Lesson 17: The Line Joining Two Distinct Points of the Graph $y = mx + b$ has Slope m

Student Outcomes

- Students show that the slope of a line joining any two distinct points of the graph of $y = mx + b$ has slope, m.
- Students transform the standard form of an equation into $y = -\frac{a}{b}x + \frac{c}{b}$.

Lesson Notes

In the previous lesson, we determined that slope can be calculated using any two points on the same line. In this lesson, we show that equations of the form $y = mx$ and $y = mx + b$ generate lines with slope m. Students need graph paper to complete some of the Exercises and Problem Set items.

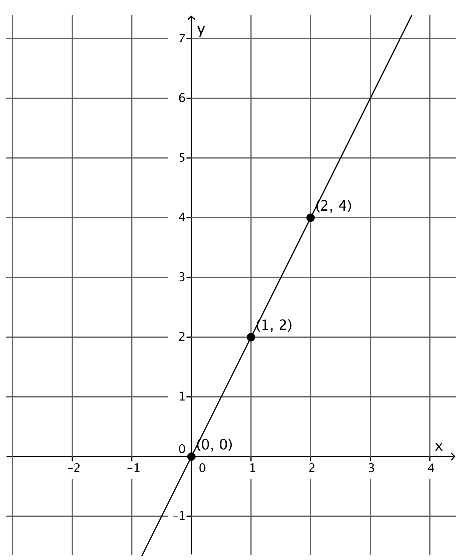
Classwork

Exercises 1–3 (8 minutes)

Students work independently to complete Exercises 1–3.

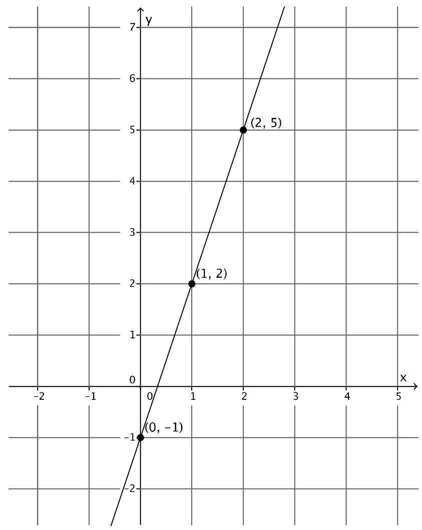
Exercises

1. **Find at least three solutions to the equation $y = 2x$, and graph the solutions as points on the coordinate plane. Connect the points to make a line. Find the slope of the line.**



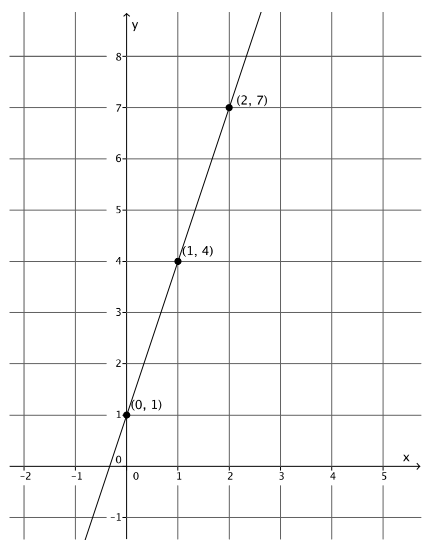
The slope of the line is 2, $m = 2$.

2. **Find at least three solutions to the equation $y = 3x - 1$, and graph the solutions as points on the coordinate plane. Connect the points to make a line. Find the slope of the line.**



The slope of the line is 3, $m = 3$.

3. **Find at least three solutions to the equation $y = 3x + 1$, and graph the solutions as points on the coordinate plane. Connect the points to make a line. Find the slope of the line.**



