



Lesson 14: More on the Angles of a Triangle

Student Outcomes

- Students know a third informal proof of the angle sum theorem.
- Students know how to find missing interior and exterior angle measures of triangles and present informal arguments to prove their answer is correct.

Lesson Notes

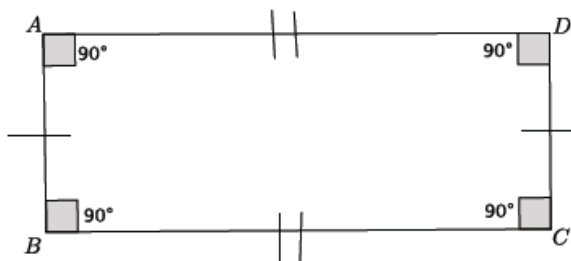
Students will see one final informal proof of the angle sum of a triangle before moving on to working with exterior angles of triangles.

Classwork

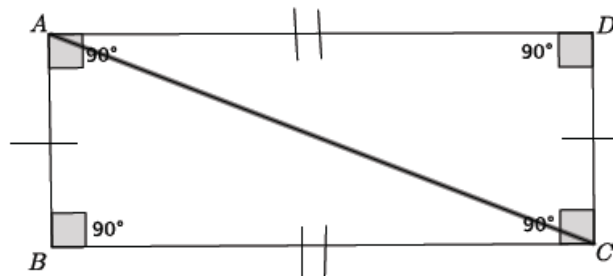
Discussion (7 minutes)

Let's look at one final proof that the sum of the degrees of the interior angles of a triangle is 180.

- Start with a rectangle. What properties do rectangles have?
 - *All four angles are right angles; opposite sides are equal in length.*



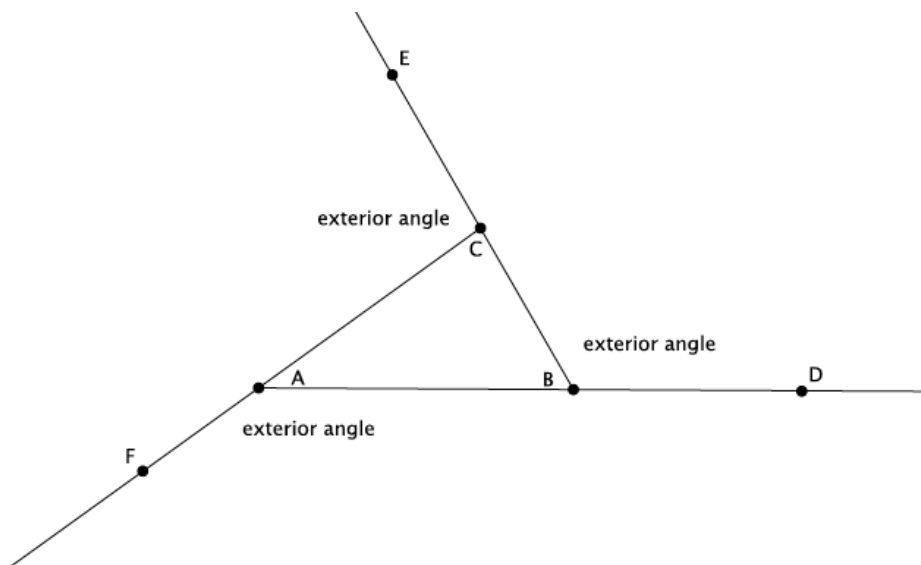
- If we draw a diagonal that connects A to C (or we could choose to connect B to D), what shapes are formed?
 - *We get two triangles.*



- What do we know about these triangles, and how do we know it?
 - *The triangles are congruent. We can trace one of the triangles and, through a sequence of basic rigid motions, map it onto the other triangle.*
- Our goal is to show that the angle sum of a triangle is 180° . We know that when we draw a diagonal through a rectangle we get two congruent triangles. How can we put this information together to show that the sum of angles in a triangle is 180° ?
 - *The rectangle has four right angles which means that the sum of the angles of the rectangle is $4(90^\circ) = 360^\circ$. Since the diagonal divides the rectangle into two congruent triangles, each triangle will have exactly half the total degrees of the rectangle. Since, $360 \div 2 = 180$, then each triangle has a sum of angles equal to 180.*

