Lesson 11: More About Similar Triangles

Student Outcomes

- Students present informal arguments as to whether or not two triangles are similar.
- Students practice finding lengths of corresponding sides of similar triangles.

Lesson Notes

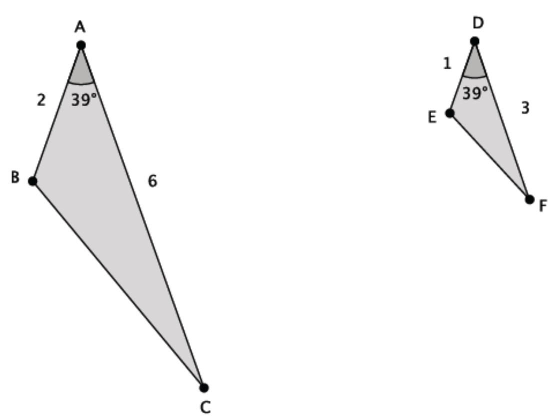
This lesson synthesizes the knowledge gained thus far in Module 3. Students use what they know about dilation, congruence, the Fundamental Theorem of Similarity (FTS), and the AA criterion to determine if two triangles are similar. In the first two examples, students use informal arguments to decide if a pair of triangles are similar. To do so, they look for pairs of corresponding angles that are equal (wanting to use the AA criterion). When they realize that information is not given, they compare lengths of corresponding sides to see if the sides could be dilations with the same scale factor. After a dilation and congruence is performed, students see that a pair of triangles are similar (or not) and then continue to give more proof as to why they must be, e.g., by FTS, a specific pair of lines are parallel, and the corresponding angles cut by a transversal must be equal; therefore, we can use AA criterion to state that two triangles are similar. Once students know how to determine if two triangles are similar or not, they apply this knowledge to finding lengths of segments of triangles that are unknown in Examples 3–5.

Classwork

Example 1 (6 minutes)

MP.1

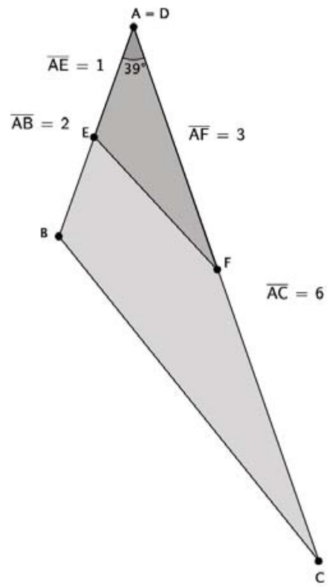
- Given the information provided, is $\triangle ABC \sim \triangle DEF$? (Give students a minute or two to discuss with a partner.)



  - *Students will likely say that they cannot tell if the triangles are similar because there is only information for one angle provided. In the previous lesson, students could determine if two triangles were similar using Angle-Angle criterion.*

MP.1

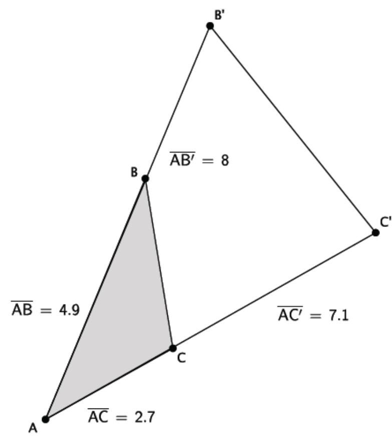
- What if we combined our knowledge of dilation and similarity? That is, we know we can translate $\triangle ABC$ so that $\angle A = \angle D$. Then our picture would look like this:



- Can we tell if the triangles are similar now?
  - *We still do not have information about the angles, but we can use what we know about dilation and the Fundamental Theorem of Similarity to find out if EF is parallel to BC. If they are, then $\triangle ABC \sim \triangle DEF$ because the corresponding angles of parallel lines are equal.*
- We do not have the information we need about corresponding angles. So let's examine the information we are provided. Compare the given side lengths to see if the ratios of corresponding sides are equal: Is $\frac{|AE|}{|AB|} = \frac{|AF|}{|AC|}$? That's the same as asking if $\frac{1}{2} = \frac{3}{6}$? Since the ratios of corresponding sides are equal, then there exists a dilation from center A with scale factor $r = \frac{1}{2}$ that maps $\triangle ABC \sim \triangle DEF$. Since the ratios of corresponding sides are equal, then by the Fundamental Theorem of Similarity, we know EF is parallel to BC and the corresponding angles of the parallel lines are also equal in measure.
- This example illustrates another way for us to determine if two triangles are similar. That is, if they have one pair of equal corresponding angles, and the ratio of corresponding sides (along each side of the given angle) are equal, then the triangles are similar.