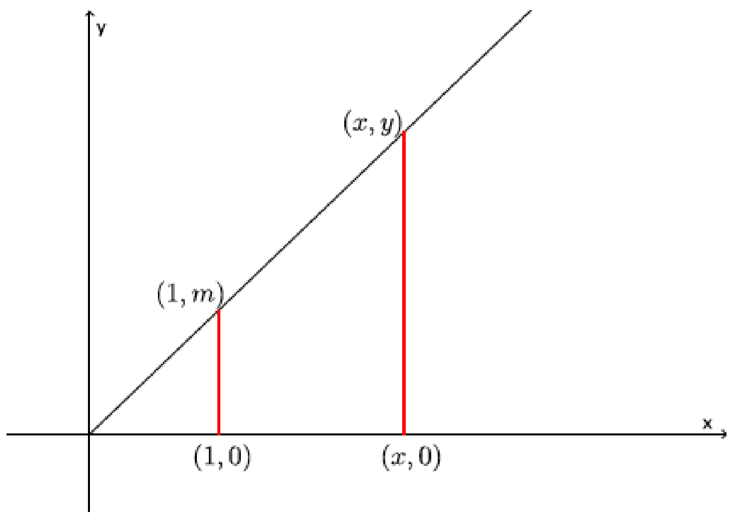
The slope of the line is 3, $m = 3$.

Discussion (12 minutes)

Recall the goal from Lesson 15: We want to prove that the graph of a linear equation is a line. To do so, we needed some tools, specifically two related to slope. Now that facts are known about slope, we will focus on showing that the line that joins two distinct points is a linear equation with slope m.

MP.2

- We know from our previous work with slope that when the horizontal distance between two points is fixed at one, then the slope of the line is the difference in the y-coordinates. We also know that when the horizontal distance is not fixed at one, we can find the slope of the line using any two points because the ratio of corresponding sides of similar triangles will be equal. We can put these two facts together to prove that the graph of the line $y = mx$ has slope m. Consider the diagram below:



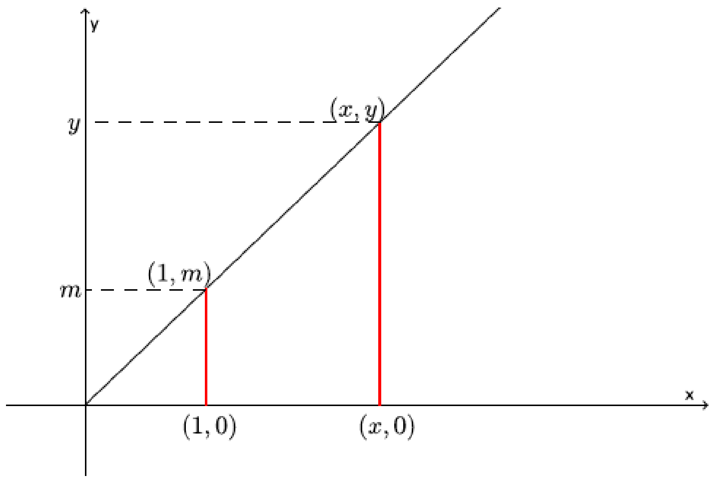
- Examine the diagram and think of how we could prove that $\frac{y}{m} = \frac{x}{1}$.

Provide students time to work independently, then time for them to discuss in pairs a possible proof of $\frac{y}{m} = \frac{x}{1}$. If necessary, use the four bullet points below to guide students' thinking.

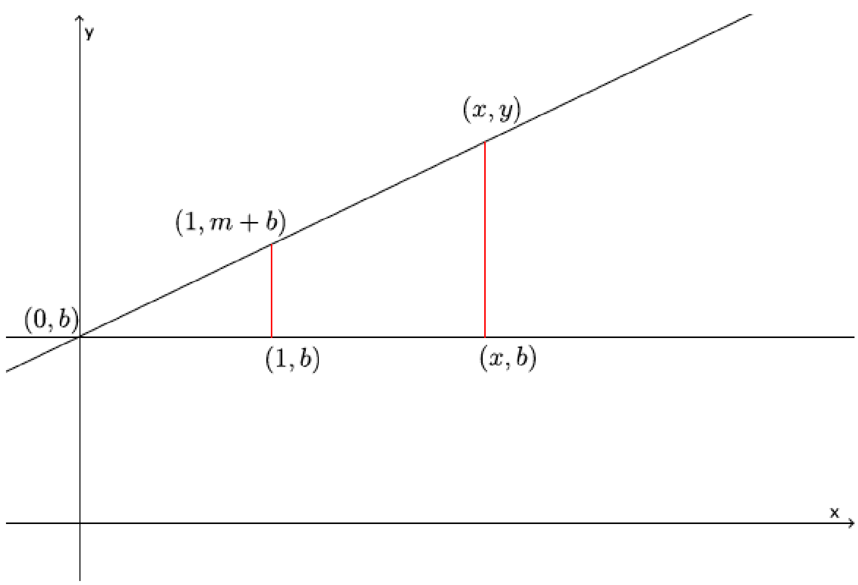
- Do we have similar triangles? Explain.
  - *Yes. Each of the triangles has a common angle at the origin, and each triangle has a right angle. By the AA criterion these triangles are similar.*
- What is the slope of the line? Explain.
  - *The slope of the line is m. By our definition of slope and the information in the diagram, when the horizontal distance between two points is fixed at one, the slope is m.*
- Write the ratio of the corresponding sides. Then solve for y.
  - $\frac{y}{m} = \frac{x}{1}$, $y = mx$
- Therefore, the slope of the graph of $y = mx$ is m.

