Lesson 5: Writing and Solving Linear Equations

Student Outcomes

- Students apply knowledge of geometry to writing and solving linear equations.

Lesson Notes

All of the problems in this lesson relate to what students have learned about geometry in recent modules and previous years. The purpose is two-fold, first, we want to reinforce what students have learned about geometry, and second, we want to demonstrate a need for writing and solving an equation in a context that is familiar. Throughout the lesson students solve mathematical problems that relate directly to what students have learned about angle relationships, congruence, and the triangle sum theorem. Encourage students to draw a diagram to represent the situation presented in the word problems.

Classwork

Example 1 (5 minutes)

- Solve the following:

> **Example 1**
>
> One angle is five less than three times the size of another angle. Together they have a sum of $143°$. What are the sizes of each angle?

MP.1

Provide students with time to make sense of the problem and persevere in solving it. They could begin their work by guessing and checking, drawing a diagram, or other methods as appropriate. Then move to the algebraic method shown below.

- What do we need to do first to solve this problem?
  - *First we need to define our variable (symbol). Let x be the size of the first angle.*
- If x is the size of the first angle, how do you represent the size of the second angle?
  - *The second angle is $3x - 5$.*
- What is the equation that represents this situation?
  - *The equation is: $x + 3x - 5 = 143$.*
- The equation that represents this situation is: $x + 3x - 5 = 143$. Solve for x, then determine the size of each angle.

> *Scaffolding:*
>
> Model for students how to use diagrams to help make sense of the problems throughout this lesson. Encourage students to use diagrams to help them understand the situation.

As students share their solutions for this and subsequent problems, ask them a variety of questions to reinforce the learning of the last few lessons. For example, you can ask students whether or not this is a linear equation and how they know, to justify their steps and explain why they chose their particular first step, what the solution means, or how they know their answer is correct.

Student work:

$$x + 3x - 5 = 143$$
$$(1 + 3)x - 5 = 143$$
$$4x - 5 = 143$$
$$4x - 5 + 5 = 143 + 5$$
$$4x = 148$$
$$x = 37$$

The size of the first angle is $37°$. *The second angle is* $3(37) - 5 = 111 - 5 = 106°$.

- Compare the method you tried at the beginning of the problem with the algebraic method. What advantage does writing and solving an equation have?
  - *Writing and solving an equation is a more direct method than the one I tried before. It allows me to find the answer more quickly.*
- Could we have defined x to be the size of the second angle? If so, what, if anything, would change?
  - *If we let x be the size of the second angle, then the equation would change, but the answers for the sizes of the angles should remain the same.*
- If x is the size of the second angle, how would we write the size of the first angle?
  - *The first angle would be $\frac{x+5}{3}$.*
- The equation that represents the situation is $x + \frac{x+5}{3} = 143$. How should we solve this equation?
  - *We could add the fractions together, then solve for x.*
  - *We could multiply every term by 3 to change the fraction to a whole number.*
- Using either method, solve the equation. Verify that the sizes of the angles are the same as before.
  -

$$x + \frac{x + 5}{3} = 143$$
$$\frac{3x}{3} + \frac{x + 5}{3} = 143$$
$$\frac{3x + x + 5}{3} = 143$$
$$3x + x + 5 = 143(3)$$
$$(3 + 1)x + 5 = 429$$
$$4x + 5 = 429$$
$$4x + 5 - 5 = 429 - 5$$
$$4x = 424$$
$$x = 106$$

> *Scaffolding:*
>
> You may need to remind students how to add fractions by rewriting term(s) as equivalent fractions then adding the numerators. Provide support as needed.

□ *The size of the second angle is* $106°$. *The measure of the first angle is* $\frac{x+5}{3} = \frac{106+5}{3} = \frac{111}{3} = 37°$.

OR

$$x + \frac{x+5}{3} = 143$$
$$3\left(x + \frac{x+5}{3} = 143\right)$$
$$3x + x + 5 = 429$$
$$4x + 5 = 429$$
$$4x + 5 - 5 = 429 - 5$$
$$4x = 424$$
$$x = 106$$

The size of the second angle is $106°$. *The measure of the first angle is* $\frac{x+5}{3} = \frac{106+5}{3} = \frac{111}{3} = 37°$.

