing $r(t)$ together with our data points confirms that our function is a good model for the given data.



c. We already found the amplitude, midline and period for $f(t)$ in part (a). Again, we have to decide if we want to use sine or cosine to model this function. Looking at the graph, we observe that the fox population has a vertical intercept at its maximum and then decreases. This suggests the use of the cosine function $r(t) = A\cos(B(t - D)) + C$, where A is the amplitude, $B = \frac{2\pi}{\text{period}}$, C is the midline, and D is the horizontal shift. Because of the choice of using a cosine function, the horizontal shift is zero.

Since $A = 50$, $C = 100$ and $B = \frac{2\pi}{\text{period}} = \frac{2\pi}{12}$, we have $f(t) = 50\cos(\frac{\pi}{6}t) + 100$.

Graphing $f(t)$ together with our data points confirms that our function is a good model for the given data.



d. Graphing both functions on the same coordinate axes, we observe the mentioned "chasing" of the two functions. While the rabbit population decreases during the winter and recovers during the summer, the fox population seems to lag behind by three months. This makes sense in the context of a predator-prey relationship. The foxes depend on the rabbits as a food source. When there are many rabbits present, the fox population grows. However, when there is a larger fox population, more rabbits are being hunted, so the rabbit population decreases. When the rabbit population is small, the fox population has a limited food supply and decreases. Once the fox population is small enough, the rabbit population can recover and this cycle continues.