Point students to their work in Exercise 1 where the graph of $y = 2x$ was a line with a slope of 2.

- We know that the graph of $y = mx$ has slope m, where m is a number. The y in the equation $y = mx$ is equal to the difference in y-coordinates as shown in the diagram below:

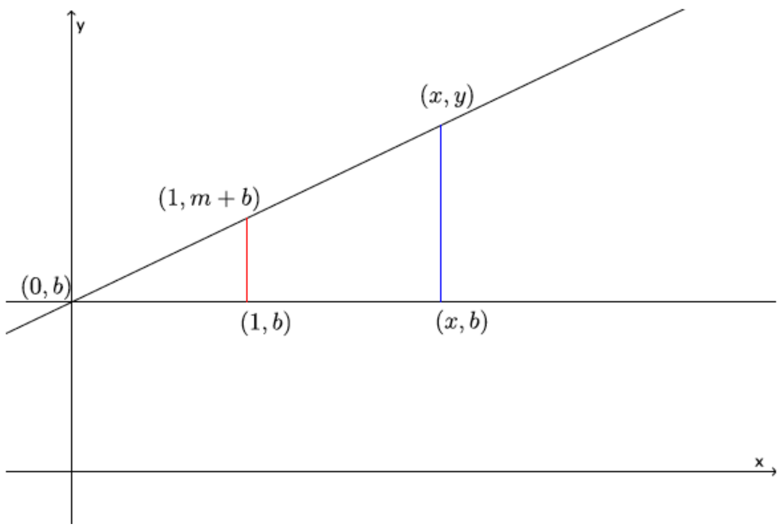


- Consider the diagram below. How does this compare to the graph of $y = mx$ that we just worked on?

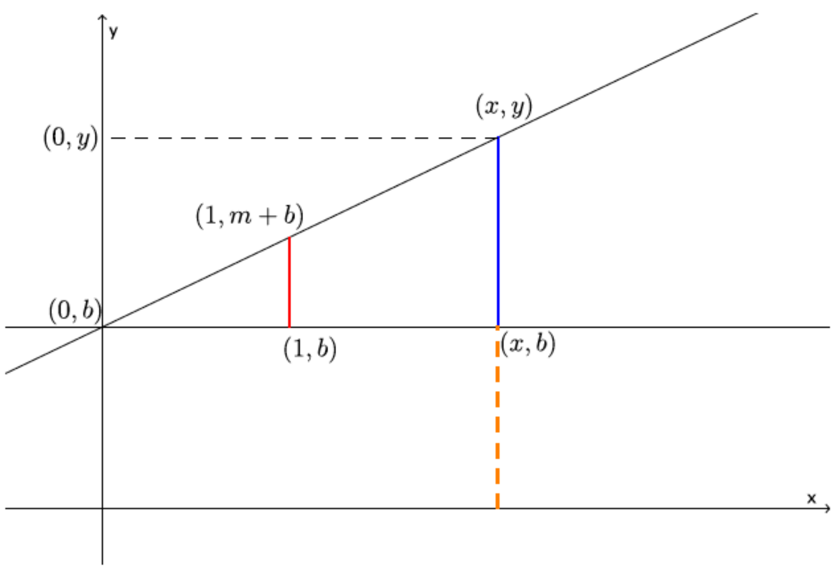


  - *The graph is the same except for the fact that the line has been translated b units up the y-axis.*

- We want to write the ratio of corresponding sides of the slope triangles. We know the lengths of two sides of the smaller slope triangle; they are 1 and m. We also know one of the lengths of the larger slope triangle, x. What we need to do now is express the length of the larger slope triangle that corresponds to side m. When our line passed through the origin, the length was simply y, but that is not the case here. How can we express the length we need, noted in blue in the diagram below?