Discussion (7 minutes)

Now let's look at what is called the *exterior angle of a triangle*. An exterior angle is formed when one of the sides of the triangle is extended. The interior angles are inside the triangle, so the exterior angle is outside of the triangle along the extended side. In triangle ABC , the exterior angles are $\angle CBD$, $\angle ECA$, and $\angle BAF$.



- What do we know about the sum of interior angles of a triangle? Name the angles.
 - *The sum of the interior angles of a triangle is 180° . $\angle ABC$, $\angle BCA$, and $\angle CAB$*
- What do we know about the degree of a straight angle?
 - *A straight angle has a measure of 180° .*
- Let's look specifically at straight angle $\angle ABD$. Name the angles that make up this straight angle.
 - *$\angle ABC$ and $\angle CBD$.*
- Because the triangle and the straight angle both have measures of 180° , we can write them as equal to one another. That is, since

$$\angle ABC + \angle BCA + \angle CAB = 180$$

and

$$\angle ABC + \angle CBD = 180$$

then,

$$\angle ABC + \angle BCA + \angle CAB = \angle ABC + \angle CBD$$

- Which angle is common to both the triangle and the straight angle?
 - $\angle ABC$
- If we subtract the measure of $\angle ABC$ from both the triangle and the straight angle, we get:

$$\begin{aligned} \angle ABC - \angle ABC + \angle BCA + \angle CAB &= \angle ABC - \angle ABC + \angle CBD \\ \angle BCA + \angle CAB &= \angle CBD \end{aligned}$$

- What kind of angle is $\angle CBD$?
 - *It is the exterior angle of the triangle.*
- We call angles $\angle BCA$ and $\angle CAB$ the remote interior angles because they are the farthest “remotest” from the exterior angles. The equation $\angle BCA + \angle CAB = \angle CBD$ means that the sum of the remote interior angles are equal to the exterior angle of the triangle.

Exercises 1–4 (8 minutes)

Students work in pairs to identify the remote interior angles and corresponding exterior angle of the triangle in Exercises 1–3. After most of the students have finished Exercises 1–3, provide the correct answers before they move on to the next exercise. In Exercise 4, students recreate the reasoning of Example 1 for another exterior angle of the triangle.

Scaffolding:
Keep the work of Example 1 visible while students work on Exercises 1–4.

Exercises 1–4

Use the diagram below to complete Exercises 1–4.

1. Name an exterior angle and the related remote interior angles.
The exterior angle is $\angle ZYP$, and the related remote interior angles are $\angle YZX$ and $\angle ZXY$.
2. Name a second exterior angle and the related remote interior angles.
The exterior angle is $\angle XZQ$, and the related remote interior angles are $\angle ZYX$ and $\angle ZXY$.
3. Name a third exterior angle and the related remote interior angles.
The exterior angle is $\angle RXY$, and the related remote interior angles are $\angle ZYX$ and $\angle XZY$.

4. Show that the measure of an exterior angle is equal to the sum of the related remote interior angles.

Triangle XYZ has interior angles $\angle XYZ$, $\angle YZX$ and $\angle ZXY$. The sum of those angles is 180° . The straight angle $\angle XYP$ also has a measure of 180° and is made up of angles $\angle XYZ$ and $\angle ZYP$. Since the triangle and the straight angle have the same number of degrees, we can write the sum of their respective angles as an equality:

$$\angle XYZ + \angle YZX + \angle ZXY = \angle XYZ + \angle ZYP$$

Both the triangle and the straight angle share $\angle XYZ$. We can subtract the measure of that angle from the triangle and the straight angle. Then we have:

