Lesson 11: Constant Rate

Student Outcomes

- Students know the definition of constant rate in varied contexts as expressed using two variables where one is t representing a time interval.
- Students graph points on a coordinate plane related to constant rate problems.

Classwork

Example 1 (6 minutes)

Give students the first question below and allow them time to work. Ask them to share their solutions with the class and then proceed with the discussion, table, and graph to finish Example 1.

> **Example 1**
>
> **Pauline mows a lawn at a constant rate. Suppose she mows a 35 square foot lawn in 2.5 minutes. What area, in square feet, can she mow in 10 minutes? t minutes?**

- What is Pauline's average rate in 2.5 minutes?
  - *Pauline's average rate in 2.5 minutes is $\frac{35}{2.5}$ square feet per minute.*
- What is Pauline's average rate in 10 minutes?
  - *Let A represent area mowed. Pauline's average rate in 10 minutes is $\frac{A}{10}$ square feet per mintute.*
- Since Pauline mows at a constant rate, C, then $\frac{35}{2.5} = C$ and $\frac{A}{10} = C$. Therefore,

$$\frac{35}{2.5} = \frac{A}{10}$$
$$350 = 2.5A$$
$$\frac{350}{2.5} = \frac{2.5}{2.5}A$$
$$140 = A$$

Pauline mows 140 square feet of lawn in 10 minutes.

- If we let y represent the number of square feet Pauline can mow in t minutes, then Pauline's average rate in t minutes is $\frac{y}{t}$ square feet per minute.
- Write the two variable equation that represents the area of lawn, y, Pauline can mow in t minutes.

Sample student work:

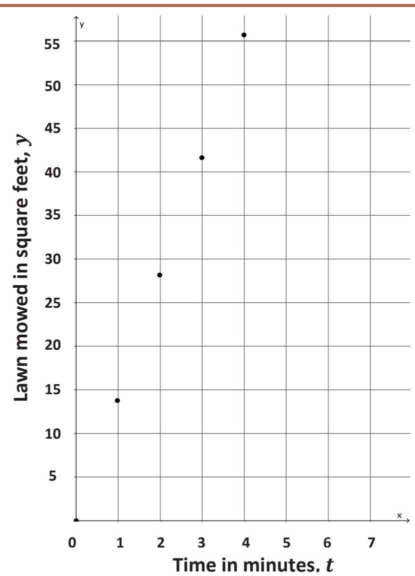
$$\frac{35}{2.5} = \frac{y}{t}$$
$$2.5y = 35t$$
$$\frac{2.5}{2.5}y = \frac{35}{2.5}t$$
$$y = \frac{35}{2.5}t$$

MP.7

- What is the meaning of $\frac{35}{2.5}$ in the equation $y = \frac{35}{2.5}t$?
  - *The number* $\frac{35}{2.5}$ *represents the rate at which Pauline can mow a lawn.*
- We can organize the information in a table:

t (time in minutes)	Linear equation: $y = \frac{35}{2.5}t$	y (area in square feet)
0	$y = \frac{35}{2.5}(0)$	0
1	$y = \frac{35}{2.5}(1)$	$\frac{35}{2.5} = 14$
2	$y = \frac{35}{2.5}(2)$	$\frac{70}{2.5} = 28$
3	$y = \frac{35}{2.5}(3)$	$\frac{105}{2.5} = 42$
4	$y = \frac{35}{2.5}(4)$	$\frac{140}{2.5} = 56$

- On a coordinate plane, we will let the x-axis represent time t, in minutes, and the y-axis represent the area of mowed lawn in square feet. Then we have the following graph:



- Because Pauline mows at a constant rate, we would expect the square feet of mowed lawn to continue to rise as the time in minutes increases.