▪ Whether we let x represent the first angle or the second angle does not change our answers. Whether we solve the equation using the first or second method does not change our answers. What matters is that we accurately write the information in the problem and correctly use the properties of equality. You may solve a problem differently than your classmates or teachers. Again, what matters most is that what you do is accurate and correct.

Example 2 (12 minutes)

▪ Solve the following:

> **Example 2**
>
> **Given a right triangle, find the size of the angles if one angle is ten more than four times the other angle and the third angle is the right angle.**

Give students time to work. As they work, walk around and identify students who are writing and solving the problem in different ways. The instructional goal of this example is to make clear that there are different ways to solve a linear equation as opposed to one "right way". Select students to share their work with the class. If students don't come up with different ways of solving the equation, talk them through the following student work samples.

Again, as students share their solutions, ask them a variety of questions to reinforce the learning of the last few lessons. For example, you can ask students whether or not this is a linear equation and how they know, or to justify their steps and explain why they chose their particular first step, what the solution means, or how they know their answer is correct.

Solution One

Let x be the size of the first angle. Then the second angle is $4x + 10$. The sum of the measures for the angles for this right triangle is as follows: $x + 4x + 10 + 90 = 180$.

$$x + 4x + 10 + 90 = 180$$
$$(1 + 4)x + 100 = 180$$
$$5x + 100 = 180$$
$$5x + 100 - 100 = 180 - 100$$
$$5x = 80$$
$$x = 16$$

The measure of the first angle is $16°$, the measure of the second angle is $4(16) + 10 = 64 + 10 = 74°$, and the third angle is $90°$.

Solution Two

Let x be the size of the first angle. Then the second angle is $4x + 10$. Since we have a right triangle we already know that one angle is $90°$ which means that the sum of the other two angles is 90: $x + 4x + 10 = 90$.

$$x + 4x + 10 = 90$$
$$(1 + 4)x + 10 = 90$$
$$5x + 10 = 90$$
$$5x + 10 - 10 = 90 - 10$$
$$5x = 80$$
$$x = 16$$

The measure of the first angle is $16°$, the measure of the second angle is $4(16) + 10 = 64 + 10 = 74°$, and the third angle is $90°$.

Solution Three

Let x be the size of the second angle. Then the first angle is $\frac{x-10}{4}$. Since we have a right triangle we already know that one angle is $90°$ which means that the sum of the other two angles is 90: $x + \frac{x-10}{4} = 90$.

$$x + \frac{x - 10}{4} = 90$$
$$4\left(x + \frac{x - 10}{4} = 90\right)$$
$$4x + x - 10 = 360$$
$$(4 + 1)x - 10 = 360$$
$$5x - 10 = 360$$
$$5x - 10 + 10 = 360 + 10$$
$$5x = 370$$
$$x = 74$$

The measure of the second angle is $74°$, the measure of the first angle is $\frac{74-10}{4} = \frac{64}{4} = 16°$, and the third angle is $90°$.

> *Solution Four*
>
> *Let x be the size of the second angle. Then the first angle is $\frac{x-10}{4}$. The sum of the three angles is as follows: $x + \frac{x-10}{4} +$*
> *$90 = 180$.*
>
> $$x + \frac{x-10}{4} + 90 = 180$$
> $$x + \frac{x-10}{4} + 90 - 90 = 180 - 90$$
> $$x + \frac{x-10}{4} = 90$$
> $$\frac{4x}{4} + \frac{x-10}{4} = 90$$
> $$\frac{4x + x - 10}{4} = 90$$
> $$4x + x - 10 = 360$$
> $$5x - 10 + 10 = 360 + 10$$
> $$5x = 370$$
> $$x = 74$$
>
> *The second angle is $74°$, the first angle is $\frac{74-10}{4} = \frac{64}{4} = 16°$, and the third angle is $90°$.*

Make sure students see at least four different methods of solving the problem. Conclude this example with the statements below.

- Each method was slightly different either in terms of how the variable was defined or how the properties of equality were used to solve the equation. The way you find the answer may be different than your classmates or your teacher.

