Lesson 4: Solving a Linear Equation

Student Outcomes

- Students extend the use of the properties of equality to solve linear equations having rational coefficients.

Classwork

Concept Development (13 minutes)

- To solve an equation means to find all of the numbers x, if they exist, so that the given equation is true.
- In some cases, some simple guess work can lead us to a solution. For example, consider the following equation:

$$4x + 1 = 13$$

What number x would make this equation true? That is, what value of x would make the left side equal to the right side? (Give students a moment to guess a solution.)

  - *When $x = 3$, we get a true statement. The left side of the equal sign is equal to 13 and so is the right side of the equal sign.*

In other cases, guessing the correct answer is not so easy. Consider the following equation:

$$3(4x - 9) + 10 = 15x + 2 + 7x$$

Can you guess a number for x that would make this equation true? (Give students a minute to guess.)

- Guessing is not always an efficient strategy for solving equations. In the last example, there are several terms in each of the linear expressions comprising the equation. This makes it more difficult to easily guess a solution. For this reason, we want to use what we know about the properties of equality to transform equations into equations with fewer terms.
- The ultimate goal of solving any equation is to get it into the form of x (or whatever symbol is being used in the equation) equal to a constant.

MP.8

Complete the activity described to remind students of the properties of equality, then proceed with the discussion that follows.

Give students the equation: $4 + 1 = 7 - 2$ and ask them the following questions.

1. Is this equation true?
2. Perform each of the following operations, and state whether or not the equation is still true:
   a. Add three to both sides of the equal sign.
   b. Add three to the left side of the equal sign, and add two to the right side of the equal sign.
   c. Subtract six from both sides of the equal sign.
   d. Subtract three from one side of the equal sign and subtract three from the other side.
   e. Multiply both sides of the equal sign by ten.
   f. Multiply the left side of the equation by ten and the right side by four.
   g. Divide both sides of the equation by two.
   h. Divide the left side of the equation by two and the right side of the equation by five.
3. What do you notice? Describe any patterns you see.

- There are four properties of equality that will allow us to transform an equation into the form we want. If A, B, and C are any rational numbers, then
  - If $A = B$, then $A + C = B + C$.
  - If $A = B$, then $A - C = B - C$.
  - If $A = B$, then $A \cdot C = B \cdot C$.
  - If $A = B$, then $\frac{A}{C} = \frac{B}{C}$, where C is not equal to zero.

  All four of the properties require us to start off with $A = B$. That is, we have to assume that a given equation has an expression on the left side that is equal to the expression on the right side. Working under that assumption, each time we use one of the properties of equality, we are transforming the equation into another equation that is also true, i.e., left side equals right side.

Example 1 (3 minutes)

- Solve the linear equation $2x - 3 = 4x$ for the number x.
- Examine the properties of equality. Choose "something" to add, subtract, multiply, or divide on both sides of the equation.

Validate the use of the properties of equality by having students share their thoughts. Then, discuss the "best" choice for the first step in solving the equation with the points below. Be sure to remind students throughout this and the other examples that our goal is to get x equal to a constant; therefore, the "best" choice is one that gets us to that goal most efficiently.

- First, we must assume that there is a number x that makes the equation true. Working under that assumption, when we use the property, If $A = B$, then $A - C = B - C$, we get an equation that is also true:

$$2x - 3 = 4x$$
$$2x - 2x - 3 = 4x - 2x$$

  Now, using the distributive property, we get another set of equations that is also true:

$$(2-2)x - 3 = (4-2)x$$
$$0x - 3 = 2x$$
$$-3 = 2x$$

  Using another property, If $A = B$, then $\frac{A}{C} = \frac{B}{C}$, we get another true equation:

$$\frac{-3}{2} = \frac{2x}{2}$$

  After simplifying the fraction $\frac{2}{2}$, we have

$$\frac{-3}{2} = x$$

  is also true.

- The last step is to check to see if $x = -\frac{3}{2}$ satisfies the equation $2x - 3 = 4x$.

  The left side of the equation is equal to $2 \cdot \left(-\frac{3}{2}\right) - 3 = -3 - 3 = -6$.