Provide students time to think and share in small groups. If necessary, show students the diagram below and ask them what the length of the dashed orange segment is. They should recognize that it is the length of b. Then ask them how they can use that information to determine the length of the blue segment.



  - *The length of the blue segment is* $y - b$.

- Now that we know the length of the blue segment, we can write ratios that represent the corresponding sides of the triangles:

$$\frac{m}{y-b} = \frac{1}{x}$$

Now we can solve for y:

$$mx = y - b$$
$$mx + b = y - b + b$$
$$mx + b = y$$

Therefore, the slope of an equation of the form $y = mx + b$ is m.

Point students to their work in Exercises 2 and 3 where the graph of $y = 3x - 1$ and $y = 3x + 1$ was a line with slope 3.

- We can show this algebraically using two distinct points on the graph of the line $y = 3x - 1$. Points $(1, 2)$ and $(2, 5)$ were on the graph of $y = 3x - 1$. Using the slope formula, we see that

$$m = \frac{5-2}{2-1}$$
$$= \frac{3}{1}$$
$$= 3$$

The next four bullet points generalize the slope between *any* two points of the graph of $y = 3x - 1$ is 3. This is an optional part of the discussion of slope between any two points. The last bullet point of this section should be shared with students as it orients them to our progress towards our ultimate goal: showing that the graph of a linear equation is a line.

- In general, let $P = (p_1, p_2)$ and $R = (r_1, r_2)$ be *any* two distinct points of the graph of $y = 3x - 1$. Recall what is known about points on a graph; it means that they are solutions to the equation. Since P is on the graph of $y = 3x - 1$, then

$$p_2 = 3p_1 - 1$$

Similarly, since R is on the graph of $y = 3x - 1$, then

$$r_2 = 3r_1 - 1$$

- By the slope formula, $m = \frac{p_2 - r_2}{p_1 - r_1}$, we can substitute the above values of p_2 and r_2 and use our properties of equality to simplify:

$$m = \frac{p_2 - r_2}{p_1 - r_1}$$

$$= \frac{(3p_1 - 1) - (3r_1 - 1)}{p_1 - r_1}$$ By substitution

$$= \frac{3p_1 - 1 - 3r_1 + 1}{p_1 - r_1}$$ By taking the opposite of the terms in the second grouping

$$= \frac{3p_1 - 3r_1}{p_1 - r_1}$$ By simplifying $(-1 + 1)$

$$= \frac{3(p_1 - r_1)}{p_1 - r_1}$$ By the Distributive Property "collecting like terms"

$$= 3$$ By division, the number $p_1 - r_1$ divided by itself

- Thus we have shown that the line passing through points P and R has a slope of 3.
- To truly generalize our work, we need only replace 3 in the calculations with m and the -1 with b.

Make clear to students that we are still working toward our goal of proving that the graph of a linear equation is a line using the summary of work below.

- Our goal, ultimately, is to prove that the graph of a linear equation is a line. In Lesson 15, we said we have to develop the following tools:
  1. We must define a number for each non-vertical line that can be used to measure the "steepness" or "slant" of the line. Once defined, this number will be called the ***slope*** of the line and is often referred to as the "rate of change".
  2. We must show that *any* two points on a non-vertical line can be used to find the slope of the line.
  3. We must show that the line joining two points on a graph of a linear equation of the form $y = mx + b$ has the slope m.
  4. We must show that there is only one line passing through a given point with a given slope.

  At this point in our work, we just finished #3. Number 4, is the topic of the next lesson.

Discussion (5 minutes)

- When an equation is in the form $y = mx + b$, the slope m is easily identifiable compared to an equation in the form $ax + by = c$. Note that b in each of the equations is unique. That is, the number represented by b in the equation $y = mx + b$ is not necessarily the same as the number b in $ax + by = c$. For example, we will solve the equation $8x + 2y = 6$ for y. Our goal is to have y equal to an expression:

$$8x + 2y = 6$$

First we use our properties of equality to remove $8x$ from the left side of the equation

$$8x - 8x + 2y = 6 - 8x$$
$$2y = 6 - 8x$$

Now we divide both sides by 2

$$\frac{2y}{2} = \frac{6 - 8x}{2}$$
$$y = \frac{6}{2} - \frac{8x}{2}$$
$$y = 3 - 4x$$
$$y = -4x + 3$$

The slope of the graph of this equation is -4.