Concept Development (6 minutes)

- In the last lesson, we learned about average speed and constant speed. Constant speed problems are just a special case of a larger variety of problems known as constant rate problems. Some of these problems such as water pouring out of a faucet into a tub, painting a house, and mowing a lawn were topics in Grade 7.
- First, we define the average rate:

  Suppose V gallons of water flow from a faucet in a given time interval t (minutes). Then the *average rate* of water flow in the given time interval is $\frac{V}{t}$ in gallons per minute.
- Then, we define the constant rate:

  Suppose the average rate of water flow is the same constant C for *any* given time interval. Then we say that the water is flowing at a *constant rate,* C.
- Similarly, suppose A square feet of lawn are mowed in a given time interval t (minutes). Then the *average rate* of lawn mowing in the given time interval is $\frac{A}{t}$ square feet per minute. If we assume that the average rate of lawn mowing is the same constant C for *any* given time interval, then we say that the lawn is mowed at a constant rate, C.
- Describe the average rate of painting a house.
  - *Suppose A square feet of house are painted in a given time interval t (minutes). Then the average rate of house painting in the given time interval is $\frac{A}{t}$ square feet per minute.*
- Describe the constant rate of painting a house.
  - *If we assume that the average rate of house painting is the same constant C over any given time interval, then we say that the wall is painted at a constant rate, C.*
- What is the difference between average rate and constant rate?
  - *Average rate is the rate in which something can be done over a specific time interval. Constant rate assumes that the average rate is the same over any time interval.*
- As you can see, the way we define average rate and constant rate for a given situation is very similar. In each case, a transcription of the given information leads to an expression in two variables.

Example 2 (8 minutes)

> **Example 2**
>
> **Water flows at a constant rate out of a faucet. Suppose the volume of water that comes out in three minutes is 10.5 gallons. How many gallons of water comes out of the faucet in t minutes?**

- Write the linear equation that represents the volume of water, V, that comes out in t minutes.

Let C represent the constant rate of water flow.

$\frac{10.5}{3} = C$ *and* $\frac{V}{t} = C$, *then* $\frac{10.5}{3} = \frac{V}{t}$.

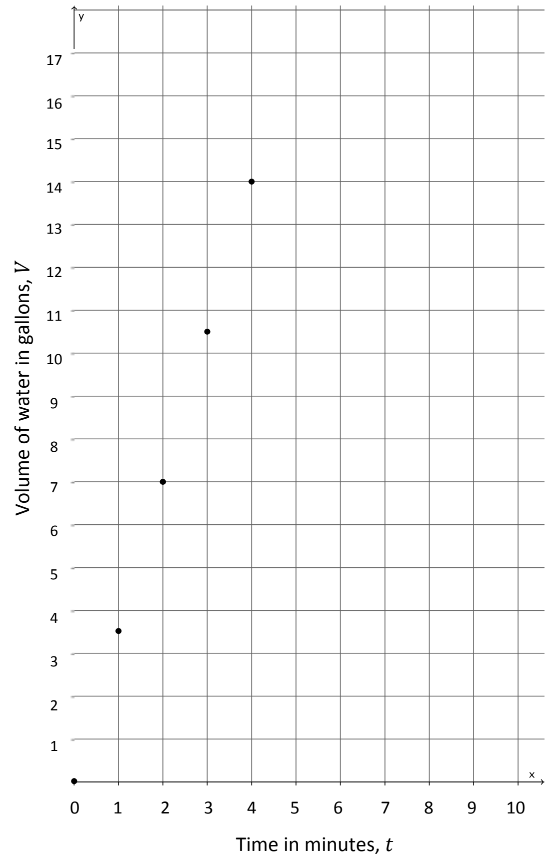
$$\frac{10.5}{3} = \frac{V}{t}$$
$$3V = 10.5t$$
$$\frac{3}{3}V = \frac{10.5}{3}t$$
$$V = \frac{10.5}{3}t$$

