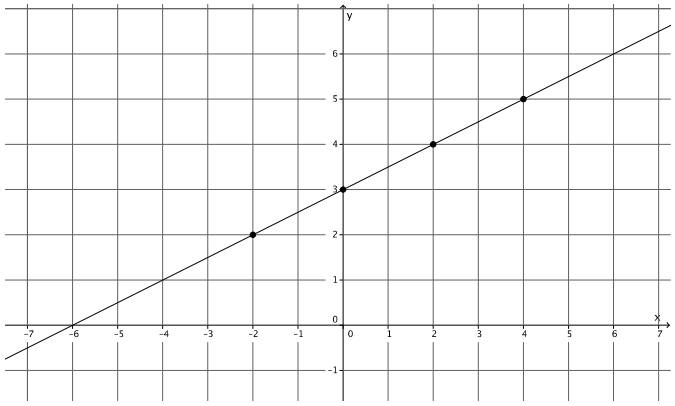
Lesson 18: There is Only One Line Passing Through a Given Point with a Given Slope

Classwork

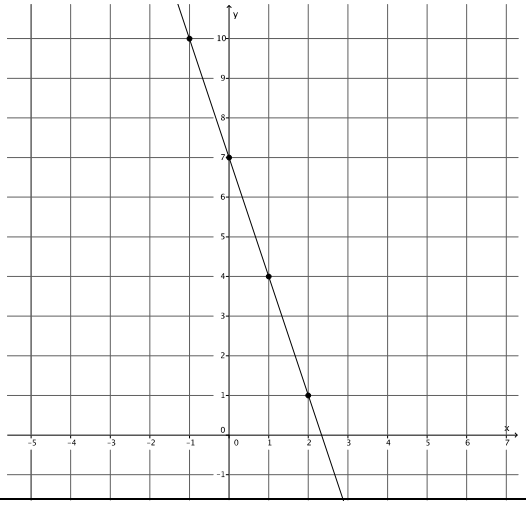
Opening Exercise

Examine each of the graphs and their equations below. Identify the coordinates of the point where the line intersects the y-axis. Describe the relationship between the point and the equation $y = mx + b$.