  The right side of the equation is equal to $4 \cdot \left(-\frac{3}{2}\right) = 2 \cdot (-3) = -6$.

  Since the left side equals the right side, we know we have found the correct number x that solves the equation $2x - 3 = 4x$.

Example 2 (4 minutes)

- Solve the linear equation $\frac{3}{5}x - 21 = 15$. Keep in mind that our goal is to transform the equation so that it is in the form of x equal to a constant. If we assume that the equation is true for some number x, which property should we use to help us reach our goal and how should we use it?

Again, provide students time to decide which property is "best" to use first.

  - *We should use: If $A = B$, then $A + C = B + C$, where the number C is* 21.

Note to teacher: If students suggest that we subtract 15 from both sides, i.e., make C be -15, then remind them that we want the form of x equal to a constant. Subtracting 15 from both sides of the equal sign puts the x and all of the constants on the same side of the equal sign. There is nothing mathematically incorrect about subtracting 15; it just doesn't get us any closer to reaching our goal.

- If we use $A + C = B + C$, then we have the true statement:

$$\frac{3}{5}x - 21 + 21 = 15 + 21$$

and

$$\frac{3}{5}x = 36.$$

Which property should we use to reach our goal, and how should we use it?

  - *We should use: If $A = B$, then $A \cdot C = B \cdot C$, where C is $\frac{5}{3}$.*

- If we use $A \cdot C = B \cdot C$, then we have the true statement:

$$\frac{3}{5}x \cdot \frac{5}{3} = 36 \cdot \frac{5}{3}$$

and by the commutative property and the cancellation law we have:

$$x = 12 \cdot 5 = 60.$$

- Does $x = 60$ satisfy the equation $\frac{3}{5}x - 21 = 15$?
  - *Yes, because the left side of the equation is equal to $\frac{180}{5} - 21 = 36 - 21 = 15$. Since the right side is also 15, then we know that $x = 60$ is a solution to $\frac{3}{5}x - 21 = 15$.*

Example 3 (5 minutes)

- The properties of equality are not the only properties we can use with equations. What other properties do we know that could make solving an equation more efficient?
  - *We know the distributive property, which allows us to expand and simplify expressions.*
  - *We know the commutative and associative properties, which allow us to rearrange and group terms within expressions.*

Now we will solve the linear equation $\frac{1}{5}x + 13 + x = 1 - 9x + 22$. Is there anything we can do to the linear expression on the left side to transform it into an expression with fewer terms?

- *Yes, we can use the commutative and distributive properties:*

$$\frac{1}{5}x + 13 + x = \frac{1}{5}x + x + 13$$
$$= \frac{6}{5}x + 13$$

- Is there anything we can do to the linear expression on the right side to transform it into an expression with fewer terms?
  - *Yes, we can use the commutative property:*

$$1 - 9x + 22 = 1 + 22 - 9x$$
$$= 23 - 9x$$

- Now we have the equation: $\frac{6}{5}x + 13 = 23 - 9x$. What should we do now to solve the equation?

Students should come up with the following four responses as to what should be done first. A case can be made for each of them being the "best" move. In this case, each first move gets us one step closer to our goal of having the solution in the form of x equal to a constant. Select one option and move forward with solving the equation (the notes that follow align to the first choice, subtracting 13 from both sides of the equal sign).

- *We should subtract* 13 *from both sides of the equal sign.*
- *We should subtract* 23 *from both sides of the equal sign.*
- *We should add* $9x$ *to both sides of the equal sign.*
- *We should subtract* $\frac{6}{5}x$ *from both sides of the equal sign.*

Note to Teacher:

There are many ways to solve this equation. Any of the actions listed below are acceptable. In fact, a student could say: "add 100 to both sides of the equal sign," and that, too, would be an acceptable action. It may not lead us directly to our answer, but it is still an action that would make a mathematically correct statement. Make clear to students that it doesn't matter which option they choose or in which order; what matters is that they use the properties of equality to make true statements that lead to a solution in the form of x equal to a constant.