$$\angle YZX + \angle ZXY = \angle ZYP$$

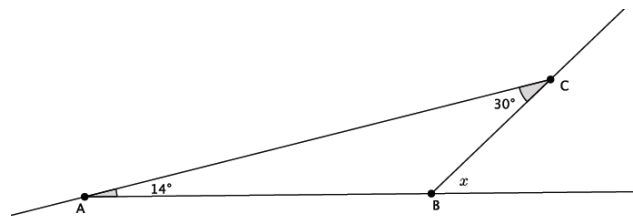
where the angle $\angle ZYP$ is the exterior angle and the angles $\angle YZX$ and $\angle ZXY$ are the related remote interior angles of the triangle. Therefore, the sum of the remote interior angles of a triangle are equal to the exterior angle.

Example 1 (2 minutes)

- Ask students what we need to do to find the measure of angle x . Then have them work on white boards and show you their answer.

Example 1

Find the measure of angle x .



We need to find the sum of the remote interior angles to find the measure of the exterior angle x :

$$14 + 30 = 44, \text{ angle } x = 44^\circ.$$

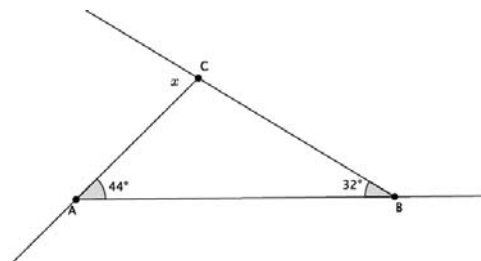
- Ask students to present an informal argument that proves they are correct.
 - We know that triangles have a sum of interior angles that is equal to 180° . We also know that straight angles are 180° . Angle $\angle ABC$ must be 136° which means that $\angle x = 44^\circ$.*

Example 2 (2 minutes)

- Ask students what we need to do to find the measure of angle x . Then have them work on white boards and show you their answer.

Example 2

Find the measure of angle x .



We need to find the sum of the remote interior angles to find the measure of the exterior angle x :

$$44 + 32 = 76, \text{ angle } x = 76^\circ.$$

- Ask students to present an informal argument that proves they are correct.
 - We know that triangles have a sum of interior angles that is equal to 180° . We also know that straight angles are 180° . Angle $\angle ACB$ must be 104° which means that $\angle x = 76^\circ$.

Example 3 (2 minutes)

Ask students what we need to do to find the measure of angle x . Then have them work on white boards and show you their answer. Make sure students see that this is not like the last two examples. They must pay attention to the information that is provided and not expect to always do the same procedure.

Example 3
Find the measure of angle x .

$180 - 121 = 59$, angle $x = 59^\circ$.

- Students should notice that we are not given the two remote interior angles associated with the exterior angle x . For that reason, we must use what we know about straight angles (or supplementary angles) to find the measure of angle x .

Example 4 (2 minutes)

Ask students what we need to do to find the measure of angle x . Then have them work on white boards and show you their answer. Make sure students see that this is not like the last three examples. They must pay attention to the information that is provided and not expect to always do the same procedure.

Example 4
Find the measure of angle x .

$129 - 45 = 84$, angle $x = 84^\circ$.

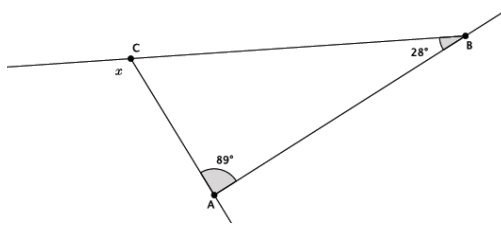
- Students should notice that we are given just one of the remote interior angle measures and the exterior angle measure. For that reason, we will need to subtract 45 from the exterior angle to find the measure of angle x .

Exercises 5–10 (6 minutes)

Students complete Exercises 5–10 independently. Check solutions once most students have finished.

