# Lesson 18: There is Only One Line Passing Through a Given Point with a Given Slope

## Student Outcomes

- Students graph equations in the form of $y = mx + b$ using information about slope and $y$-intercept.
- Students know that if they have two straight lines with the same slope and a common point that the lines are the same.

## Lesson Notes

The Opening Exercise requires students to examine part (f) from the problem set of Lesson 17. Each part (f) requires students to identify the point where the graph of the line intersects the $y$-axis. Knowing that this point represents the $y$-intercept and that it is the point $(0, b)$ in the equation $y = mx + b$ is integral for the content in the lesson.

To keep consistent, throughout the lesson students form the slope triangle with points $P$, $Q$, and $R$. Since slope has been defined in terms of lengths of sides of similar triangles, there is notation and verbiage about numbers in terms of distances. As you work through the examples, make clear that the lengths of the slope triangle are positive distances, but that does not necessarily mean that the slope must be positive (see Example 2). Remind students that graphs of lines with positive slopes are *left-to-right inclining* and graphs of lines with negative slopes are *left-to-right declining.*

Coordinate planes are provided for students in the exercises of this lesson, but they will need graph paper to complete the Problem Set.

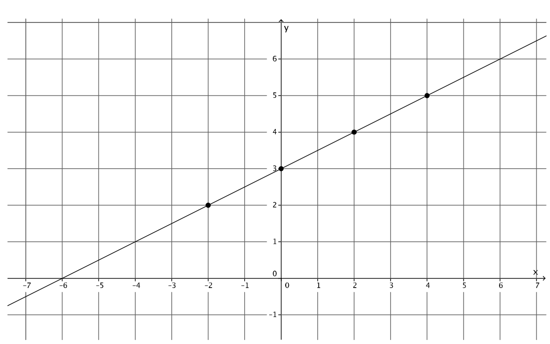
## Classwork

### Opening Exercise (5 minutes)

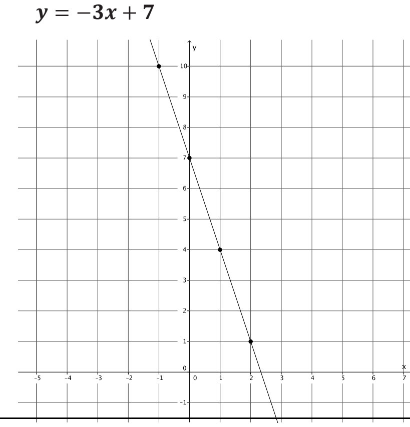
MP.8

**Examine each of the graphs and their equations below. Identify the coordinates of the point where the line intersects the $y$-axis. Describe the relationship between the point and the equation $y = mx + b$.**