- As long as you write accurately and do what is mathematically correct, you will find the correct answer.

Example 3 (4 minutes)

- A pair of alternate interior angles are described as follows. One angle is fourteen more than half a number. The other angle is six less than half a number. Are the angles congruent?

- We will begin by assuming that the angles are congruent. If the angles are congruent, what does that mean about their measures?

    ▫ *It means that they are equal in measure.*

- Write an expression that describes each angle.

    ▫ *One angle is $\frac{x}{2} + 14$ and the other angle is $\frac{x}{2} - 6$.*

- If the angles are congruent, we can write the equation is $\frac{x}{2} + 14 = \frac{x}{2} - 6$. We know that our properties of equality allow us to transform equations while making sure that they remain true.

$$\frac{x}{2} + 14 = \frac{x}{2} - 6$$
$$\frac{x}{2} - \frac{x}{2} + 14 = \frac{x}{2} - \frac{x}{2} - 6,$$
$$14 \neq -6.$$

Therefore, our assumption was not correct and the angles are not congruent.

Exercises 1–6 (16 minutes)

Students complete Exercises 1–6 independently or in pairs.

Exercises 1–6

For each of the following problems, write an equation and solve.

1. **A pair of congruent angles are described as follows: the measure of one angle is three more than twice a number and the other angle's measure is 54.5 less than three times the number. Determine the size of the angles.**

    Let x be the number. Then the measure of one angle is $3 + 2x$ and the measure of the other angle is $3x - 54.5$. Because the angles are congruent, their measures are equal. Therefore,

    $$3 + 2x = 3x - 54.5$$
    $$3 + 2x - 2x = 3x - 2x - 54.5$$
    $$3 = x - 54.5$$
    $$3 + 54.5 = x - 54.5 + 54.5$$
    $$57.5 = x$$

    Then each angle is $3 + 2(57.5) = 3 + 115 = 118°$.

2. **The measure of one angle is described as twelve more than four times a number. Its supplement is twice as large. Find the measure of each angle.**

    Let x be the number. Then the measure of one angle is $4x + 12$. The other angle is $2(4x + 12) = 8x + 24$. Since the angles are supplementary, their sum must be 180:

    $$4x + 12 + 8x + 24 = 180$$
    $$12x + 36 = 180$$
    $$12x + 36 - 36 = 180 - 36$$
    $$12x = 144$$
    $$x = 12$$

    One angle is $4(12) + 12 = 48 + 12 = 60°$ and the other angle is $180 - 60 = 120°$.

3. **A triangle has angles described as follows: the first angle is four more than seven times a number, another angle is four less than the first and the third angle is twice as large as the first. What are the sizes of each of the angles?**

    Let x be the number. The measure of the first angle is $7x + 4$. The measure of the second angle is $7x + 4 - 4 = 7x$. The measure of the third angle is $2(7x + 4) = 14x + 8$. The sum of the angles of a triangle must be $180°$.

    $$7x + 4 + 7x + 14x + 8 = 180$$
    $$28x + 12 = 180$$
    $$28x + 12 - 12 = 180 - 12$$
    $$28x = 168$$
    $$x = 6$$

    The measure of the first angle is $7(6) + 4 = 42 + 4 = 46°$. The measure of the second angle is $7(6) = 42°$. The measure of the third angle is $2(46) = 92°$.

4. **One angle measures nine more than six times a number. A sequence of rigid motions maps the angle onto another angle that is described as being thirty less than nine times the number. What is the measure of the angles?**

    Let x be the number. Then the measure of one angle is $6x + 9$. The measure of the other angle is $9x - 30$. Since rigid motions preserve the measures of angles, then the measure of these angles is equal.

    $$6x + 9 = 9x - 30$$
    $$6x + 9 - 9 = 9x - 30 - 9$$
    $$6x = 9x - 39$$
    $$6x - 9x = 9x - 9x - 39$$
    $$-3x = -39$$
    $$x = 13$$

    The angles measure $6(13) + 9 = 78 + 9 = 87°$.