By convention (an agreed upon way of doing things) we place the term mx before the term b. This is a version of the standard form of a linear equation that is referred to as the "slope-intercept" form. It is called "slope-intercept" form because it makes clear the number that describes slope, i.e., m, and the y-intercept, which is something that will be discussed later. Also, notice the value of b is different in both forms of the equation.

Exercises 4–11 (11 minutes)

Students work independently or in pairs to identify the slope from an equation and to transform the standard form of an equation into slope-intercept form.

4. **The graph of the equation $y = 7x - 3$ has what slope?**

   The slope is 7.

5. **The graph of the equation $y = -\frac{3}{4}x - 3$ has what slope?**

   The slope is $-\frac{3}{4}$.

6. **You have $\$20$ in savings at the bank. Each week, you add $\$2$ to your savings. Let y represent the total amount of money you have saved at the end of x weeks. Write an equation to represent this situation and identify the slope of the equation. What does that number represent?**

   $y = 2x + 20$

   The slope is 2. It represents how much money is saved each week.

7. A friend is training for a marathon. She can run 4 miles in 28 minutes. Assume she runs at a constant rate. Write an equation to represent the total distance, y, your friend can run in x minutes. Identify the slope of the equation. What does that number represent?

$$\frac{y}{x} = \frac{4}{28}$$
$$y = \frac{4}{28}x$$
$$y = \frac{1}{7}x$$

The slope is $\frac{1}{7}$. It represents the rate at which my friend can run, one mile in seven minutes.

8. Four boxes of pencils cost $\$5$. Write an equation that represents the total cost, y, for x boxes of pencils. What is the slope of the equation? What does that number represent?

$$y = \frac{5}{4}x$$

The slope is $\frac{5}{4}$. It represents the cost of one box of pencils, $\$1.25$.

9. Solve the following equation for y: $9x - 3y = 15$, then identify the slope of the line.

$$9x - 3y = 15$$
$$9x - 9x - 3y = 15 - 9x$$
$$-3y = 15 - 9x$$
$$\frac{-3}{-3}y = \frac{15 - 9x}{-3}$$
$$y = \frac{15}{-3} - \frac{9x}{-3}$$
$$y = -5 + 3x$$
$$y = 3x - 5$$

The slope of the line is 3.

10. Solve the following equation for y: $5x + 9y = 6$, then identify the slope of the line.

$$5x + 9y = 8$$
$$5x - 5x + 9y = 8 - 5x$$
$$9y = 8 - 5x$$
$$\frac{9}{9}y = \frac{8 - 5x}{9}$$
$$y = \frac{8}{9} - \frac{5}{9}x$$
$$y = -\frac{5}{9}x + \frac{8}{9}$$

The slope of the line is $-\frac{5}{9}$.

11. **Solve the following equation for y: $ax + by = c$, then identify the slope of the line.**

$$ax + by = c$$
$$ax - ax + by = c - ax$$
$$by = c - ax$$
$$\frac{b}{b}y = \frac{c - ax}{b}$$
$$y = \frac{c}{b} - \frac{ax}{b}$$
$$y = \frac{c}{b} - \frac{a}{b}x$$
$$y = -\frac{a}{b}x + \frac{c}{b}$$

The slope of the line is $-\frac{a}{b}$.

Closing (4 minutes)

Summarize, or ask students to summarize, the main points from the lesson:

- We know that the line that joins any two distinct points of the graph $y = mx + b$ has slope m.
- We know how to identify the slope for any equation in the form $y = mx + b$.
- We know how to transform the standard form of a linear equation into another form to more easily identify the slope.

Lesson Summary

The line joining two distinct points of the graph of the linear equation $y = mx + b$ has slope m.

The m of $y = mx + b$ is the number that describes the slope. For example, in the equation $y = -2x + 4$, the slope of the graph of the line is -2.