**a.** $y = \frac{1}{2}x + 3$

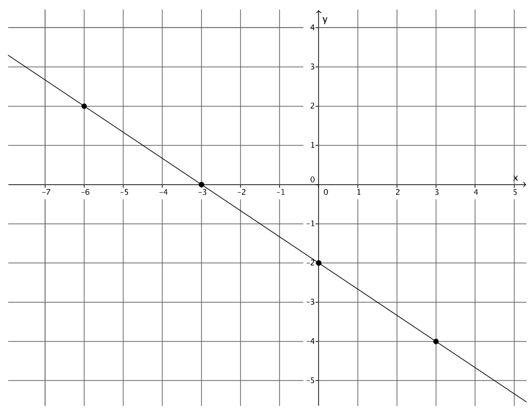


**b.** $y = -3x + 7$

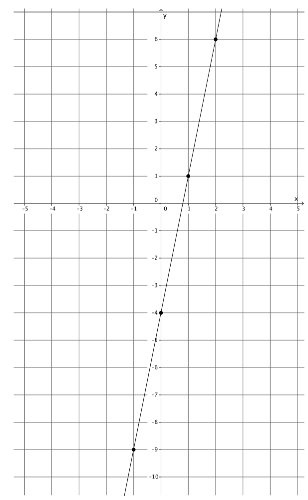


MP.8

c. $y = -\frac{2}{3}x - 2$



d. $y = 5x - 4$



***Points noted in graphs above where the line intersects the $y$-axis:***

- ***a.*** $\boldsymbol{y = \frac{1}{2}x + 3}$, $\boldsymbol{(0, 3)}$
- ***b.*** $\boldsymbol{y = -3x + 7}$, $\boldsymbol{(0, 7)}$
- ***c.*** $\boldsymbol{y = -\frac{2}{3}x - 2}$, $\boldsymbol{(0, -2)}$
- ***d.*** $\boldsymbol{y = 5x - 4}$, $\boldsymbol{(0, -4)}$

***In each equation, the number $b$ was the $y$-coordinate of the point where the line intersected the $y$-axis.***

## Discussion (5 minutes)

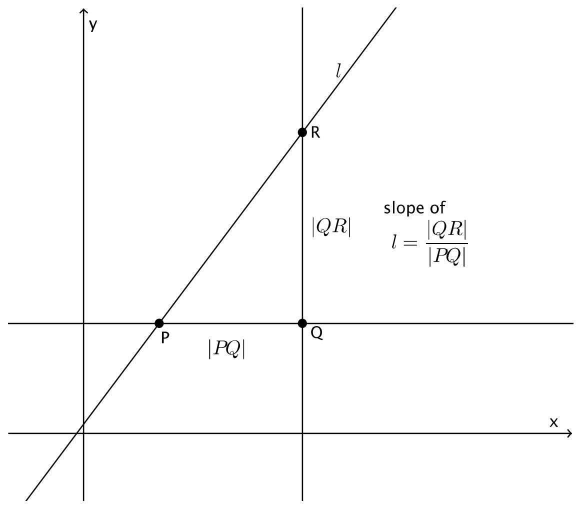
- In the last lesson, we transformed the standard form of a linear equation $ax + by = c$, into what was referred to as the slope-intercept form $y = mx + b$. We know that the slope is represented by $m$, but we did not discuss the meaning of $b$. In the Opening Exercise you were asked to note the location (ordered pair) that describes where the line intersected the $y$-axis.

MP.7

- What do you notice about the value of $b$ in relation to the point where the graph of the equation intersected the $y$-axis?
  - *The value of $b$ was the same number as the $y$-coordinate of each location.*
- When a linear equation is in the form $y = mx + b$, it is known as the slope-intercept form because this form provides information about the slope, $m$, and $y$-intercept, $(0, b)$ of the graph. The $y$-intercept is defined as the location on the graph where a line intersects the $y$-axis.

MP.3

- In this lesson, we develop the last tool that we need in order to prove that the graph of a linear equation in two variables $ax + by = c$, where $a$, $b$, and $c$ are constants is a straight line. We will show that if two straight lines have the same slope and pass through the same point, then they are the same line.
- Since an equation of the form $y = mx + b$ provides information about both the $y$-intercept and slope, we will use this equation to graph lines.
- Recall that we began discussing slope graphically, $m = \frac{|QR|}{|PQ|}$.

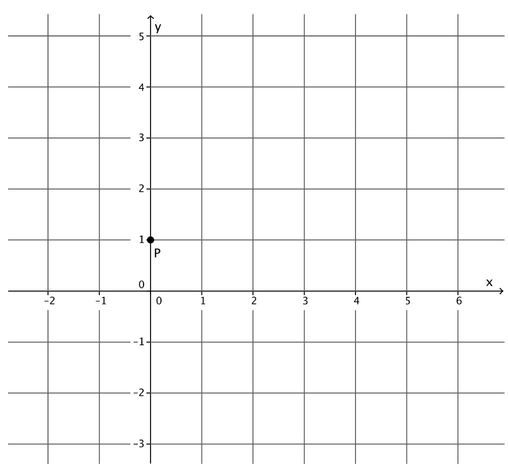



## Example 1 (6 minutes)

Graph equation in the form of $y = mx + b$.