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5. A right triangle is described as having an angle of size "six less than negative two times a number," another angle that is "three less than negative one-fourth the number", and a right angle. What are the measures of the angles?

Let x be a number. Then one angle is $-2x - 6$. The other angle is $-\frac{x}{4} - 3$. The sum of the two angles must be $90°$.

$$-2x - 6 + \left(-\frac{x}{4}\right) - 3 = 90$$
$$\left(-\frac{8x}{4}\right) + \left(-\frac{x}{4}\right) - 9 = 90$$
$$\left(-\frac{9x}{4}\right) - 9 + 9 = 90 + 9$$
$$-\frac{9x}{4} = 99$$
$$-9x = 396$$
$$x = -44$$

One of the angles is $-2(-44) - 6 = 88 - 6 = 82°$. The other angle is $90 - 82 = 8°$.

6. One angle is one less than six times the size of another. The two angles are complementary angles. Find the size of each angle.

Let x be the measure of the first angle. Then the measure of the second angle is $6x - 1$. The sum of the measures will be 90 because the angles are complementary.

$$x + 6x - 1 = 90$$
$$7x - 1 = 90$$
$$7x - 1 + 1 = 90 + 1$$
$$7x = 91$$
$$x = 13$$

One angle is $13°$ and the other angle is $6(13) - 1 = 78 - 1 = 77°$.

Closing (4 minutes)

Summarize, or ask students to summarize, the main points from the lesson:

▪ We know that an algebraic method for solving equations is more efficient than guessing and checking.

▪ We know how to write and solve equations that relate to angles, triangles, and geometry in general.

▪ We know that drawing a diagram can sometimes make it easier to understand a problem and that there is more than one way to solve an equation.

Exit Ticket (4 minutes)

Name _____ Date _____

Lesson 5: Writing and Solving Linear Equations

Exit Ticket

For each of the following problems, write an equation and solve.

1. Given a right triangle, find the measures of all the angles if one angle is a right angle and the measure of the second angle is six less than seven times the measure of the third angle.

2. In a triangle, the measure of the first angle is six times a number. The measure of the second angle is nine less than the first angle. The measure of the third angle is three times the number more than the measure of the first angle. Determine the measures of each of the angles.

Exit Ticket Sample Solutions

1. **Given a right triangle, find the measures of all of the angles if one angle is a right angle and the measure of a second angle is six less than seven times the measure of the third angle.**

   Let x represent the measure of the third angle. Then $7x - 6$ can represent the measure of the second angle. The sum of the two angles in the right triangles will be $90°$.

$$7x - 6 + x = 90$$
$$8x - 6 = 90$$
$$8x - 6 + 6 = 90 + 6$$
$$8x = 96$$
$$\frac{8}{8}x = \frac{96}{8}$$
$$x = 12$$

   The measure of the third angle is $12°$ and the measure of the second angle is $7(12) - 6 = 84 - 6 = 78°$,. The measure of the third angle is $90°$.

2. **In a triangle, the measure of the first angle is six times a number. The measure of the second angle is nine less than the first angle. The measure of the third angle is three times the number more than the measure of the first angle. Determine the measures of each of the angles.**

   Let x be the number. Then the measure of the first angles is $6x$, the measure of the second angle is $6x - 9$, and the measure of the third angle is $3x + 6x$. The sum of the measures of the angles in a triangle is $180°$.

$$6x + 6x - 9 + 3x + 6x = 180$$
$$21x - 9 = 180$$
$$21x - 9 + 9 = 180 + 9$$
$$21x = 189$$
$$\frac{21}{21}x = \frac{189}{21}$$
$$x = 9$$

   The measure of the first angle is $6(9) = 54°$. The measure of the second angle is $54 - 9 = 45°$. The measure of the third angle is $54 + 3(9) = 54 + 27 = 81°$.

Note to teacher: There are several ways to solve problems like these. For example, a student may let x be the size of the first angle, and write the size of the other angles accordingly. Either way, make sure that students are defining their symbols and correctly using the properties of equality to solve.

Problem Set Sample Solutions

Students practice writing and solving linear equations.

For each of the following problems, write an equation and solve.