MP.7

- What is the meaning of the number $\frac{10.5}{3}$ in the equation $V = \frac{10.5}{3}t$?
  - *The number* $\frac{10.5}{3}$ *represents the rate at which water flows from a faucet.*
- Using the linear equation $V = \frac{10.5}{3}t$, complete the table:

t (time in minutes)	Linear equation: $V = \frac{10.5}{3}t$	V (in gallons)
0	$V = \frac{10.5}{3}(0)$	0
1	$V = \frac{10.5}{3}(1)$	$\frac{10.5}{3} = 3.5$
2	$V = \frac{10.5}{3}(2)$	$\frac{21}{3} = 7$
3	$V = \frac{10.5}{3}(3)$	$\frac{31.5}{3} = 10.5$
4	$V = \frac{10.5}{3}(4)$	$\frac{42}{3} = 14$

- On a coordinate plane, we will let the x-axis represent time t in minutes and the y-axis represent the volume of water. Graph the data from the table.



- Using the graph, about how many gallons of water do you think would flow after $1\frac{1}{2}$ minutes? Explain.
  - *After $1\frac{1}{2}$ minutes, between $3\frac{1}{2}$ and 7 gallons of water will flow. Since the water is flowing at a constant rate, we can expect the volume of water to rise between 1 and 2 minutes. The number of gallons that flows after $1\frac{1}{2}$ minutes then would have to be between the number of gallons that flows out after 1 minute and 2 minutes.*
- Using the graph, about how long would it take for 15 gallons of water to flow out of the faucet? Explain.
  - *It would take between 4 and 5 minutes for 15 gallons of water to flow out of the faucet. It takes 4 minutes for 14 gallons to flow; therefore, it must take more than 4 minutes for 15 gallons to come out. It must take less than 5 minutes because $3\frac{1}{2}$ gallons flow out every minute.*
- Graphing proportional relationships like these last two constant rate problems provide us more information than simply solving an equation and calculating one value. The graph provides information that is not so obvious in an equation.

Exercises 1–3 (15 minutes)

Students complete Exercises 1–3 independently.

Exercises 1–3

1. **Jesus types at a constant rate. He can type a full page of text in $3\frac{1}{2}$ minutes. We want to know how many pages, p, Jesus can type after t minutes.**

   a. **Write the linear equation in two variables that represents the number of pages Jesus types in any given time interval.**

   Let C represent the constant rate that Jesus types. Then,

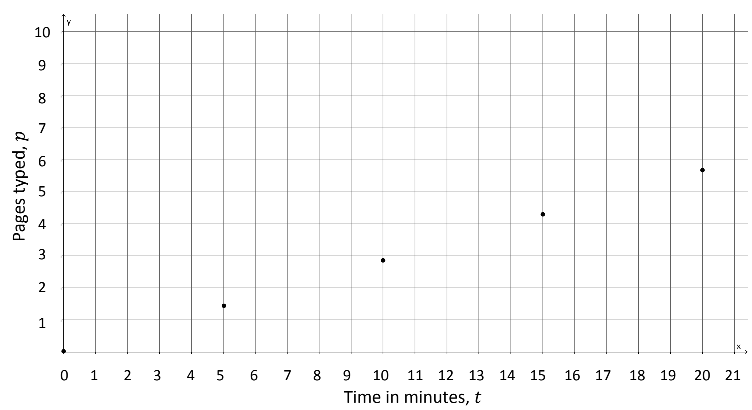
   $\frac{1}{3.5} = C$ *and* $\frac{p}{t} = C$, *therefore* $\frac{1}{3.5} = \frac{p}{t}$.

$$\frac{1}{3.5} = \frac{p}{t}$$
$$3.5p = t$$
$$\frac{3.5}{3.5}p = \frac{1}{3.5}t$$
$$p = \frac{1}{3.5}t$$

   b. **Complete the table below. Use a calculator and round answers to the tenths place.**

t (time in minutes)	Linear equation: $p = \frac{1}{3.5}t$	p (pages typed)
0	$p = \frac{1}{3.5}(0)$	0
5	$p = \frac{1}{3.5}(5)$	$\frac{5}{3.5} \cong 1.4$
10	$p = \frac{1}{3.5}(10)$	$\frac{10}{3.5} \cong 2.9$
15	$p = \frac{1}{3.5}(15)$	$\frac{15}{3.5} \cong 4.3$
20	$p = \frac{1}{3.5}(20)$	$\frac{20}{3.5} \cong 5.7$