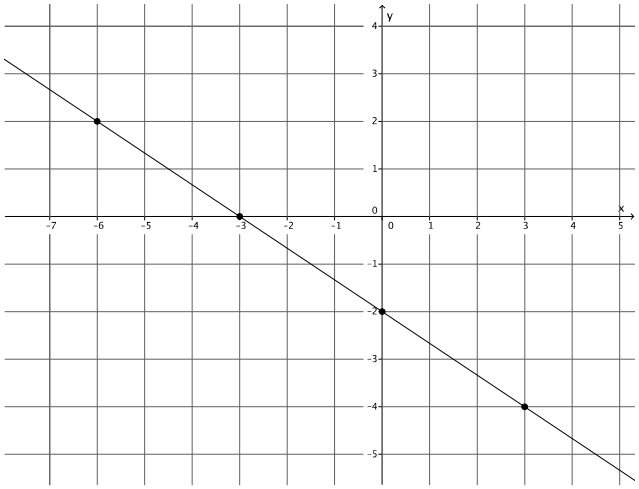
a. $y = \frac{1}{2}x + 3$



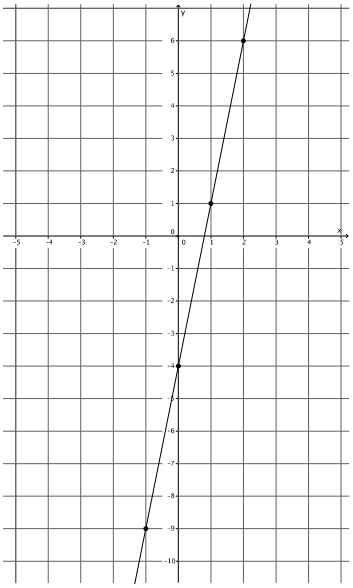
b. $y = -3x + 7$



c. $y = -\frac{2}{3}x - 2$



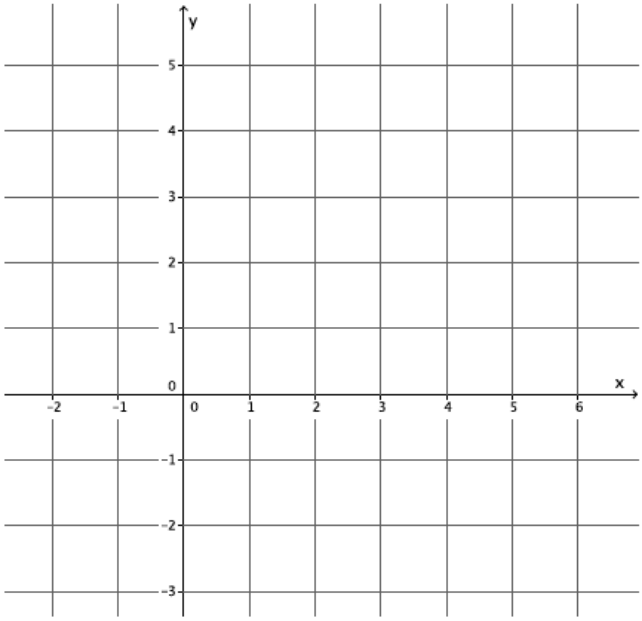
d. $y = 5x - 4$



Opening Exercise

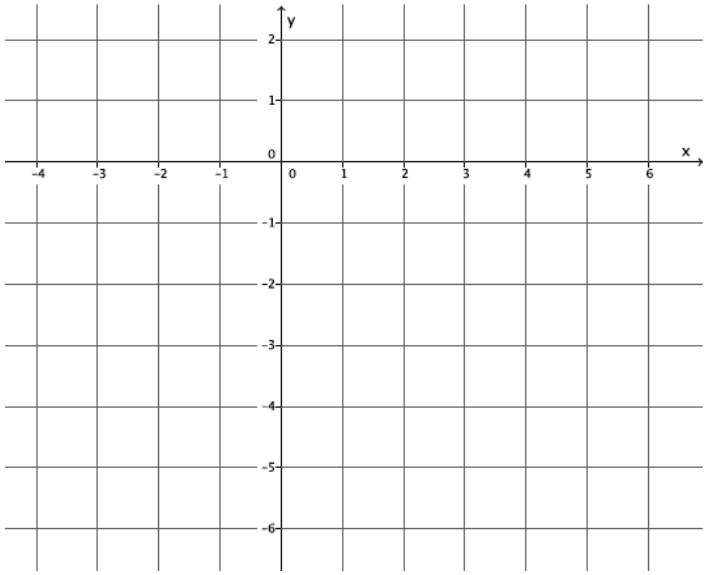
Example 1

Graph the equation $y = \frac{2}{3}x + 1$. Name the slope and y-intercept.



Example 2

Graph the equation $y = -\frac{3}{4}x - 2$. Name the slope and y-intercept.

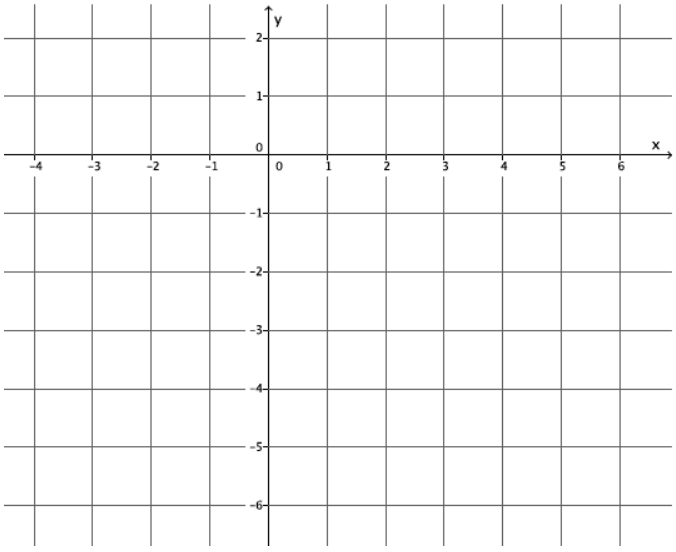


Exercises

1. Graph the equation $y = \frac{5}{2}x - 4$.

   a. Name the slope and the y-intercept.