- Let's choose to subtract 13 from both sides of the equal sign. Though all options were generally equal with respect to being the "best" first step, I chose this one because when I subtract 13 on both sides, the value of the constant on the left side is positive. I prefer to work with positive numbers. Then we have

$$\frac{6}{5}x + 13 - 13 = 23 - 13 - 9x$$
$$\frac{6}{5}x = 10 - 9x$$

- What should we do next? Why?
  - *We should add* $9x$ *to both sides of the equal sign. We want our solution in the form of* x *equal to a constant, and this move puts all terms with an* x *on the same side of the equal sign.*
- Adding $9x$ to both sides of the equal sign, we have:

$$\frac{6}{5}x + 9x = 10 - 9x + 9x$$
$$\frac{51}{5}x = 10$$

Note to Teacher:

We still have options. If students say we should subtract $\frac{6}{5}x$ from both sides of the equal sign, remind them of our goal of obtaining the x equal to a constant.

- What do we need to do now?
  - *We should multiply $\frac{5}{51}$ on both sides of the equal sign.*
- Then we have:

$$\frac{51}{5}x \cdot \frac{5}{51} = 10 \cdot \frac{5}{51}$$

By the commutative property and the fact that $\frac{5}{51} \times \frac{51}{5} = 1$, we have

$$x = \frac{50}{51}.$$

- Since all transformed versions of the original equation are true, we can select any of them to check our answer. However, it is best to check the solution in the original equation because we may have made a mistake transforming the equation.

$$\frac{1}{5}x + 13 + x = 1 - 9x + 22$$
$$\frac{1}{5}\left(\frac{50}{51}\right) + 13 + \frac{50}{51} = 1 - 9\left(\frac{50}{51}\right) + 22$$
$$\frac{6}{5}\left(\frac{50}{51}\right) + 13 = 23 - 9\left(\frac{50}{51}\right)$$
$$\frac{300}{255} + 13 = 23 - \frac{450}{51}$$
$$\frac{3615}{255} = \frac{723}{51}$$
$$\frac{723}{51} = \frac{723}{51}$$

Since both sides of our equation equal $\frac{723}{51}$, then we know our answer is correct.

Exercises 1–5 (10 minutes)

Students work on Exercises 1–5 independently.

Exercises 1–5

For each problem, show your work and check that your solution is correct.

1. **Solve the linear equation: $x + x + 2 + x + 4 + x + 6 = -28$. State the property that justifies your first step and why you chose it.**

   The left side of the equation can be transformed from $x + x + 2 + x + 4 + x + 6$ to $4x + 12$ using the commutative and distributive properties. Using these properties decreases the number of terms of the equation. Now we have the equation:

$$4x + 12 = -28$$
$$4x + 12 - 12 = -28 - 12$$
$$4x = -40$$
$$\frac{1}{4} \cdot 4x = -40 \cdot \frac{1}{4}$$
$$x = -10$$

   The left side of the equation is equal to $(-10) + (-10) + 2 + (-10) + 4 + (-10) + 6$ which is -28. Since the left side is equal to the right side, then $x = -10$ is the solution to the equation.

   Note: Students could use the division property in the last step to get the answer.

2. Solve the linear equation: $2(3x+2) = 2x - 1 + x$. State the property that justifies your first step and why you chose it.

Both sides of equation can be rewritten using the distributive property. I have to use it on the left side to expand the expression. I have to use it on the right side to collect like terms.

The left side is:

$$2(3x+2) = 6x + 4$$

The right side is:

$$\begin{aligned} 2x - 1 + x &= 2x + x - 1 \\ &= 3x - 1 \end{aligned}$$

The equation is:

$$\begin{aligned} 6x + 4 &= 3x - 1 \\ 6x + 4 - 4 &= 3x - 1 - 4 \\ 6x &= 3x - 5 \\ 6x - 3x &= 3x - 3x - 5 \\ 3x &= -5 \\ \frac{1}{3} \cdot 3x &= \frac{1}{3} \cdot (-5) \\ x &= -\frac{5}{3} \end{aligned}$$

The left side of the equation is equal to $2(-5+2) = 2(-3) = -6$. The right side of the equation is equal to $-5 - 1 = -6$. Since both sides are equal to -6, then $x = -\frac{5}{3}$ is a solution to $2(3x+2) = 2x - 1 + x$.