   c. **Graph the data on a coordinate plane.**



d. **About how long would it take Jesus to type a 5-page paper? Explain.**

It would take him between 15 and 20 minutes. After 15 minutes he will have typed 4.3 pages. In 20 minutes he can type 5.7 pages. Since 5 pages is between 4.3 and 5.7, then it will take him between 15 and 20 minutes.

2. **Emily paints at a constant rate. She can paint 32 square feet in five minutes. What area, A, can she paint in t minutes?**

a. **Write the linear equation in two variables that represents the number of square feet Emily can paint in any given time interval.**

Let C be the constant rate that Emily paints. Then,

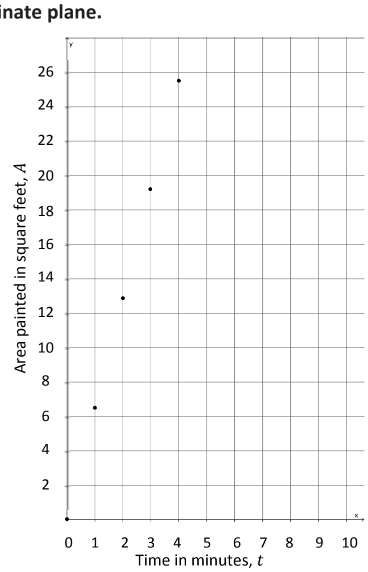
$\frac{32}{5} = C$ *and* $\frac{A}{t} = C$, *therefore* $\frac{32}{5} = \frac{A}{t}$.

$$\frac{32}{5} = \frac{A}{t}$$
$$5A = 32t$$
$$\frac{5}{5}A = \frac{32}{5}t$$
$$A = \frac{32}{5}t$$

b. **Complete the table below. Use a calculator and round answers to the tenths place.**

t (time in minutes)	Linear equation: $A = \frac{32}{5}t$	A (area painted in square feet)
0	$A = \frac{32}{5}(0)$	0
1	$A = \frac{32}{5}(1)$	$\frac{32}{5} = 6.4$
2	$A = \frac{32}{5}(2)$	$\frac{64}{5} = 12.8$
3	$A = \frac{32}{5}(3)$	$\frac{96}{5} = 19.2$
4	$A = \frac{32}{5}(4)$	$\frac{128}{5} = 25.6$

c. **Graph the data on a coordinate plane.**



d. About how many square feet can Emily paint in $2\frac{1}{2}$ minutes? Explain.

Emily can paint between 12.8 and 19.2 square feet in $2\frac{1}{2}$ minutes. After 2 minutes she paints 12.8 square feet and at 3 minutes she will have painted 19.2 square feet.

3. Joseph walks at a constant speed. He walked to the store, one-half mile away, in 6 minutes. How many miles, m, can he walk in t minutes?

a. Write the linear equation in two variables that represents the number of miles Joseph can walk in any given time interval, t.

Let C be the constant rate that Joseph walks. Then,

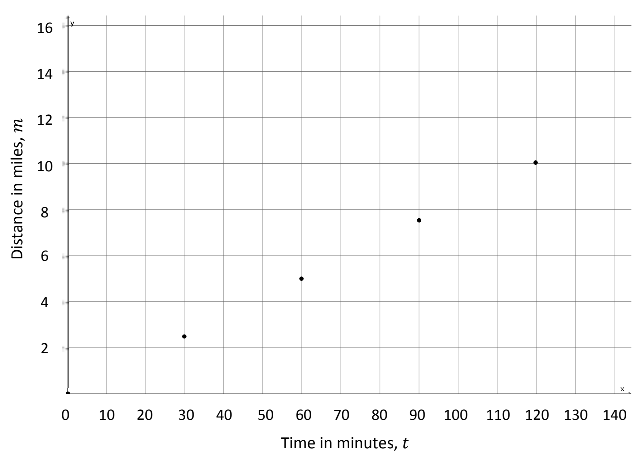
$\frac{0.5}{6} = C$ *and* $\frac{m}{t} = C$, *therefore* $\frac{0.5}{6} = \frac{m}{t}$.