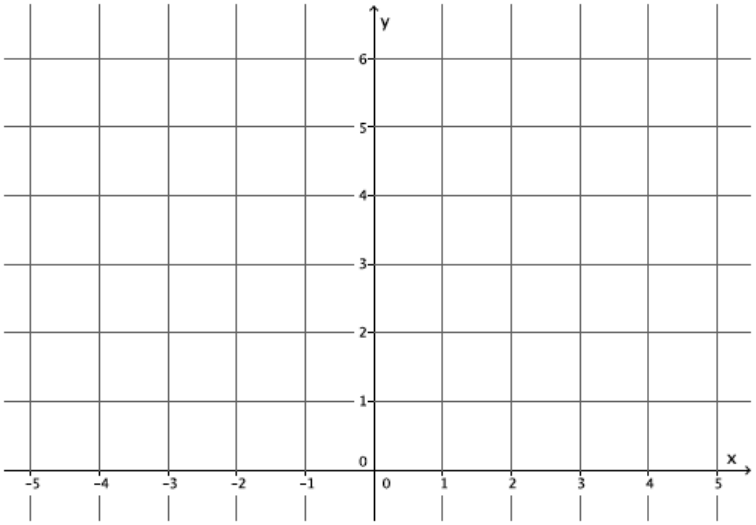
   b. Graph the known point, then use the slope to find a second point before drawing the line.



2. Graph the equation $y = -3x + 6$.

   a. Name the slope and the y-intercept.

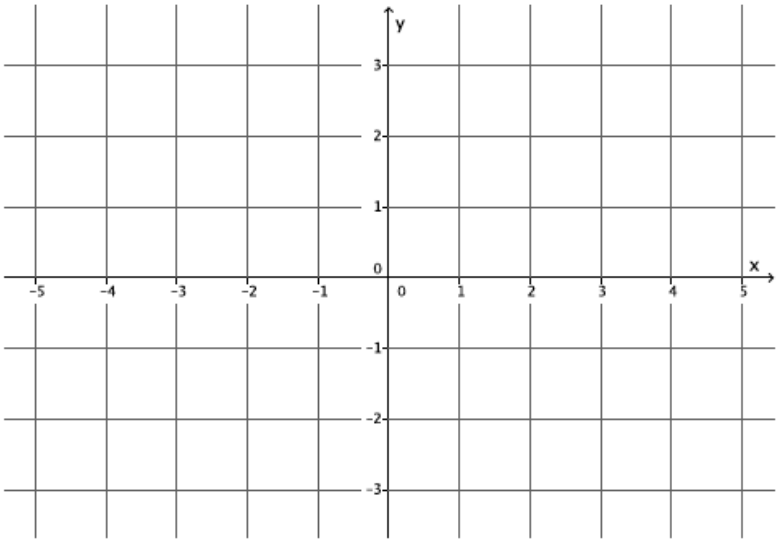
   b. Graph the known point, then use the slope to find a second point before drawing the line.



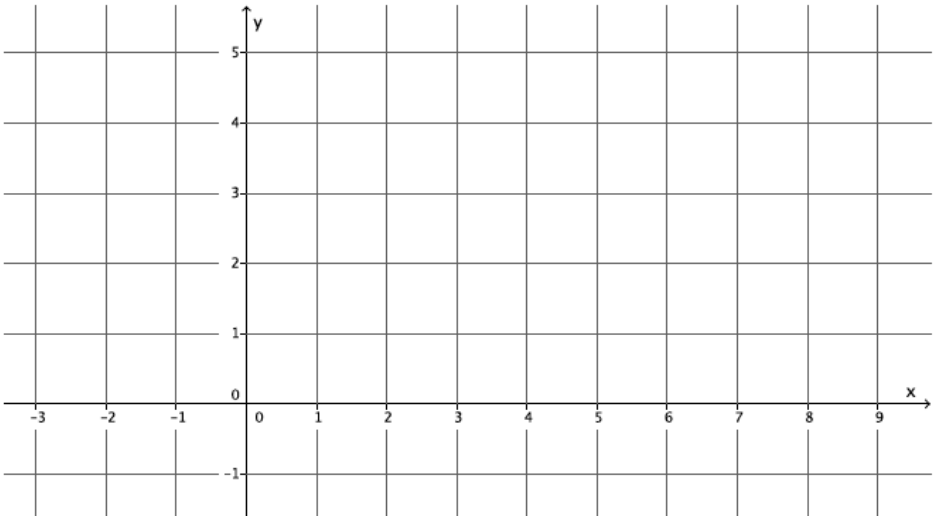
3. The equation $y = 1x + 0$ can be simplified to $y = x$. Graph the equation $y = x$.