Note: Students could use the division property in the last step to get the answer.

3. Solve the linear equation: $x - 9 = \frac{3}{5}x$. State the property that justifies your first step and why you chose it.

I chose to use the subtraction property of equality to get all terms with an x on one side of the equal sign.

$$\begin{aligned} x - 9 &= \frac{3}{5}x \\ x - x - 9 &= \frac{3}{5}x - x \\ -9 &= -\frac{2}{5}x \\ -\frac{5}{2} \cdot (-9) &= -\frac{5}{2} \cdot -\frac{2}{5}x \\ \frac{45}{2} &= x \end{aligned}$$

The left side of the equation is $\frac{45}{2} - \frac{18}{2} = \frac{27}{2}$. The right side is $\frac{3}{5} \cdot \frac{45}{2} = \frac{3}{1} \cdot \frac{9}{2} = \frac{27}{2}$. Since both sides are equal to the same number, then $x = \frac{45}{2}$ is a solution to $x - 9 = \frac{3}{5}x$.

4. Solve the linear equation: $29 - 3x = 5x + 5$. State the property that justifies your first step and why you chose it.

I chose to use the addition property of equality to get all terms with an x on one side of the equal sign.

$$\begin{aligned} 29 - 3x &= 5x + 5 \\ 29 - 3x + 3x &= 5x + 3x + 5 \\ 29 &= 8x + 5 \\ 29 - 5 &= 8x + 5 - 5 \\ 24 &= 8x \\ \frac{1}{8} \cdot 24 &= \frac{1}{8} \cdot 8x \\ 3 &= x \end{aligned}$$

The left side of the equal sign is $29 - 3(3) = 29 - 9 = 20$. The right side is equal to $5(3) + 5 = 15 + 5 = 20$. Since both sides are equal, $x = 3$ is a solution to $29 - 3x = 5x + 5$.

Note: Students could use the division property in the last step to get the answer.

5. Solve the linear equation: $\frac{1}{3}x - 5 + 171 = x$. State the property that justifies your first step and why you chose it.

I chose to combine the constants -5 and 171. Then, I used the subtraction property of equality to get all terms with an x on one side of the equal sign.

$$\frac{1}{3}x - 5 + 171 = x$$
$$\frac{1}{3}x + 166 = x$$
$$\frac{1}{3}x - \frac{1}{3}x + 166 = x - \frac{1}{3}x$$
$$166 = \frac{2}{3}x$$
$$166 \cdot \frac{3}{2} = \frac{3}{2} \cdot \frac{2}{3}x$$
$$83 \times 3 = x$$
$$249 = x$$

The left side of the equation is $\frac{1}{3} \cdot 249 - 5 + 171 = 83 - 5 + 171 = 78 + 171 = 249$, which is exactly equal to the right side. Therefore, $x = 249$ is a solution to $\frac{1}{3}x - 5 + 171 = x$.

Closing (5 minutes)

Summarize, or ask students to summarize, the main points from the lesson:

- We know that properties of equality, when used to transform equations, make equations with fewer terms that are simpler to solve.
- When solving an equation, we want the answer to be in the form of the symbol x equal to a constant.

Lesson Summary

The properties of equality, shown below, are used to transform equations into simpler forms. If A, B, C are rational numbers, then

- **If $A = B$, then $A + C = B + C$** — **Addition Property of Equality**
- **If $A = B$, then $A - C = B - C$** — **Subtraction Property of Equality**
- **If $A = B$, then $A \cdot C = B \cdot C$** — **Multiplication Property of Equality**
- **If $A = B$, then $\frac{A}{C} = \frac{B}{C}$, where C is not equal to zero** — **Division Property of Equality**

To solve an equation, transform the equation until you get to the form of x equal to a constant ($x = 5$, for example).