Example 2 (4 minutes)

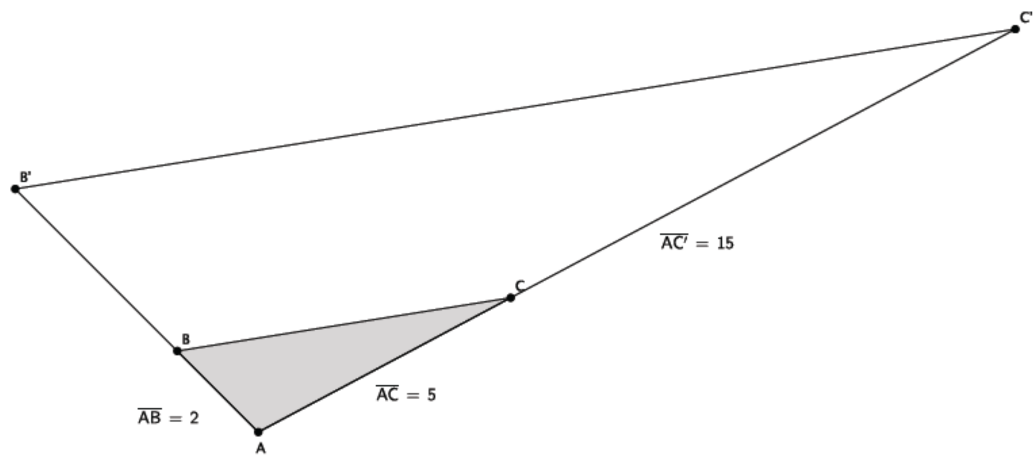
- Given the information provided, is $\triangle ABC \sim \triangle AB'C'$? Explain. (Give students a minute or two to discuss with a partner.)



- If students say that the triangles are not similar because lines BC and $B'C'$ are not parallel, ask them how they know this. If they say "They don't look parallel", tell students that the way they look is not good enough. They must prove mathematically that the lines are not parallel. For that reason, the following response is more legitimate.
  - *We do not have information about two pairs of corresponding angles, so we will need to examine the ratios of corresponding side lengths. If the ratios are equal, then the triangles are similar.*
- If the ratios of the corresponding sides are equal, it means that the lengths were dilated by the same scale factor. Write the ratios of the corresponding sides.
  - *The ratios of corresponding sides are* $\frac{|AC'|}{|AC|} = \frac{|AB'|}{|AB|}$.
- Does $\frac{|AC'|}{|AC|} = \frac{|AB'|}{|AB|}$? That is the same as asking if $\frac{7.1}{2.7}$ and $\frac{8}{4.9}$ are equivalent fractions. One possible way of verifying if the fractions are equal is by multiplying the numerator of each fraction by the denominator of the other. If the products are equal, then we know the fractions are equivalent.
  - *The products are* 34.79 *and* 21.6. *Since* $34.79 \neq 21.6$, *the fractions are not equivalent, and the triangles are not similar.*

Example 3 (4 minutes)

- Given that $\triangle ABC \sim \triangle AB'C'$, could we determine the length of AB'? What does it mean to say that $\triangle ABC \sim \triangle AB'C'$? (Give students a minute or two to discuss with a partner.)



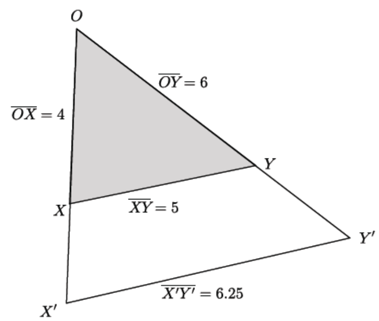
  - *It means that corresponding angles are equal, the ratio of corresponding sides are equal, i.e.,* $\frac{|AC'|}{|AC|} = \frac{|AB'|}{|AB|} = \frac{|B'C'|}{|BC|}$, *and lines* BC *and* $B'C'$ *are parallel.*

- How can we use what we know about similar triangles to determine the length of AB'?
  - *The corresponding sides are supposed to be equal in ratio:* $\frac{|AC'|}{|AC|} = \frac{|AB'|}{|AB|}$ *is the same as* $\frac{15}{5} = \frac{|AB'|}{2}$.

