Illustrative Mathematics

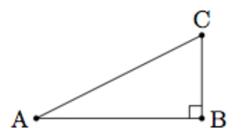
F-TF Trigonometric Ratios and the Pythagorean Theorem

Alignments to Content Standards

- Alignment: F-TF.C.8

Tags

- *This task is not yet tagged.*



a. In the triangle pictured above show that

$$\left(\frac{|AB|}{|AC|}\right)^2 + \left(\frac{|BC|}{|AC|}\right)^2 = 1$$

b. Deduce that $\sin^2\theta + \cos^2\theta = 1$ for any acute angle θ.

c. If θ is in the second quadrant and $\sin\theta = \frac{8}{17}$ what can you say about $\cos\theta$? Draw a picture and explain.

Commentary

The purpose of this task is to use the Pythagorean Theorem to establish the fundamental trigonometric identity $\sin^2\theta + \cos^2\theta = 1$ for an acute angle θ. The reasoning behind this identity is then applied to calculate $\cos\theta$ for a given obtuse angle. In order to successfully complete part (c) students must be familiar with the definitions of trigonometric functions for arbitrary angles using the unit circle (F-TF.2).

The Pythagorean Theorem requires $\triangle ABC$ to be a right triangle and so $\angle A$ must be acute. So the reasoning in parts a and b of this task establish the identity

$$\sin^2\theta + \cos^2\theta = 1$$

for acute angles only. Using the unit circle as in part c, however, shows that this identity is true for all angles. More about this can be found in http://www.illustrativemathematics.org/tasks/1692 .

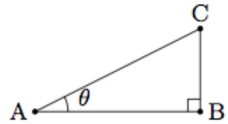
Solutions

Solution: 1

a. The Pythagorean Theorem says that if $\triangle ABC$ is a right triangle with right angle B then $|AB|^2 + |BC|^2 = |AC|^2$. Dividing both sides by $|AC|^2$ gives

$$\left(\frac{|AB|}{|AC|}\right)^2 + \left(\frac{|BC|}{|AC|}\right)^2 = 1.$$

b. If $0 < m(\theta) < 90$, then we can make a right triangle ABC, as pictured in the problem statement, so that $m(\angle BAC) = \theta$.

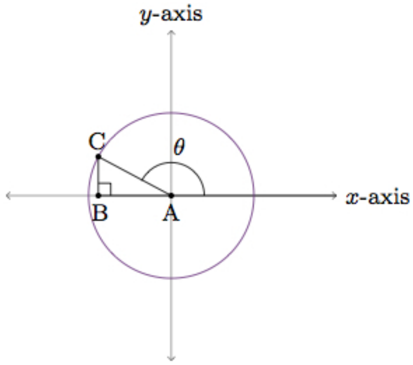


Then from part (a) we have

$$\left(\frac{|AB|}{|AC|}\right)^2 + \left(\frac{|BC|}{|AC|}\right)^2 = 1.$$

We also know that $\frac{|AB|}{|AC|} = \cos\theta$ and $\frac{|BC|}{|AC|} = \sin\theta$ so we have $\sin^2\theta + \cos^2\theta = 1$.

c. Below is a picture of an angle θ in the second quadrant with $\sin\theta = \frac{8}{17}$:



In the picture, the purple circle is the unit circle. The coordinates of C are $(\cos\theta, \sin\theta)$ and since C lies on the unit circle we have

$$\sin^2\theta + \cos^2\theta = 1.$$

Since $\sin\theta = \frac{8}{17}$ we can solve for $\cos\theta$ and we find $\cos\theta = \pm\frac{15}{17}$. Since we are in the second quadrant $\cos\theta = -\frac{15}{17}$.