   a. Name the slope and the y-intercept.

   b. Graph the known point, then use the slope to find a second point before drawing the line.



4. Graph the point $(0, 2)$.



a. Find another point on the graph using the slope, $m = \frac{2}{7}$.

b. Connect the points to make the line.

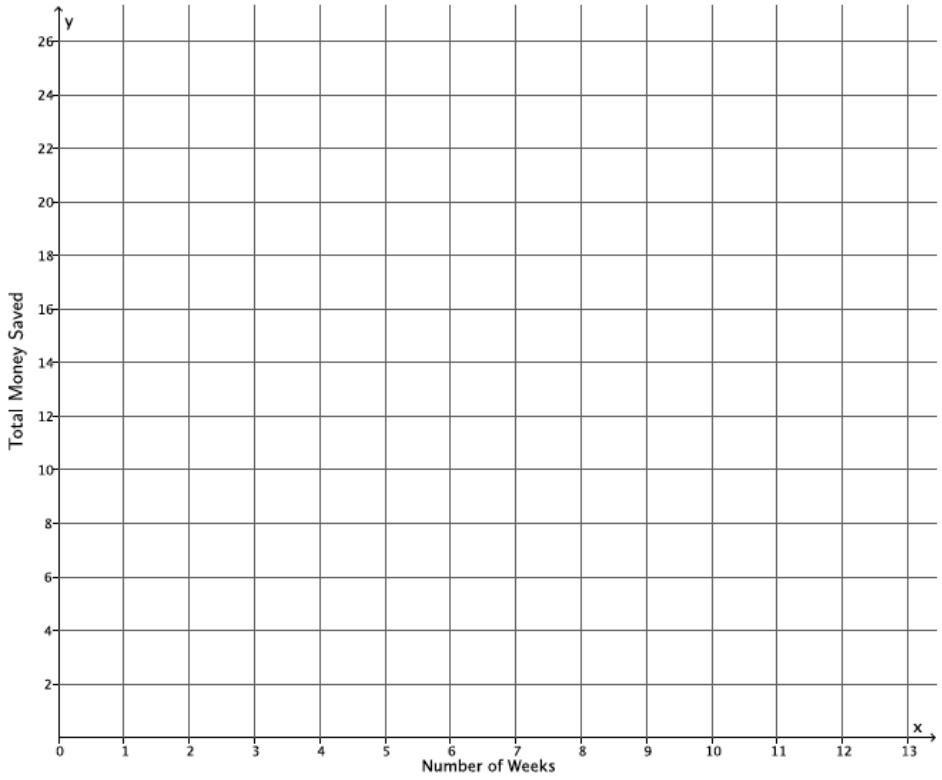
c. Draw a different line that goes through the point $(0, 2)$ with slope $m = \frac{2}{7}$. What do you notice?

5. A bank put $\$10$ into a savings account when you opened the account. Eight weeks later you have a total of $\$24$. Assume you saved the same amount every week.

a. If y is the total amount of money in the savings account and x represents the number of weeks, write an equation in the form $y = mx + b$ that describes the situation.

b. Identify the slope and the y-intercept. What do these numbers represent?

c. Graph the equation on a coordinate plane.