$$\frac{0.5}{6} = \frac{m}{t}$$
$$6m = 0.5t$$
$$\frac{6}{6}m = \frac{0.5}{6}t$$
$$m = \frac{0.5}{6}t$$

b. Complete the table below. Use a calculator and round answers to the tenths place.

t (time in minutes)	Linear equation: $m = \frac{0.5}{6}t$	m (distance in miles)
0	$m = \frac{0.5}{6}(0)$	0
30	$m = \frac{0.5}{6}(30)$	$\frac{15}{6} = 2.5$
60	$m = \frac{0.5}{6}(60)$	$\frac{30}{6} = 5$
90	$m = \frac{0.5}{6}(90)$	$\frac{45}{6} = 7.5$
120	$m = \frac{0.5}{6}(120)$	$\frac{60}{6} = 10$

c. Graph the data on a coordinate plane.



d. **Joseph's friend lives 4 miles away from him. About how long would it take Joseph to walk to his friend's house? Explain.**

It will take Joseph a little less than an hour to walk to his friend's house. Since it takes 30 minutes for him to walk 2.5 miles, and 60 minutes to walk 5 miles, and 4 is closer to 5 than 2.5, it will take Joseph less than an hour to walk the 4 miles.

Closing (5 minutes)

Summarize, or ask students to summarize, the main points from the lesson:

- Constant rate problems appear in a variety of contexts like painting a house, typing, walking, water flow, etc.
- We can express the constant rate as a two variable equation representing proportional change.
- We can graph the constant rate situation by completing a table to compute data points.

Lesson Summary

When constant rate is stated for a given problem, then you can express the situation as a two variable equation. The equation can be used to complete a table of values that can then be graphed on a coordinate plane.

Exit Ticket (5 minutes)

Name ____ Date ____________________

Lesson 11: Constant Rate

Exit Ticket

1. Vicky reads at a constant rate. She can read 5 pages in 9 minutes. We want to know how many pages, p, Vicky can read after t minutes.

   a. Write a linear equation in two variables that represents the number of pages Vicky reads in any given time interval.

   b. Complete the table below. Use a calculator and round answers to the tenths place.

t (time in minutes)	Linear equation:	p (pages read)
0		
20		
40		
60		

   c. About how long would it take Vicky to read 25 pages? Explain.

Exit Ticket Sample Solutions

1. Vicky reads at a constant rate. She can read 5 pages in 9 minutes. We want to know how many pages, p, Vicky can read after t minutes.

   a. Write a linear equation in two variables that represents the number of pages Vicky reads in any given time interval.

   Let C represent the constant rate that Vicky reads. Then,

   $\frac{5}{9} = C$ *and* $\frac{p}{t} = C$*, therefore* $\frac{5}{9} = \frac{p}{t}$.

$$\frac{5}{9} = \frac{p}{t}$$
$$9p = 5t$$
$$\frac{9}{9}p = \frac{5}{9}t$$
$$p = \frac{5}{9}t$$

   b. Complete the table below. Use a calculator and round answers to the tenths place.

t (time in minutes)	Linear equation: $p = \frac{5}{9}t$	p (pages read)
0	$p = \frac{5}{9}(0)$	0
20	$p = \frac{5}{9}(20)$	$\frac{100}{9} \cong 11.1$
40	$p = \frac{5}{9}(40)$	$\frac{200}{9} \cong 22.2$
60	$p = \frac{5}{9}(60)$	$\frac{300}{9} \cong 33.3$

   c. About how long would it take Vicky to read 25 pages? Explain.