Exit Ticket (5 minutes)

Name ______________________________ Date ____________________

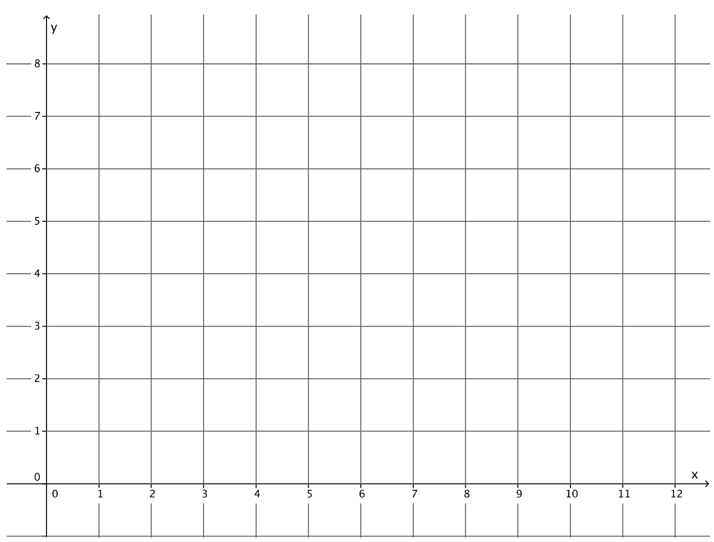
Lesson 17: The Line Joining Two Distinct Points of the Graph $y = mx + b$ has Slope m

Exit Ticket

1. Solve the following equation for y: $35x - 7y = 49$.

2. What is the slope of the equation in problem 1?

3. Show, using similar triangles, why the graph of an equation of the form $y = mx$ is a line with slope m.



Exit Ticket Sample Solutions

1. Solve the following equation for y: $35x - 7y = 49$.

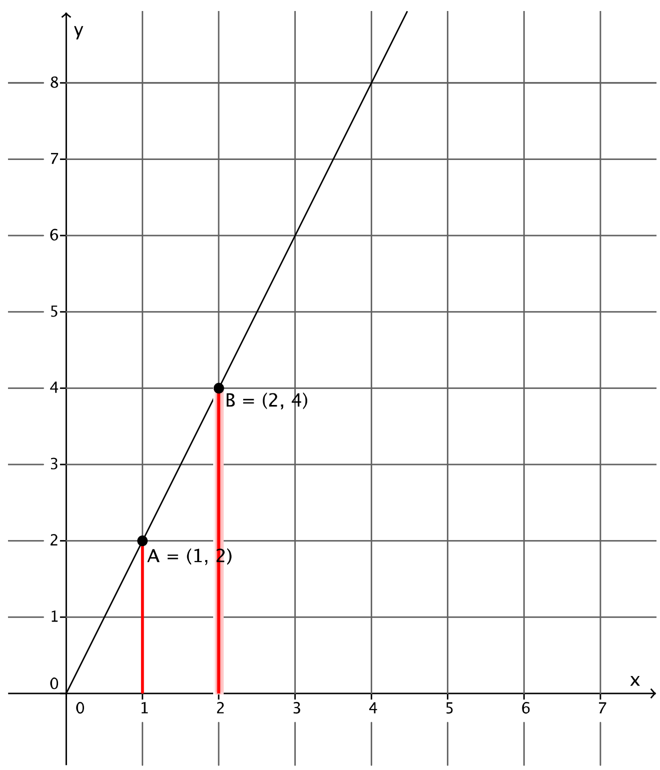
$$35x - 7y = 49$$
$$35x - 35x - 7y = 49 - 35x$$
$$-7y = 49 - 35x$$
$$\frac{-7}{-7}y = \frac{49}{-7} - \frac{35}{-7}x$$
$$y = -7 - (-5x)$$
$$y = 5x - 7$$

2. What is the slope of the equation in problem 1?

The slope of $y = 5x - 7$ is 5.

3. Show, using similar triangles, why the graph of an equation of the form $y = mx$ is a line with slope m.

Solutions will vary. Sample solution is shown below.



The line shown has slope 2. When we compare the corresponding side lengths of the similar triangles we have the ratios $\frac{2}{1} = \frac{4}{2} = 2$. In general, the ratios would be $\frac{x}{1} = \frac{y}{m}$, equivalently $y = mx$, which is a line with slope m.

Problem Set Sample Solutions

Students practice transforming equations from standard form into slope-intercept form and showing that the line joining two distinct points of the graph $y = mx + b$ has slope m. Students graph the equation and informally note the y-intercept.