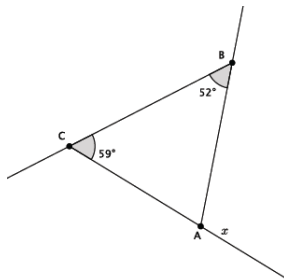
Exercise 5–10

5. Find the measure of angle x . Present an informal argument showing that your answer is correct.



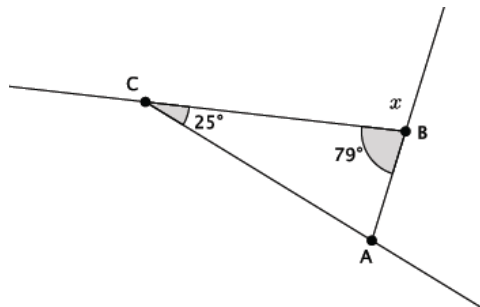
$89 + 28 = 117$, the measure of angle x is 117° . We know that triangles have a sum of interior angles that is equal to 180° . We also know that straight angles are 180° . Angle $\angle ACB$ must be 63° which means that $\angle x = 117^\circ$.

6. Find the measure of angle x . Present an informal argument showing that your answer is correct.



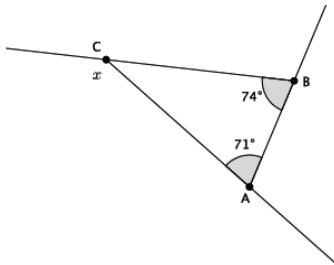
$59 + 52 = 111$, the measure of angle x is 111° . We know that triangles have a sum of interior angles that is equal to 180° . We also know that straight angles are 180° . Angle $\angle CAB$ must be 69° which means that $\angle x = 111^\circ$.

7. Find the measure of angle x . Present an informal argument showing that your answer is correct.



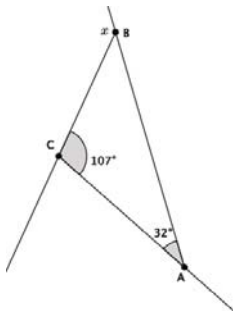
$180 - 79 = 101$, the measure of angle x is 101° . We know that straight angles are 180° , and the straight angle in the diagram is made up of angle $\angle ABC$ and angle $\angle x$. Angle $\angle ABC$ is 79° which means that $\angle x = 101^\circ$.

8. Find the measure of angle x . Present an informal argument showing that your answer is correct.



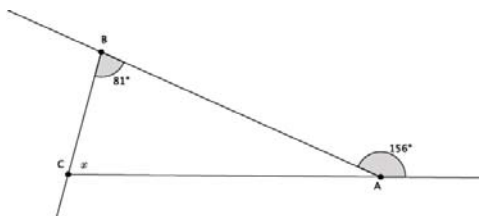
$71 + 74 = 145$, the measure of angle x is 145° . We know that triangles have a sum of interior angles that is equal to 180° . We also know that straight angles are 180° . Angle $\angle ACB$ must be 35° which means that $\angle x = 145^\circ$.

9. Find the measure of angle x . Present an informal argument showing that your answer is correct.



$107 + 32 = 139$, the measure of angle x is 139° . We know that triangles have a sum of interior angles that is equal to 180° . We also know that straight angles are 180° . Angle $\angle CBA$ must be 41° , which means that $\angle 139^\circ$.

10. Find the measure of angle x . Present an informal argument showing that your answer is correct.



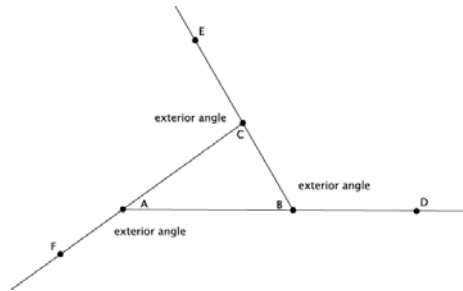
$156 - 81 = 75$, the measure of angle x is 75° . We know that triangles have a sum of interior angles that is equal to 180° . We also know that straight angles are 180° . Angle $\angle BAC$ must be 24° because it is part of the straight angle. Then $\angle x = 180^\circ - (81^\circ + 24^\circ) = 75^\circ$.