1. **The measure of one angle is thirteen less than five times the measure of another angle. The sum of the measures of the two angles is $140°$. Determine the measures of each of the angles.**

   Let x be the size of the one angle. Then the measure of the other angle is $5x - 13$.

$$x + 5x - 13 = 140$$
$$6x - 13 = 140$$
$$6x - 13 + 13 = 140 + 13$$
$$6x = 153$$
$$x = 25.5$$

   The measure of one angle is $25.5°$ and the measure of the other angle is $140 - 25.5 = 114.5°$.

2. An angle measures seventeen more than three times a number. Its supplement is three more than seven times the number. What is the measure of each angle?

Let x be the number. Then one angle is $3x + 17$. The other angle is $7x + 3$. Since the angles are supplementary, the sum of their measures will be 180.

$$3x + 17 + 7x + 3 = 180$$
$$10x + 20 = 180$$
$$10x + 20 - 20 = 180 - 20$$
$$10x = 160$$
$$x = 16$$

One angle is $3(16) + 17 = 65°$. The other angle is $180 - 65 = 115°$.

3. The angles of a triangle are described as follows: $\angle A$ is the largest angle, its measure is twice the measure of $\angle B$. The measure of $\angle C$ is 2 less than half the measure of $\angle B$. Find the measures of the three angles.

Let x be the measure of $\angle B$. Then the measure of $\angle A = 2x$ and $\angle C = \frac{x}{2} - 2$. The sum of the measures of the angles must be 180.

$$x + 2x + \frac{x}{2} - 2 = 180$$
$$3x + \frac{x}{2} - 2 + 2 = 180 + 2$$
$$3x + \frac{x}{2} = 182$$
$$\frac{6x}{2} + \frac{x}{2} = 182$$
$$\frac{7x}{2} = 182$$
$$7x = 364$$
$$x = 52$$

The measures of the angles are as follows: $\angle A = 104°, \angle B = 52°,$ and $\angle C = \frac{52}{2} - 2 = 26 - 2 = 24°$.

4. A pair of corresponding angles are described as follows: the measure of one angle is five less than seven times a number and the measure of the other angle is eight more than seven times the number. Are the angles congruent? Why or why not?

Let x be the number. Then measure of one angle is $7x - 5$ and the measure of the other angle is $7x + 8$. Assume they are congruent, which means their measures are equal.

$$7x - 5 = 7x + 8$$
$$7x - 7x - 5 = 7x - 7x + 8$$
$$-5 \neq 8$$

Since $-5 \neq 8$, the angles are not congruent.

5. The measure of one angle is eleven more than four times a number. Another angle is twice the first angle's measure. The sum of the measures of the angles is $195°$. What is the measure of each angle?

Let x be the number. The measure of one angle can be represented with $4x + 11$ and the other angle's measure can be represented as $2(4x + 11) = 8x + 22$.

$$4x + 11 + 8x + 22 = 195$$
$$12x + 33 = 195$$
$$12x + 33 - 33 = 195 - 33$$
$$12x = 162$$
$$x = 13.5$$

The measure of one angle is $4(13.5) + 11 = 54 + 11 = 65°$ and the measure of the other angle is $130°$.

6. Three angles are described as follows: $\angle B$ is half the size of $\angle A$. The measure of $\angle C$ is equal to one less than 2 times the measure of $\angle B$. The sum of $\angle A$ and $\angle B$ is 114. Can the three angles form a triangle? Why or why not?

Let x represent the measure of $\angle A$. Then the measure of $\angle B = \frac{x}{2}$ and the measure of $\angle C = 2\left(\frac{x}{2}\right) - 1 = x - 1$.

The sum of $\angle A + \angle B = 114$.

$$x + \frac{x}{2} = 114$$
$$\frac{3x}{2} = 114$$
$$3x = 228$$
$$x = 76$$

The measure of $\angle A = 76°$, $\angle B = \frac{76}{2} = 38°$, and $\angle C = 75°$. The sum of the three angles is $76 + 38 + 75 = 189$.

Since the sum of the measures of the interior angles of a triangle must sum to $180°$, these angles do not form a triangle. Their sum is too large.