   It would take her a little over 40 minutes. After 40 minutes she can read about 22.2 pages, and after 1 hour she can read about 33.3 pages. Since 25 pages is between 22.2 and 33.3, then it will take her between 40 and 60 minutes to read 25 pages.

Problem Set Sample Solutions

Students practice writing two variable equations that represent a constant rate.

1. A train travels at a constant rate of 45 miles per hour.

   a. What is the distance, d, in miles, that the train travels in t hours?

   Let C be the constant rate the train travels. Then, $\frac{45}{1} = C$ *and* $\frac{d}{t} = C$*; therefore,* $\frac{45}{1} = \frac{d}{t}$.

$$\frac{45}{1} = \frac{d}{t}$$
$$d = 45t$$

b. How many miles will it have traveled in 2.5 hours?

$$d = 45(2.5)$$
$$= 112.5$$

The train will travel 112.5 miles in 2.5 hours.

2. Water is leaking from a faucet at a constant rate of $\frac{1}{3}$ gallons per minute.

a. What is the amount of water, w, that is leaked from the faucet after t minutes?

Let C be the constant rate the water leaks from the faucet. Then,

$\frac{\frac{1}{3}}{1} = C$ *and* $\frac{w}{t} = C$; *therefore,* $\frac{\frac{1}{3}}{1} = \frac{w}{t}$.

$$\frac{\frac{1}{3}}{1} = \frac{w}{t}$$
$$w = \frac{1}{3}t$$

b. How much water is leaked after an hour?

$$w = \frac{1}{3}t$$
$$= \frac{1}{3}(60)$$
$$= 20$$

The faucet will leak 20 gallons in one hour.

3. A car can be assembled on an assembly line in 6 hours. Assume that the cars are assembled at a constant rate.

a. How many cars, y, can be assembled in t hours?

Let C be the constant rate the cars are assembled. Then,

$\frac{1}{6} = C$ *and* $\frac{y}{t} = C$, *therefore* $\frac{1}{6} = \frac{y}{t}$.

$$\frac{1}{6} = \frac{y}{t}$$
$$6y = t$$
$$\frac{6}{6}y = \frac{1}{6}t$$
$$y = \frac{1}{6}t$$

b. How many cars can be assembled in a week?

A week is $24 \times 7 = 268$ hours. So $y = \frac{1}{6}(168) = 28$. Twenty-eight cars can be assembled in a week.

4. A copy machine makes copies at a constant rate. The machine can make 80 copies in $2\frac{1}{2}$ minutes.

a. Write an equation to represent the number of copies, n, that can be made over any time interval, t.

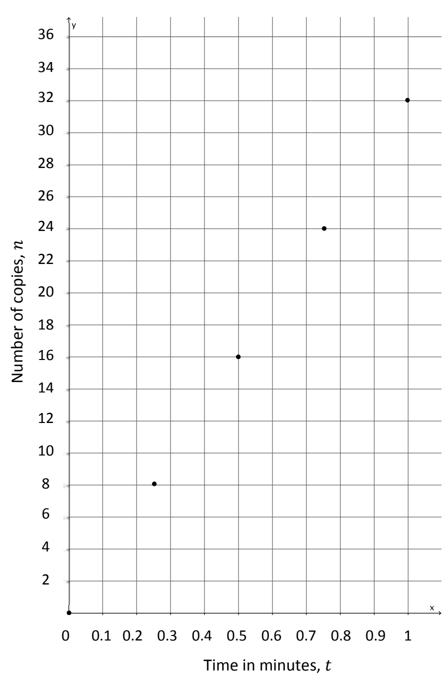
Let C be the constant rate that copies can be made. Then $\frac{80}{2\frac{1}{2}} = C$ and $\frac{n}{t} = C$, therefore $\frac{80}{2\frac{1}{2}} = \frac{n}{t}$.