1. **Solve the following equation for y: $-4x + 8y = 24$.**

$$-4x + 8y = 24$$
$$-4x + 4x + 8y = 24 + 4x$$
$$8y = 24 + 4x$$
$$\frac{8}{8}y = \frac{24}{8} + \frac{4}{8}x$$
$$y = 3 + \frac{1}{2}x$$
$$y = \frac{1}{2}x + 3$$

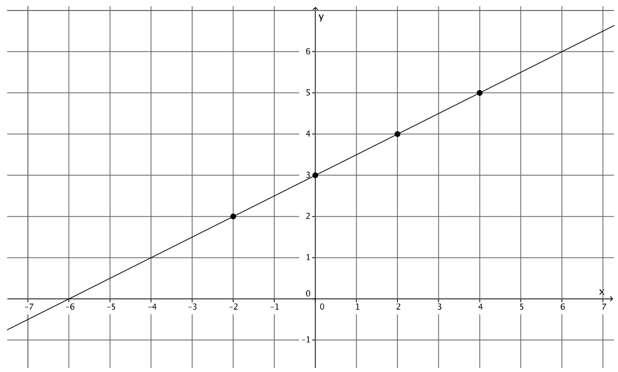
a. **Based on your transformed equation, what is the slope of the linear equation $-4x + 8y = 24$?**

The slope is $\frac{1}{2}$.

b. **Complete the table to find solutions to the linear equation.**

x	Transformed linear equation: $y = \frac{1}{2}x + 3$	y
-2	$y = \frac{1}{2}(-2) + 3$ $= -1 + 3$ $= 2$	2
0	$y = \frac{1}{2}(0) + 3$ $= 0 + 3$ $= 3$	3
2	$y = \frac{1}{2}(2) + 3$ $= 1 + 3$ $= 4$	4
4	$y = \frac{1}{2}(4) + 3$ $= 2 + 3$ $= 5$	5

c. **Graph the points on the coordinate plane.**



d. **Find the slope between any two points.**

Using points $(0, 3)$ *and* $(2, 4)$*:*

$$m = \frac{4-3}{2-0}$$
$$= \frac{1}{2}$$

e. **The slope you found in part (d) should be equal to the slope you noted in part (a). If so, connect the points to make the line that is the graph of an equation of the form $y = mx + b$ that has slope m.**

f. **Note the location (ordered pair) that describes where the line intersects the y-axis.**

$(0, 3)$ is the location where the line intersects the y-axis.

2. **Solve the following equation for y: $9x + 3y = 21$.**

$$9x + 3y = 21$$
$$9x - 9x + 3y = 21 - 9x$$
$$3y = 21 - 9x$$
$$\frac{3}{3}y = \frac{21}{3} - \frac{9}{3}x$$
$$y = 7 - 3x$$
$$y = -3x + 7$$

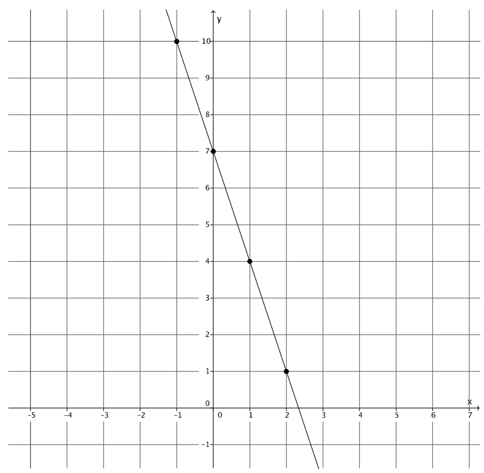
a. **Based on your transformed equation, what is the slope of the linear equation $9x + 3y = 21$?**

The slope is -3.

b. **Complete the table to find solutions to the linear equation.**

x	Transformed linear equation: $y = -3x + 7$	y
-1	$y = -3(-1) + 7$ $= 3 + 7$ $= 10$	10
0	$y = -3(0) + 7$ $= 0 + 7$ $= 7$	7
1	$y = -3(1) + 7$ $= -3 + 7$ $= 4$	4
2	$y = -3(2) + 7$ $= -6 + 7$ $= 1$	1

c. **Graph the points on the coordinate plane.**



d. **Find the slope between any two points.**

Using points $(1, 4)$ *and* $(2, 1)$*:*

$$m = \frac{4-1}{1-2} = \frac{3}{-1} = -3$$

e. **The slope you found in part (d) should be equal to the slope you noted in part (a). If so, connect the points to make the line that is the graph of an equation of the form $y = mx + b$ that has slope m.**

f. **Note the location (ordered pair) that describes where the line intersects the y-axis.**

$(0, 7)$ is the location where the line intersects the y-axis.