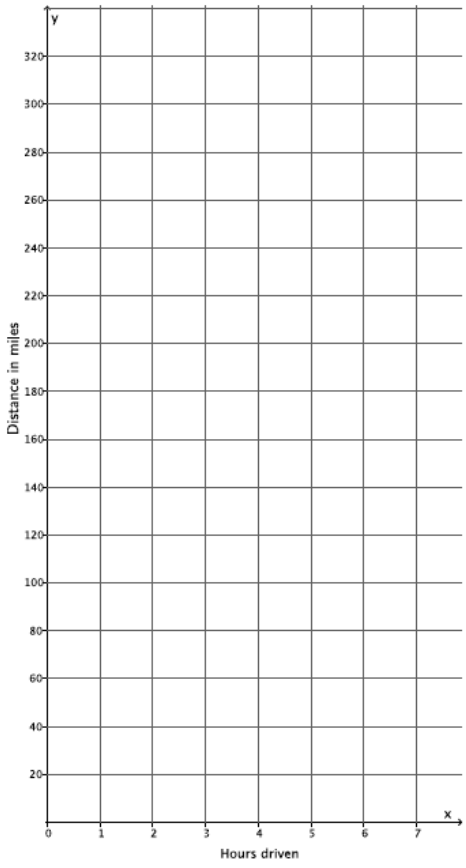
d. Could any other line represent this situation? For example, could a line through point $(0,10)$ with slope $\frac{7}{5}$ represent the amount of money you save each week? Explain.

6. A group of friends are on a road trip. So far they have driven 120 miles. They continue their trip and drive at a constant rate of 50 miles per hour.

   a. Let y represent the total distance traveled in x hours. Write an equation to represent the total number of miles driven in x hours.

   b. Identify the slope and the y-intercept. What do these numbers represent?

   c. Graph the equation on a coordinate plane.



   d. Could any other line represent this situation? For example, could a line through point $(0, 120)$ with slope 75 represent the total distance the friends drive? Explain.

Lesson Summary

The equation $y = mx + b$ is in slope-intercept form. The number m represents the slope of the graph and the point $(0, b)$ is the location where the graph of the line intersects the y-axis.

To graph a line from the slope-intercept form of a linear equation, begin with the known point, $(0, b)$, then use the slope to find a second point. Connect the points to graph the equation.

There is only one line passing through a given point with a given slope.

Problem Set

Graph each equation on a separate pair of x and y axes.

1. Graph the equation $y = \frac{4}{5}x - 5$.
   a. Name the slope and the y-intercept.
   b. Graph the known point, and then use the slope to find a second point before drawing the line.

2. Graph the equation $y = x + 3$.
   a. Name the slope and the y-intercept.
   b. Graph the known point, and then use the slope to find a second point before drawing the line.

3. Graph the equation $y = -\frac{4}{3}x + 4$.
   a. Name the slope and the y-intercept.
   b. Graph the known point, and then use the slope to find a second point before drawing the line.

4. Graph the equation $y = \frac{5}{2}x$.
   a. Name the slope and the y-intercept.
   b. Graph the known point, and then use the slope to find a second point before drawing the line.

5. Graph the equation $y = 2x - 6$.
   a. Name the slope and the y-intercept.
   b. Graph the known point, and then use the slope to find a second point before drawing the line.

6. Graph the equation $y = -5x + 9$.
   a. Name the slope and the y-intercept.
   b. Graph the known point, and then use the slope to find a second point before drawing the line.

7. Graph the equation $y = \frac{1}{3}x + 1$.
   a. Name the slope and the y-intercept.
   b. Graph the known point, and then use the slope to find a second point before drawing the line.

8. Graph the equation $5x + 4y = 8$. (Hint: transform the equation so that it is of the form $y = mx + b$.)
   a. Name the slope and the y-intercept.
   b. Graph the known point, and then use the slope to find a second point before drawing the line.

9. Graph the equation $-2x + 5y = 30$.
   a. Name the slope and the y-intercept.
   b. Graph the known point, and then use the slope to find a second point before drawing the line.

10. Let l and l' be two lines with the same slope m passing through the same point P. Show that there is only one line with a slope m, where $m < 0$, passing through the given point P. Draw a diagram if needed.