- Since we know that for equivalent fractions, when we multiply the numerator of each fraction by the denominator of the other fraction, the products are equal, we can use that fact to find the length of AB'. Let x represent the length of AB'; then $\frac{15}{5} = \frac{|AB'|}{2}$ is the same as $\frac{15}{5} = \frac{x}{2}$. Equivalently, we get $30 = 5x$. The value of x that makes the statement true is $x = 6$. Therefore, the length of AB' is 6.

Example 4 (4 minutes)

- If we suppose XY is parallel to $X'Y'$, can we use the information provided to determine if $\triangle OXY \sim \triangle OX'Y'$? Explain. (Give students a minute or two to discuss with a partner.)



- *Since we assume $XY || X'Y'$, then we know we have similar triangles because each triangle shares $\angle O$ and the corresponding angles are congruent: $\angle OXY \cong \angle OX'Y'$, and $\angle OYX \cong \angle OY'X'$. By the AA criterion, we can conclude that $\triangle OXY \sim \triangle OX'Y'$.*

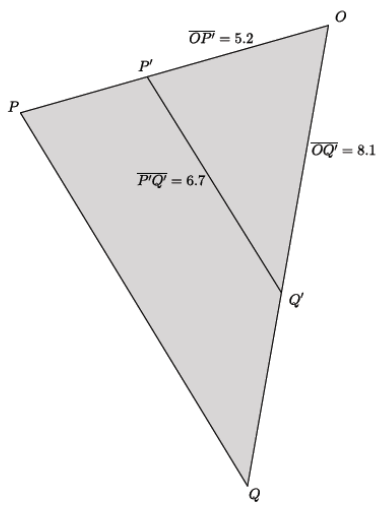
- Now that we know the triangles are similar, can we determine the length of OX'? Explain.
  - *By the converse of the Fundamental Theorem of Similarity, since we are given parallel lines and the lengths of the corresponding sides XY and $X'Y'$, we can write the ratio that represents the scale factor and compute using the fact that cross products must be equal to determine the length of OX'.*

- Write the ratio for the known side lengths XY and $X'Y'$ and the ratio that would contain the side length we are looking for. Then use the cross products to find the length of OX'.
  - $\frac{|X'Y'|}{|XY|} = \frac{|OX'|}{|OX|}$ *is the same as* $\frac{6.25}{5} = \frac{|OX'|}{4}$. *Let z represent the length of OX', then we have* $\frac{6.25}{5} = \frac{z}{4}$ *or equivalently,* $5z = 25$ *and* $z = 5$. *Therefore, the length of OX' is* 5.

- Now find the length of OY'.
  - $\frac{|X'Y'|}{|XY|} = \frac{|OY'|}{|OY|}$ *is the same as* $\frac{6.25}{5} = \frac{|OY'|}{6}$. *Let z represent the length of OY', then we have* $\frac{6.25}{5} = \frac{z}{6}$ *or equivalently,* $5z = 37.5$ *and* $z = 7.5$. *Therefore, the length of OY' is* 7.5.

Example 5 (3 minutes)

- Given the information provided, can you determine if $\triangle OPQ \sim \triangle OP'Q'$? Explain. (Give students a minute or two to discuss with a partner.)



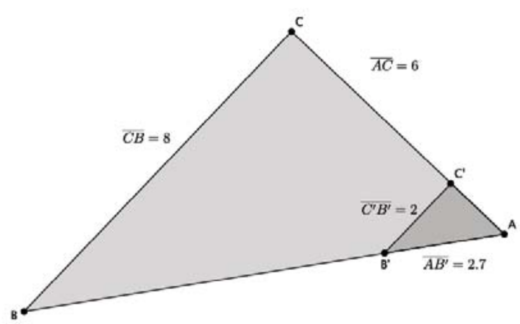
- *No, in order to determine if $\triangle OPQ \sim \triangle OP'Q'$ we need information about two pairs of corresponding angles. As is, we only know that the two triangles have one equal angle, the common angle at O. We would have corresponding angles that were equal if we knew that $PQ \parallel P'Q'$. Our other option is to compare the ratio of the sides that comprise the common angle. However, we do not have information about the lengths OP or OQ. For that reason, we cannot determine whether or not $\triangle OPQ \sim \triangle OP'Q'$.*

Exercises 1–3 (14 minutes)

Students can work independently or in pairs to complete Exercises 1–3.