$$\frac{80}{2\frac{1}{2}} = \frac{n}{t}$$
$$2\frac{1}{2}n = 80t$$
$$\frac{5}{2}n = 80t$$
$$\frac{2}{5}\cdot\frac{5}{2}n = \frac{2}{5}\cdot 80t$$
$$n = 32t$$

b. Complete the table below.

t (time in minutes)	Linear equation: $n = 32t$	n (number of copies)
0	$n = 32(0)$	0
0.25	$n = 32(0.25)$	8
0.5	$n = 32(0.5)$	16
0.75	$n = 32(0.75)$	24
1	$n = 32(1)$	32

c. Graph the data on a coordinate plane.



d. The copy machine runs for 20 seconds, then jams. About how many copies were made before the jam occurred? Explain.

Since 20 seconds is 0.3 of a minute, then the number of copies made will be between 8 and 16 because 0.3 is between 0.25 and 0.5.

5. Connor runs at a constant rate. It takes him 34 minutes to run four miles.

a. Write the linear equation in two variables that represents the number of miles Connor can run in any given time interval, t.

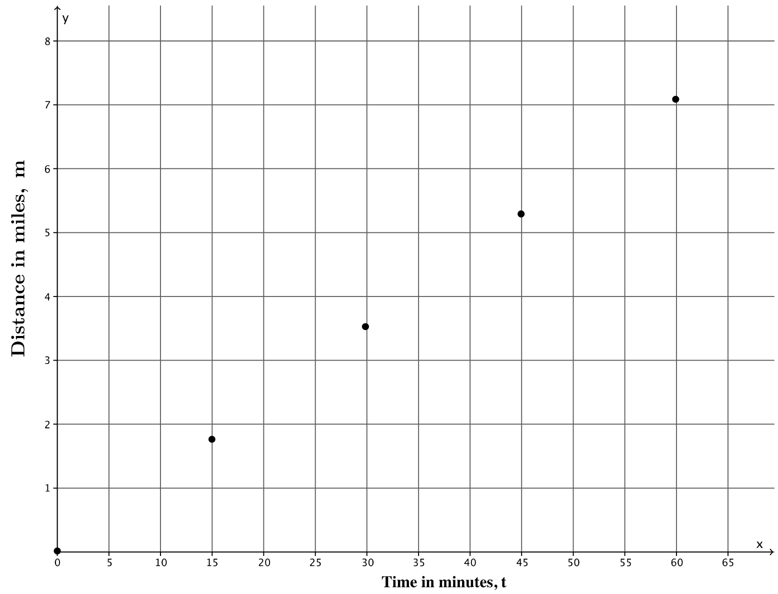
Let C be the constant rate that Connor runs and m represent the number of miles. Then $\frac{4}{34} = C$ and $\frac{m}{t} = C$, therefore $\frac{4}{34} = \frac{m}{t}$.

$$\frac{4}{34} = \frac{m}{t}$$
$$34m = 4t$$
$$\frac{34}{34}m = \frac{4}{34}t$$
$$m = \frac{4}{34}t$$
$$m = \frac{2}{17}t$$

b. Complete the table below. Use a calculator and round answers to the tenths place.

t (time in minutes)	Linear equation: $m = \frac{2}{17}t$	m (distance in miles)
0	$m = \frac{2}{17}(0)$	0
15	$m = \frac{2}{17}(15)$	$\frac{30}{17} \cong 1.8$
30	$m = \frac{2}{17}(30)$	$\frac{60}{17} \cong 3.5$
45	$m = \frac{2}{17}(45)$	$\frac{90}{17} \cong 5.3$
60	$m = \frac{2}{17}(60)$	$\frac{120}{17} \cong 7.1$

c. **Graph the data on a coordinate plane.**



d. **Connor ran for 40 minutes before tripping and spraining his ankle. About how many miles did he run before he had to stop? Explain.**

Since Connor ran for 40 minutes, that means that he ran more than 3.5 miles, but less than 5.3 miles. Since 40 is between 30 and 45, then we can use those reference points to make an estimate of how many miles he ran in 40 minutes, probably about 5 miles.