Closing (4 minutes)

Summarize, or have students summarize, the lesson.

- We learned another proof as to why the interior angles of a triangle are equal to 180 with respect to a triangle being exactly half of a rectangle.
- We learned the definitions of exterior angles and remote interior angles.
- We know that the sum of the remote interior angles of a triangle is equal to the measure of the exterior angle.

Lesson Summary



The sum of the remote interior angles of a triangle is equal to the measure of the exterior angle. For example, $\angle CAB + \angle ABC = \angle ACE$.

Exit Ticket (5 minutes)

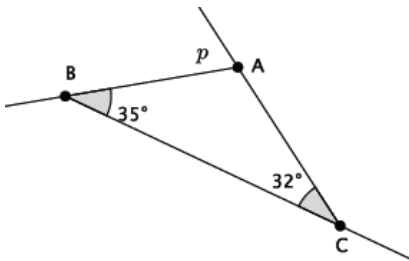
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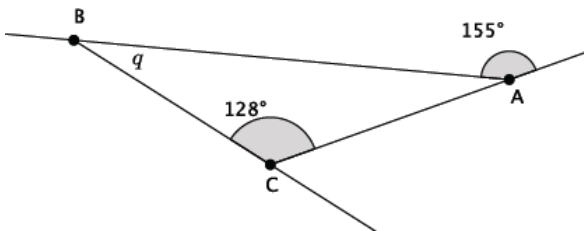
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Exit Ticket

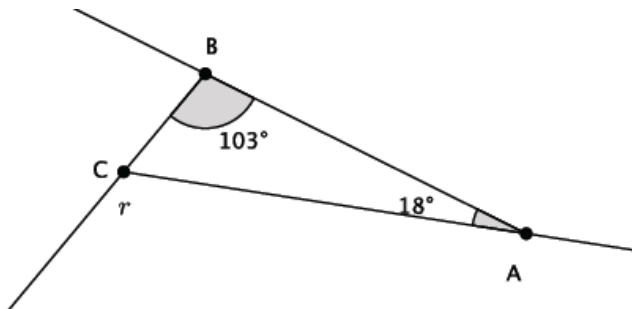
- Find the measure of angle p . Present an informal argument showing that your answer is correct.



- Find the measure of angle q . Present an informal argument showing that your answer is correct.

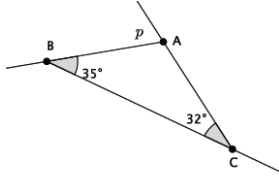


- Find the measure of angle r . Present an informal argument showing that your answer is correct.



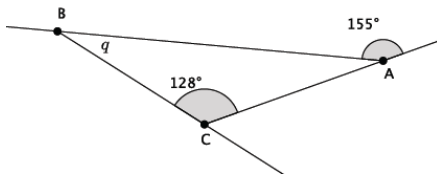
Exit Ticket Sample Solutions

1. Find the measure of angle p . Present an informal argument showing that your answer is correct.



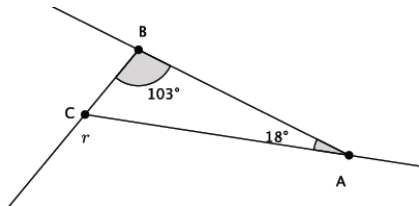
The measure of angle p is 67° . We know that triangles have a sum of interior angles that is equal to 180° . We also know that straight angles are 180° . Angle $\angle BAC$ must be 113° which means that $\angle p = 67^\circ$.

2. Find the measure of angle q . Present an informal argument showing that your answer is correct.



The measure of angle q is 27° . We know that triangles have a sum of interior angles that is equal to 180° . We also know that straight angles are 180° . Angle $\angle CAB$ must be 25° which means that $\angle q = 27^\circ$.