3. Solve the following equation for y: $2x + 3y = -6$.

$$2x + 3y = -6$$
$$2x - 2x + 3y = -6 - 2x$$
$$3y = -6 - 2x$$
$$\frac{3}{3}y = \frac{-6}{3} - \frac{2}{3}x$$
$$y = -2 - \frac{2}{3}x$$
$$y = -\frac{2}{3}x - 2$$

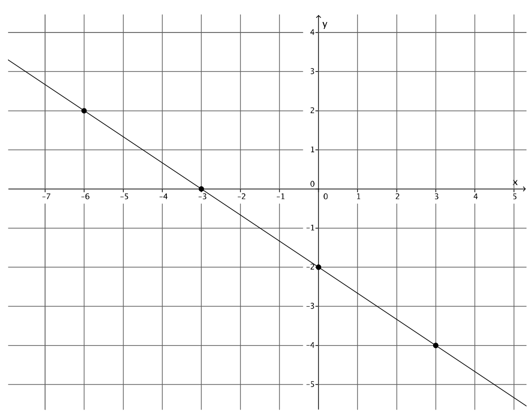
a. Based on your transformed equation, what is the slope of the linear equation $2x + 3y = -6$?

The slope is $-\frac{2}{3}$.

b. Complete the table to find solutions to the linear equation.

x	Transformed linear equation: $y = -\frac{2}{3}x - 2$	y
-6	$y = -\frac{2}{3}(-6) - 2$ $= 4 - 2$ $= 2$	2
-3	$y = -\frac{2}{3}(-3) - 2$ $= 2 - 2$ $= 0$	0
0	$y = -\frac{2}{3}(0) - 2$ $= 0 - 2$ $= -2$	-2
3	$y = -\frac{2}{3}(3) - 2$ $= -2 - 2$ $= -4$	-4

c. Graph the points on the coordinate plane.



d. **Find the slope between any two points.**

Using points $(-6,2)$ *and* $(3,-4)$*:*

$$m = \frac{2-(-4)}{-6-3} = \frac{6}{-9} = -\frac{2}{3}$$

e. **The slope you found in part (d) should be equal to the slope you noted in part (a). If so, connect the points to make the line that is the graph of an equation of the form $y = mx + b$ that has slope m.**

f. **Note the location (ordered pair) that describes where the line intersects the y-axis.**

$(0,-2)$ is the location where the line intersects the y-axis.

4. **Solve the following equation for y: $5x - y = 4$.**

$$\begin{aligned} 5x - y &= 4 \\ 5x - 5x - y &= 4 - 5x \\ -y &= 4 - 5x \\ y &= -4 + 5x \\ y &= 5x - 4 \end{aligned}$$

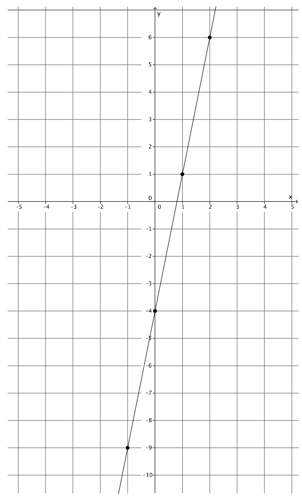
a. **Based on your transformed equation, what is the slope of the linear equation $5x - y = 4$?**

The slope is 5.

b. **Complete the table to find solutions to the linear equation.**

x	Transformed linear equation: $y = 5x - 4$	y
-1	$y = 5(-1) - 4$ $= -5 - 4$ $= -9$	-9
0	$y = 5(0) - 4$ $= 0 - 4$ $= -4$	-4
1	$y = 5(1) - 4$ $= 5 - 4$ $= 1$	1
2	$y = 5(2) - 4$ $= 10 - 4$ $= 6$	6

c. **Graph the points on the coordinate plane.**



d. **Find the slope between any two points.**

Using points $(0,-4)$ and $(1,1)$:

$$m=\frac{-4-1}{0-1}$$
$$=\frac{-5}{-1}$$
$$=5$$

e. **The slope you found in part (d) should be equal to the slope you noted in part (a). If so, connect the points to make the line that is the graph of an equation of the form $y=mx+b$ that has slope m.**

f. **Note the location (ordered pair) that describes where the line intersects the y-axis.**

$(0,-4)$ is the location where the line intersects the y-axis.