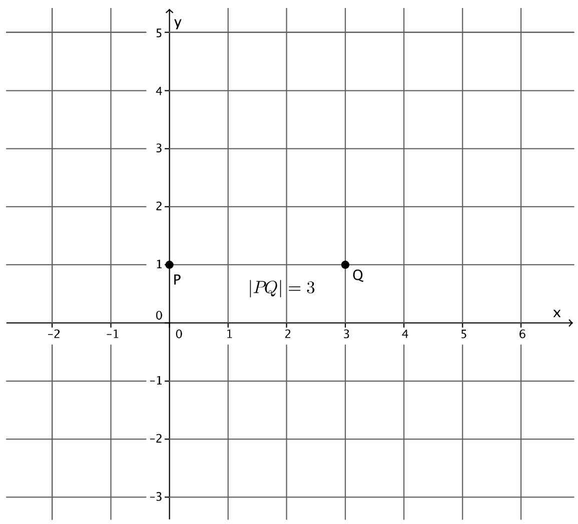
> **Example 1**
>
> **Graph the equation $y = \frac{2}{3}x + 1$. Name the slope and $y$-intercept.**
>
> *The slope, $m = \frac{2}{3}$ and the $y$-intercept is $(0, 1)$.*

- To graph the equation, we must begin with the known point. In this case the $y$-intercept. We cannot begin with the slope because the slope describes the rate of change between two points. That means we need a point to begin with. On a graph, we plot the point $(0, 1)$.

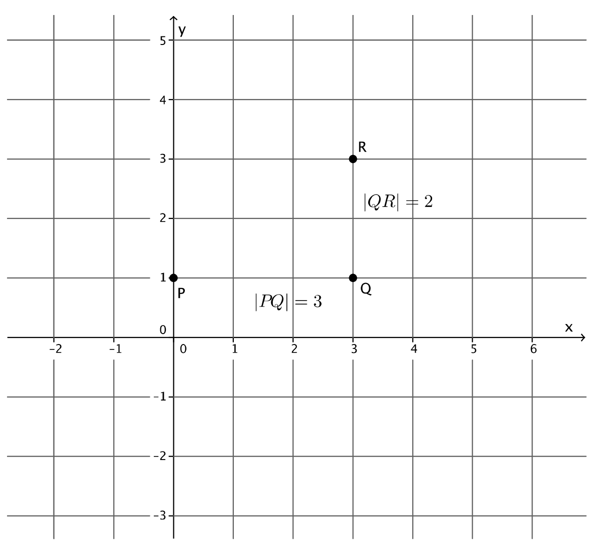


Next, we use the slope to find the second point. The slope tells us exactly how many units to go to the right to find point $Q$, and then how many vertical units we need to go from $Q$ to find point $R$. How many units will we go to the right in order to find point $Q$? How do you know?

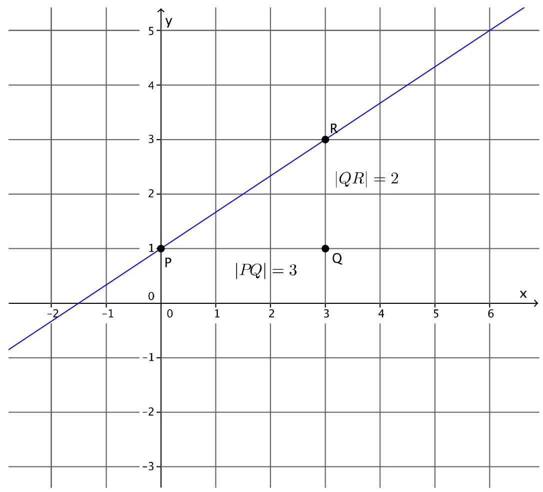
- *We need to go* $3$ *units to the right of point* $P$ *to find* $Q$. *We go* $3$ *units because* $|PQ| = 3$.