Exercises

1. In the diagram below, you have $\triangle ABC$ and $\triangle AB'C'$. Use this information to answer parts (a)–(d).



a. Based on the information given, is $\triangle ABC \sim \triangle AB'C'$? Explain.

There is not enough information provided to determine if the triangles are similar. We would need information about a pair of corresponding angles or more information about the side lengths of each of the triangles.

b. Assume line BC is parallel to line $B'C'$. With this information, can you say that $\triangle ABC \sim \triangle AB'C'$? Explain.

If line BC is parallel to line $B'C'$, then $\triangle ABC \sim \triangle AB'C'$. Both triangles share $\angle A$. Another pair of equal angles is $\angle AB'C'$ and $\angle ABC$. They are equal because they are corresponding angles of parallel lines. By the AA criterion, $\triangle ABC \sim \triangle AB'C'$.

c. Given that $\triangle ABC \sim \triangle AB'C'$, determine the length of AC'.

Let x represent the length of AC'.

$$\frac{x}{6} = \frac{2}{8}$$

We are looking for the value of x that makes the fractions equivalent. Therefore, $8x = 12$, and $x = 1.5$. The length of AC' is 1.5.

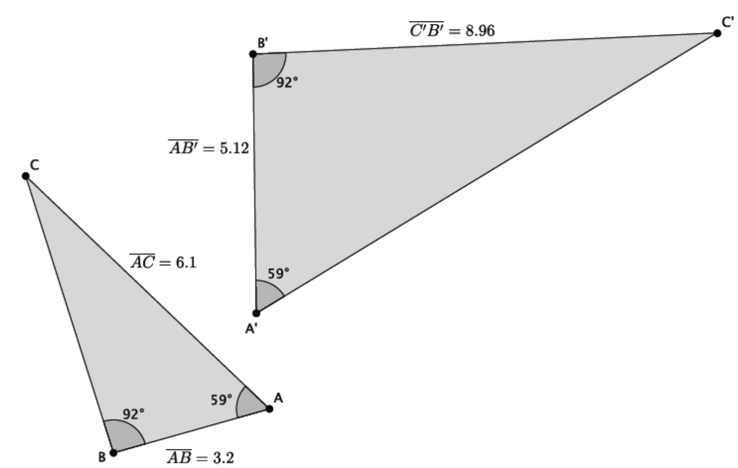
d. Given that $\triangle ABC \sim \triangle AB'C'$, determine the length of AB.

Let x represent the length of AB.

$$\frac{2.7}{x} = \frac{2}{8}$$

We are looking for the value of x that makes the fractions equivalent. Therefore, $2x = 21.6$ and $x = 10.8$. The length of AB is 10.8.

2. In the diagram below, you have $\triangle ABC$ and $\triangle A'B'C'$. Use this information to answer parts (a)–(c).



a. Based on the information given, is $\triangle ABC \sim \triangle A'B'C'$? Explain.

Yes, $\triangle ABC \sim \triangle A'B'C'$. There are two pairs of corresponding angles that are equal in measure. Namely, $\angle A = \angle A' = 59°$, and $\angle B = \angle B' = 92°$. By the AA criterion, these triangles are similar.

b. Given that $\triangle ABC \sim \triangle A'B'C'$, determine the length of $A'C'$.

Let x represent the length of $A'C'$.

$$\frac{x}{6.1} = \frac{5.12}{3.2}$$

We are looking for the value of x that makes the fractions equivalent. Therefore, $3.2x = 31.232$, and $x = 9.76$. The length of $A'C'$ is 9.76.

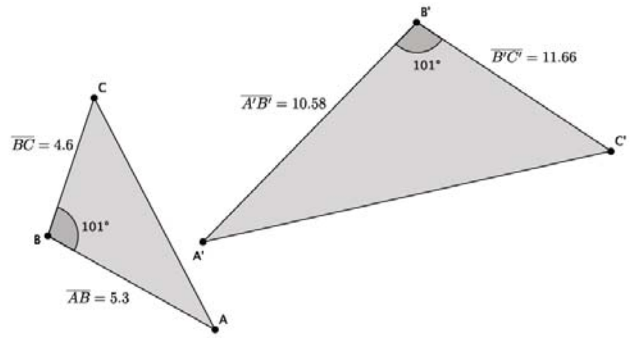
c. **Given that $\triangle ABC \sim \triangle A'B'C'$, determine the length of BC.**

Let x represent the length of BC.

$$\frac{8.96}{x} = \frac{5.12}{3.2}$$

We are looking for the value of x that makes the fractions equivalent. Therefore, $5.12x = 28.672$, and $x = 5.6$. The length of BC is 5.6.