3. Find the measure of angle r . Present an informal argument showing that your answer is correct.



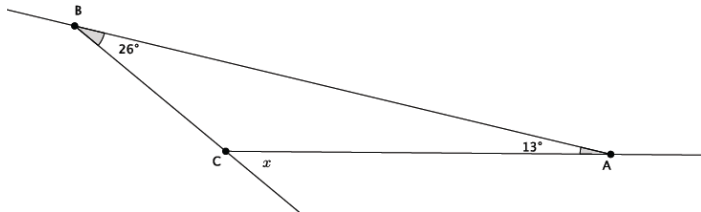
The measure of angle r is 121° . We know that triangles have a sum of interior angles that is equal to 180° . We also know that straight angles are 180° . Angle $\angle BCA$ must be 59° which means that $\angle r = 121^\circ$.

Problem Set Sample Solutions

Students practice finding missing angle measures of triangles.

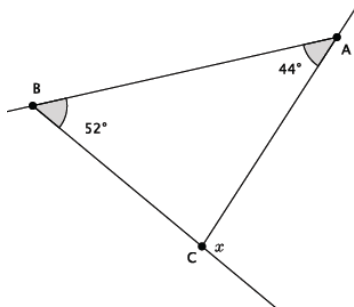
For each of the problems below, use the diagram to find the missing angle measure. Show your work.

- Find the measure of angle x . Present an informal argument showing that your answer is correct.



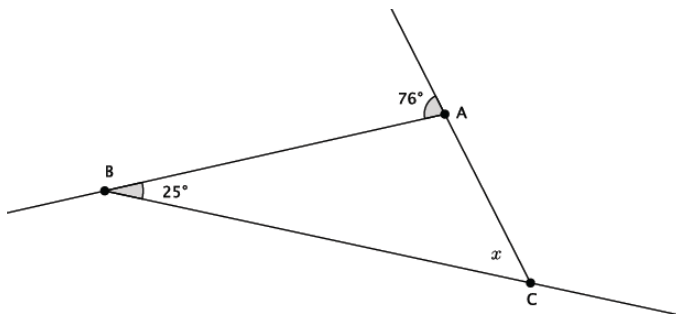
$26 + 13 = 39$, the measure of angle x is 39° . We know that triangles have a sum of interior angles that is equal to 180° . We also know that straight angles are 180° . Angle $\angle BCA$ must be 141° which means that $\angle x = 39^\circ$.

- Find the measure of angle x .



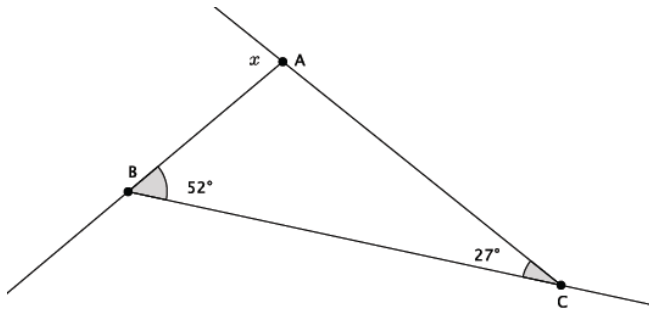
$52 + 44 = 96$, the measure of angle x is 96° .

- Find the measure of angle x . Present an informal argument showing that your answer is correct.