Exit Ticket (5 minutes)

Name ____ Date____________________

Lesson 4: Solving a Linear Equation

Exit Ticket

1. Guess a number for x that would make the equation true. Check your solution.

$$5x - 2 = 8$$

2. Use the properties of equality to solve the equation: $7x - 4 + x = 12$. State which property justifies your first step and why you chose it. Check your solution.

3. Use the properties of equality to solve the equation: $3x + 2 - x = 11x + 9$. State which property justifies your first step and why you chose it. Check your solution.

Exit Ticket Sample Solutions

1. Guess a number for x that would make the equation true. Check your solution.

$$5x - 2 = 8$$

When $x = 2$, the left side of the equation is 8, which is the same as the right side. Therefore, $x = 2$ is the solution to the equation.

2. Use the properties of equality to solve the equation: $7x - 4 + x = 12$. State which property justifies your first step and why you chose it. Check your solution.

I used the commutative and distributive properties on the left side of the equal sign to simply the expression to fewer terms.

$$\begin{aligned} 7x - 4 + x &= 12 \\ 8x - 4 &= 12 \\ 8x - 4 + 4 &= 12 + 4 \\ 8x &= 16 \\ \frac{8}{8}x &= \frac{16}{8} \\ x &= 2 \end{aligned}$$

The left side of the equation is $7(2) - 4 + 2 = 14 - 4 + 2 = 12$. The right side is also 12. Since the left side equals the right side, $x = 2$ is the solution to the equation.

3. Use the properties of equality to solve the equation: $3x + 2 - x = 11x + 9$. Check your solution.

I used the commutative and distributive properties on the left side of the equal sign to simplify the expression to fewer terms.

$$\begin{aligned} &3x + 2 - x = 11x + 9 \\ &2x + 2 = 11x + 9 \\ &2x - 2x + 2 = 11x - 2x + 9 \\ &2 = 9x + 9 \\ &2 - 9 = 9x + 9 - 9 \\ &-7 = 9x \\ &\frac{-7}{9} = \frac{9}{9}x \\ &-\frac{7}{9} = x \end{aligned}$$

The left side of the equation is $3\left(\frac{-7}{9}\right) + 2 - \frac{-7}{9} = -\frac{21}{9} + \frac{18}{9} + \frac{7}{9} = \frac{4}{9}$. The right side is $11\left(-\frac{7}{9}\right) + 9 = \frac{-77}{9} + \frac{81}{9} = \frac{4}{9}$.

Since the left side equals the right side, $x = -\frac{7}{9}$ is the solution to the equation.

Problem Set Sample Solutions

Students solve equations using properties of equality.

For each problem, show your work and check that your solution is correct.

1. **Solve the linear equation: $x + 4 + 3x = 72$. State the property that justifies your first step and why you chose it.**

   I used the commutative and distributive properties on the left side of the equal sign to simplify the expression to fewer terms.

$$\begin{aligned} x + 4 + 3x &= 72 \\ 4x + 4 &= 72 \\ 4x + 4 - 4 &= 72 - 4 \\ 4x &= 68 \\ \frac{4}{4}x &= \frac{68}{4} \\ x &= 17 \end{aligned}$$

   The left side is equal to $17 + 4 + 3(17) = 21 + 51 = 72$, which is what the right side is. Therefore, $x = 17$ is a solution to the equation $x + 4 + 3x = 72$.

2. **Solve the linear equation: $x + 3 + x - 8 + x = 55$. State the property that justifies your first step and why you chose it.**

   I used the commutative and distributive properties on the left side of the equal sign to simplify the expression to fewer terms.

$$\begin{aligned} x + 3 + x - 8 + x &= 55 \\ 3x - 5 &= 55 \\ 3x - 5 + 5 &= 55 + 5 \\ 3x &= 60 \\ \frac{3}{3}x &= \frac{60}{3} \\ x &= 20 \end{aligned}$$

   The left side is equal to $20 + 3 + 20 - 8 + 20 = 43 - 8 + 20 = 35 + 20 = 55$, which is equal to the right side. Therefore, $x = 20$ is a solution to $x + 3 + x - 8 + x = 55$.