3. In the diagram below you have $\triangle ABC$ and $\triangle A'B'C'$. Use this information to answer the question below.



Based on the information given, is $\triangle ABC \sim \triangle A'B'C'$? Explain.

No, $\triangle ABC$ is not similar to $\triangle A'B'C'$. Since there is only information about one pair of corresponding angles, then we must check to see that the corresponding sides have equal ratios. That is, the following must be true:

$$\frac{10.58}{5.3} = \frac{11.66}{4.6}$$

When we compare products of each numerator with the denominator of the other fraction, we see that $48.668 \neq 61.798$. Since the corresponding sides do not have equal ratios, then the fractions are not equivalent, and the triangles are not similar.

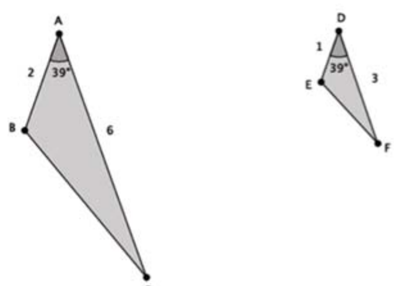
Closing (5 minutes)

Summarize, or ask students to summarize, the main points from the lesson:

- We know that if we are given just one pair of corresponding angles as equal, we can use the side lengths along the given angle to determine if triangles are in fact similar.
- If we know that we are given similar triangles, then we can use the fact that ratios of corresponding sides are equal to find any missing measurements.

Lesson Summary

Given just one pair of corresponding angles of a triangle as equal, use the side lengths along the given angle to determine if triangles are in fact similar.



$|\angle A| = |\angle D|$ and $\frac{1}{2} = \frac{3}{6} = r$; therefore, $\triangle ABC \sim \triangle DEF$.

Given similar triangles, use the fact that ratios of corresponding sides are equal to find any missing measurements.

Exit Ticket (5 minutes)

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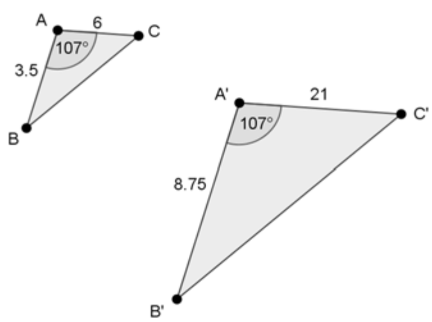
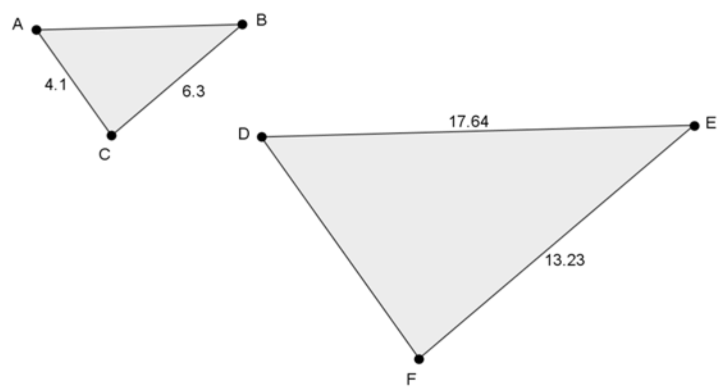
Lesson 11: More About Similar Triangles

Exit Ticket

1. In the diagram below, you have $\triangle ABC$ and $\triangle A'B'C'$. Based on the information given, is $\triangle ABC \sim \triangle A'B'C'$? Explain.