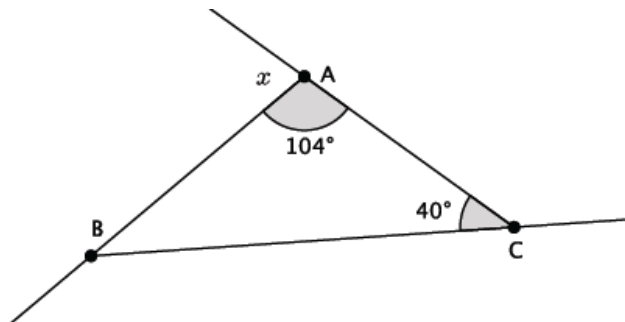
$76 - 25 = 51$, the measure of angle x is 51° . We know that triangles have a sum of interior angles that is equal to 180° . We also know that straight angles are 180° . Angle $\angle BAC$ must be 104° because it is part of the straight angle. Then $x = 180^\circ - (104^\circ + 25^\circ) = 51^\circ$

4. Find the measure of angle x .



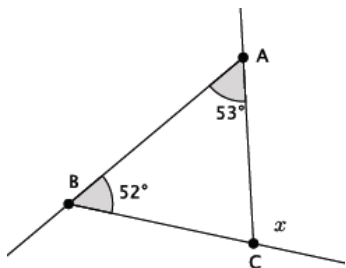
$27 + 52 = 79$, the measure of angle x is 79° .

5. Find the measure of angle x .



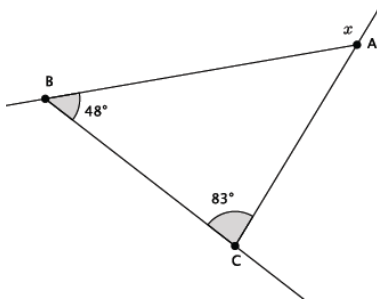
$180 - 104 = 76$, the measure of angle x is 76° .

6. Find the measure of angle x .



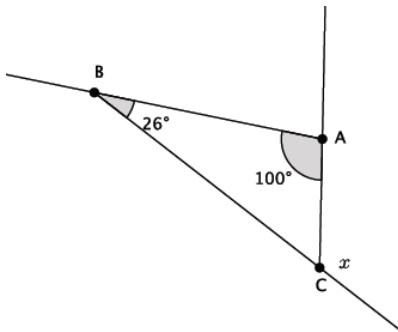
$52 + 53 = 105$, the measure of angle x is 105° .

7. Find the measure of angle x .



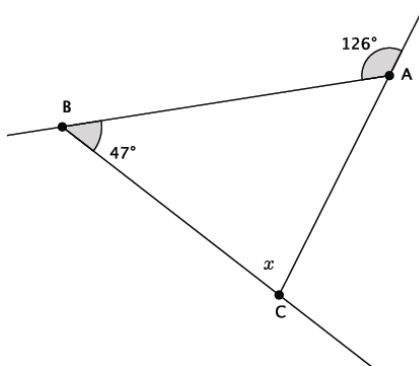
$48 + 83 = 131$, the measure of angle x is 131° .

8. Find the measure of angle x .



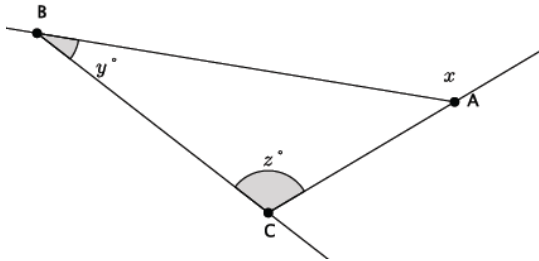
$100 + 26 = 126$, the measure of angle x is 126° .

9. Find the measure of angle x .



$126 - 47 = 79$, the measure of angle x is 79° .

10. Write an equation that would allow you to find the measure of angle x . Present an informal argument showing that your answer is correct.



$y + z = x$, the measure of angle x is $(y + z)^\circ$. We know that triangles have a sum of interior angles that is equal to 180° . We also know that straight angles are 180° . Then $\angle y + \angle z + \angle BAC = 180^\circ$ and $\angle x + \angle BAC = 180^\circ$. Since both equations are equal to 180° , then $\angle y + \angle z + \angle BAC = \angle x + \angle BAC$. Subtract $\angle BAC$ from each side of the equation and you get $\angle y + \angle z = \angle x$.