3. **Solve the linear equation: $\frac{1}{2}x + 10 = -\frac{1}{4}x + 54$. State the property that justifies your first step and why you chose it.**

   I chose to use the subtraction property of equality to get all of the constants on one side of the equal sign.

$$\begin{aligned} \frac{1}{2}x + 10 &= \frac{1}{4}x + 54 \\ \frac{1}{2}x + 10 - 10 &= \frac{1}{4}x + 54 - 10 \\ \frac{1}{2}x &= -\frac{1}{4}x + 44 \\ \frac{1}{2}x - \frac{1}{4}x &= \frac{1}{4}x - \frac{1}{4}x + 44 \\ \frac{1}{4}x &= 44 \\ 4 \cdot \frac{1}{4}x &= 4 \cdot 44 \\ x &= 176 \end{aligned}$$

   The left side of the equation is $\frac{1}{2}(176) + 10 = 88 + 10 = 98$. The right side of the equation is $\frac{1}{4}(176) + 54 = 44 + 54 = 98$. Since both sides equal 98, $x = 176$ is a solution to the equation $\frac{1}{2}x + 10 = \frac{1}{4}x + 54$.

4. Solve the linear equation: $\frac{1}{4}x + 18 = x$. State the property that justifies your first step and why you chose it.

I chose to use the subtraction property of equality to get all terms with an x on one side of the equal sign.

$$\begin{aligned} \frac{1}{4}x + 18 &= x \\ \frac{1}{4}x - \frac{1}{4}x + 18 &= x - \frac{1}{4}x \\ 18 &= \frac{3}{4}x \\ \frac{4}{3} \cdot 18 &= \frac{4}{3} \cdot \frac{3}{4}x \\ 24 &= x \end{aligned}$$

The left side of the equation is $\frac{1}{4}(24) + 18 = 6 + 18 = 24$, which is what the right side is equal to. Therefore, $x = 24$ is a solution to $\frac{1}{4}x + 18 = x$.

5. Solve the linear equation: $17 - x = \frac{1}{3} \cdot 15 + 6$. State the property that justifies your first step and why you chose it.

The right side of the equation can be simplified to 11. Then the equation is

$$17 - x = 11$$

and $x = 6$. Both sides of the equation equal 11; therefore, $x = 6$ is a solution to the equation $17 - x = \frac{1}{3} \cdot 15 + 6$. I was able to solve the equation mentally without using the properties of equality.

6. Solve the linear equation: $\frac{x+x+2}{4} = 189.5$. State the property that justifies your first step and why you chose it.

$$\begin{aligned} \frac{x + x + 2}{4} &= 189.5 \\ x + x + 2 &= 4(189.5) \\ 2x + 2 &= 758 \\ 2x + 2 - 2 &= 758 - 2 \\ 2x &= 756 \\ \frac{2}{2}x &= \frac{756}{2} \\ x &= 378 \end{aligned}$$

The left side of the equation is $\frac{378+378+2}{4} = \frac{758}{4} = 189.5$, which is equal to the right side of the equation. Therefore, $x = 378$ is a solution to $\frac{x+x+2}{4} = 189.5$.

7. Alysha solved the linear equation: $2x - 3 - 9x = 14 + x - 1$. Her work is shown below. When she checked her answer, the left side of the equation did not equal the right side. Find and explain Alysha's error, and then solve the equation correctly.

$$2x - 3 - 9x = 14 + x - 1.$$
$$-6x - 3 = 13 + 2x$$
$$-6x - 3 + 3 = 13 + 3 + 2x$$
$$-6x = 16 + 2x$$
$$-6x + 2x = 16$$
$$-4x = 16$$
$$\frac{-4}{-4}x = \frac{16}{-4}$$
$$x = -4$$

Alysha made a mistake on the 5^{th} line. She added $2x$ to the left side of the equal sign and subtracted $2x$ on the right side of the equal sign. To use the property correctly, she should have subtracted $2x$ on both sides of the equal sign, making the equation at that point:

$$-6x - 2x = 16 + 2x - 2x$$
$$-8x = 16$$
$$\frac{-8}{-8}x = \frac{16}{-8}$$
$$x = -2$$