2. In the diagram below, $\triangle ABC \sim \triangle DEF$. Use the information to answer parts (a)–(b).

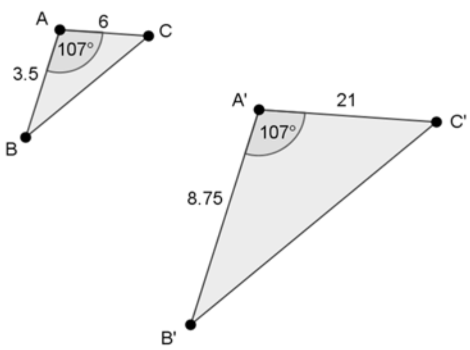


a. Determine the length of AB. Show work that leads to your answer.

b. Determine the length of DF. Show work that leads to your answer.

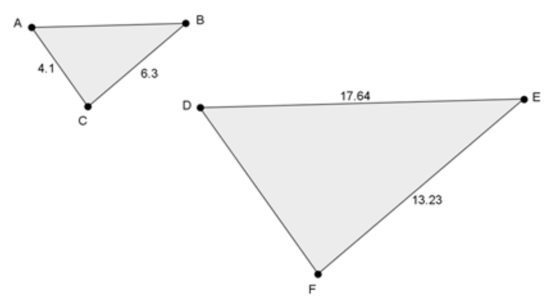
Exit Ticket Sample Solutions

1. In the diagram below, you have $\triangle ABC$ and $\triangle A'B'C'$. Based on the information given, is $\triangle ABC \sim \triangle A'B'C'$? Explain.



Since there is only information about one pair of corresponding angles, we need to check to see if corresponding sides have equal ratios. That is, does $\frac{|AB|}{|A'B'|} = \frac{|AC|}{|A'C'|}$, or does $\frac{3.5}{8.75} = \frac{6}{21}$? The products are not equal: $73.5 \neq 52.5$. Since the corresponding sides do not have equal ratios, the triangles are not similar.

2. In the diagram below, $\triangle ABC \sim \triangle DEF$. Use the information to answer parts (a)–(b).



a. Determine the length of AB. Show work that leads to your answer.

Let x represent the length of AB.

Then $\frac{x}{17.64} = \frac{6.3}{13.23}$. We are looking for the value of x that makes the fractions equivalent. Therefore,

$111.132 = 13.23x$, and $x = 8.4$. The length of AB is 8.4.

b. Determine the length of DF. Show work that leads to your answer.

Let y represent the length of DF.

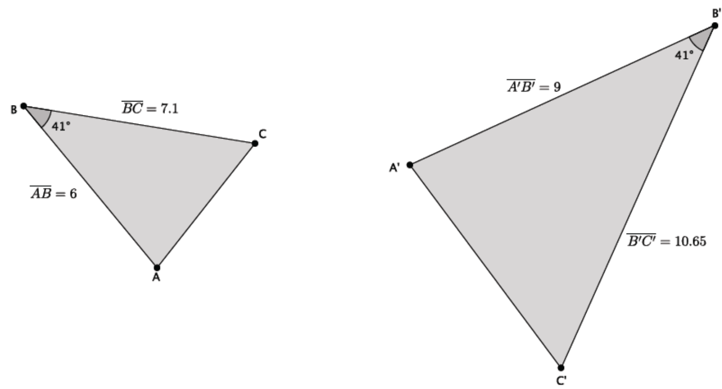
Then $\frac{4.1}{y} = \frac{6.3}{13.23}$. We are looking for the value of y that makes the fractions equivalent. Therefore,

$54.243 = 6.3y$, and $8.61 = y$. The length of DF is 8.61.

Problem Set Sample Solutions

Students practice presenting informal arguments as to whether or not two given triangles are similar. Students practice finding measurements of similar triangles.

1. In the diagram below, you have $\triangle ABC$ and $\triangle A'B'C'$. Use this information to answer parts (a)–(b).



a. Based on the information given, is $\triangle ABC \sim \triangle A'B'C'$? Explain.

Yes, $\triangle ABC \sim \triangle A'B'C'$. Since there is only information about one pair of corresponding angles being equal, then the corresponding sides must be checked to see if their ratios are equal:

$$\frac{10.65}{7.1} = \frac{9}{6}$$

$63.9 = 63.9$. *Since the cross products are equal, the triangles are similar.*

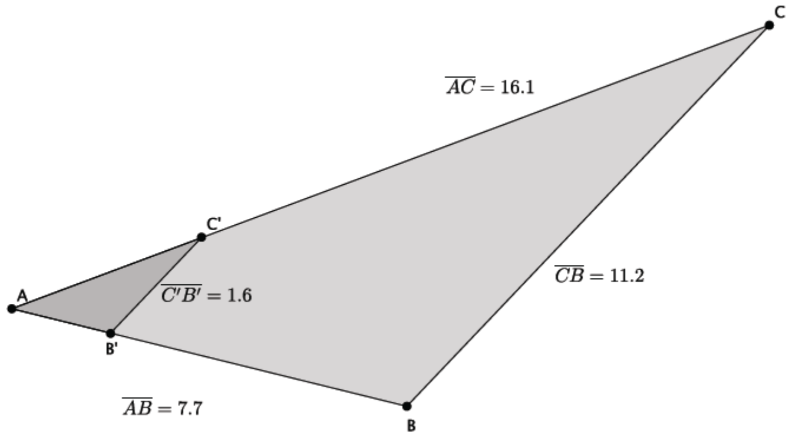
b. Assume the length of AC is 4.3. What is the length of $A'C'$?

Let x represent the length of $A'C'$.

$$\frac{x}{4.3} = \frac{9}{6}$$

We are looking for the value of x that makes the fractions equivalent. Therefore, $6x = 38.7$, and $x = 6.45$. The length of $A'C'$ is 6.45.

2. In the diagram below you have $\triangle ABC$ and $\triangle AB'C'$. Use this information to answer parts (a)–(d).



a. Based on the information given, is $\triangle ABC \sim \triangle AB'C'$? Explain.

There is not enough information provided to determine if the triangles are similar. We would need information about a pair of corresponding angles or more information about the side lengths of each of the triangles.

b. Assume line BC is parallel to line $B'C'$. With this information, can you say that $\triangle ABC \sim \triangle AB'C'$? Explain.

If line BC is parallel to line $B'C'$, then $\triangle ABC \sim \triangle AB'C'$. Both triangles share $\angle A$. Another pair of equal angles is $\angle AB'C'$ and $\angle ABC$. They are equal because they are corresponding angles of parallel lines. By the AA criterion, $\triangle ABC \sim \triangle AB'C'$.

c. Given that $\triangle ABC \sim \triangle AB'C'$, determine the length of AC'.

Let x represent the length of AC'.

$$\frac{x}{16.1} = \frac{1.6}{11.2}$$

We are looking for the value of x that makes the fractions equivalent. Therefore, $11.2x = 25.76$, and $x = 2.3$. The length of AC' is 2.3.

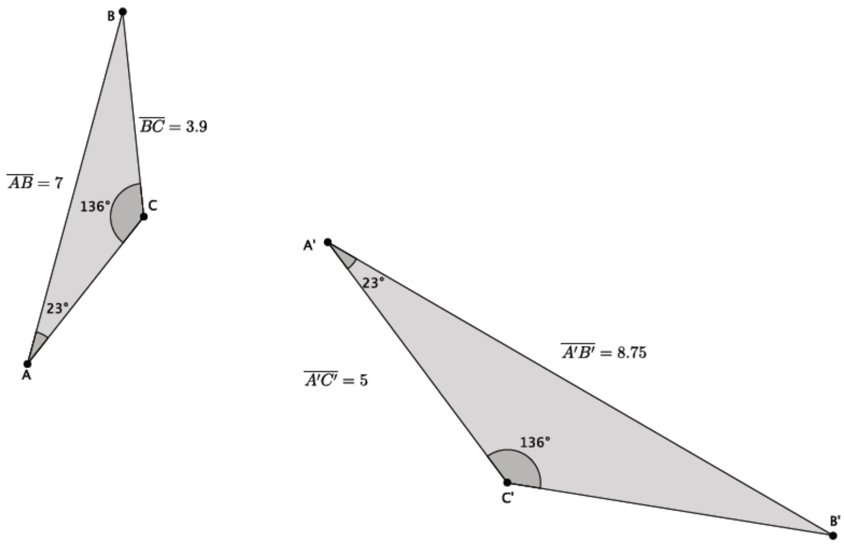
d. Given that $\triangle ABC \sim \triangle AB'C'$, determine the length of AB'.

Let x represent the length of AB'.

$$\frac{x}{7.7} = \frac{1.6}{11.2}$$

We are looking for the value of x that makes the fractions equivalent. Therefore, $11.2x = 12.32$, and $x = 1.1$. The length of AB' is 1.1.

3. In the diagram below you have $\triangle ABC$ and $\triangle A'B'C'$. Use this information to answer parts (a)–(c).



a. Based on the information given, is $\triangle ABC \sim \triangle A'B'C'$? Explain.

Yes, $\triangle ABC \sim \triangle A'B'C'$. There are two pairs of corresponding angles that are equal in measure. Namely, $\angle A = \angle A' = 23°$, and $\angle C = \angle C' = 136°$. By the AA criterion, these triangles are similar.

b. Given that $\triangle ABC \sim \triangle A'B'C'$, determine the length of $B'C'$.

Let x represent the length of $B'C'$.

$$\frac{x}{3.9} = \frac{8.75}{7}$$

We are looking for the value of x that makes the fractions equivalent. Therefore, $7x = 34.125$, and $x = 4.875$. The length of $B'C'$ is 4.875.

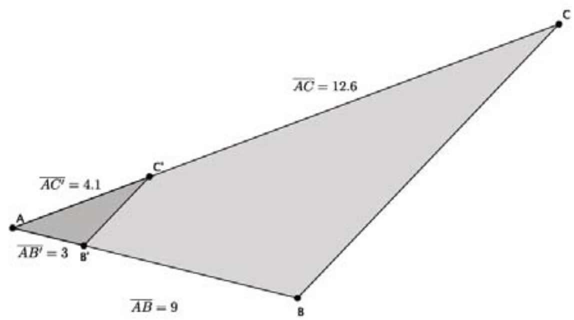
c. Given that $\triangle ABC \sim \triangle A'B'C'$, determine the length of AC.

Let x represent the length of AC.

$$\frac{5}{x} = \frac{8.75}{7}$$

We are looking for the value of x that makes the fractions equivalent. Therefore, $8.75x = 35$, and $x = 4$. The length of AC is 4.

4. In the diagram below you have $\triangle ABC$ and $\triangle AB'C'$. Use this information to answer the question below.



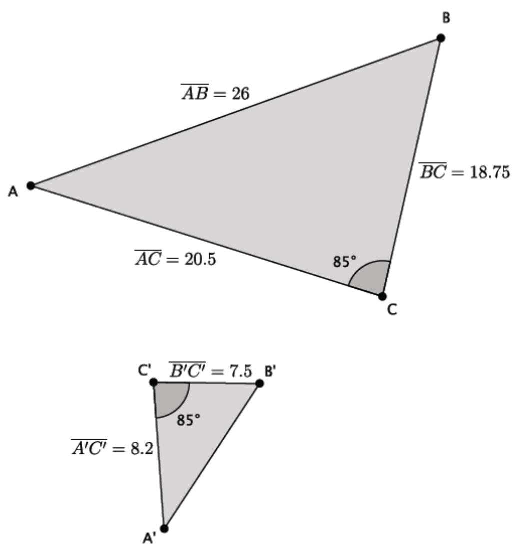
Based on the information given, is $\triangle ABC \sim \triangle AB'C'$? Explain.

No, $\triangle ABC$ is not similar to $\triangle AB'C'$. Since there is only information about one pair of corresponding angles, then we must check to see that the corresponding sides have equal ratios. That is, the following must be true:

$$\frac{9}{3} = \frac{12.6}{4.1}$$

When we compare products of each numerator with the denominator of the other fraction, we see that $36.9 \neq 37.8$. Since the corresponding sides do not have equal ratios, the fractions are not equivalent, and the triangles are not similar.

5. In the diagram below, you have $\triangle ABC$ and $\triangle A'B'C'$. Use this information to answer parts (a)–(b).



a. Based on the information given, is $\triangle ABC \sim \triangle A'B'C'$? Explain.

Yes, $\triangle ABC \sim \triangle A'B'C'$. Since there is only information about one pair of corresponding angles being equal, then the corresponding sides must be checked to see if their ratios are equal:

$$\frac{8.2}{20.5} = \frac{7.5}{18.75}$$

When we compare products of each numerator with the denominator of the other fraction, we see that $153.75 = 153.75$. Since the products are equal, the fractions are equivalent, and the triangles are similar.

b. Given that $\triangle ABC \sim \triangle A'B'C'$, determine the length of $A'B'$.

Let x represent the length of $A'B'$.

$$\frac{x}{26} = \frac{7.5}{18.75}$$

We are looking for the value of x that makes the fractions equivalent. Therefore, $18.75x = 195$, and $x = 10.4$. The length of $A'B'$ is 10.4.