- How many vertical units from point $Q$ must we go to find point $R$? How do you know?
  - *We need to go* $2$ *units from point Q to find* $R$. *We go* $2$ *units because* $|QR| = 2$.
- Will we go up from point $Q$ or down from point $Q$ to find $R$? How do you know?
  - *We need to go up because the slope is positive. That means that the line will be left-to-right inclining.*



- Since we know that the line joining two distinct points of the form $y = mx + b$ has slope $m$, and we specifically constructed points $P$ and $R$ with the slope in mind, we can join the points with a line.




## Example 2 (4 minutes)

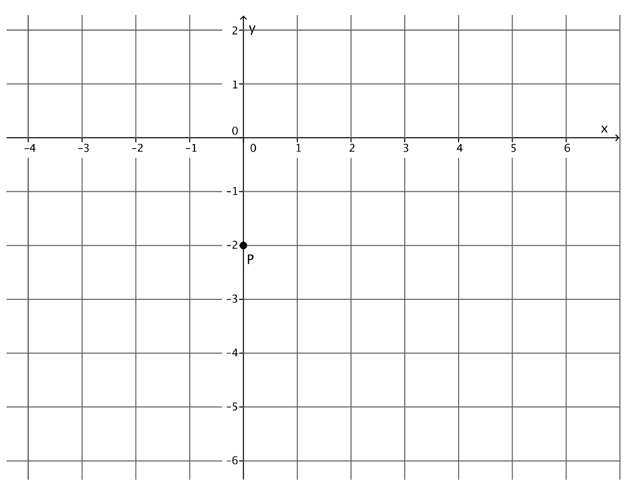
Graph equation in the form of $y = mx + b$.

**Example 2**

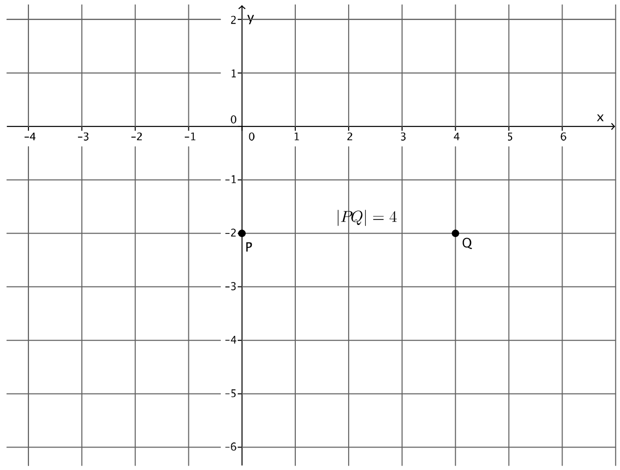
**Graph the equation $y = -\frac{3}{4}x - 2$. Name the slope and $y$-intercept.**

***The slope, $m = -\frac{3}{4}$ and the $y$-intercept is $(0, -2)$.***

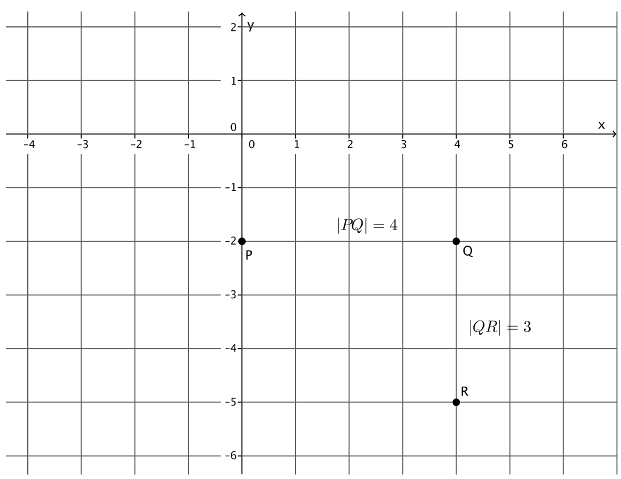
- How do we begin?
  - *We must begin by putting a known point on the graph,* $(0, -2)$.



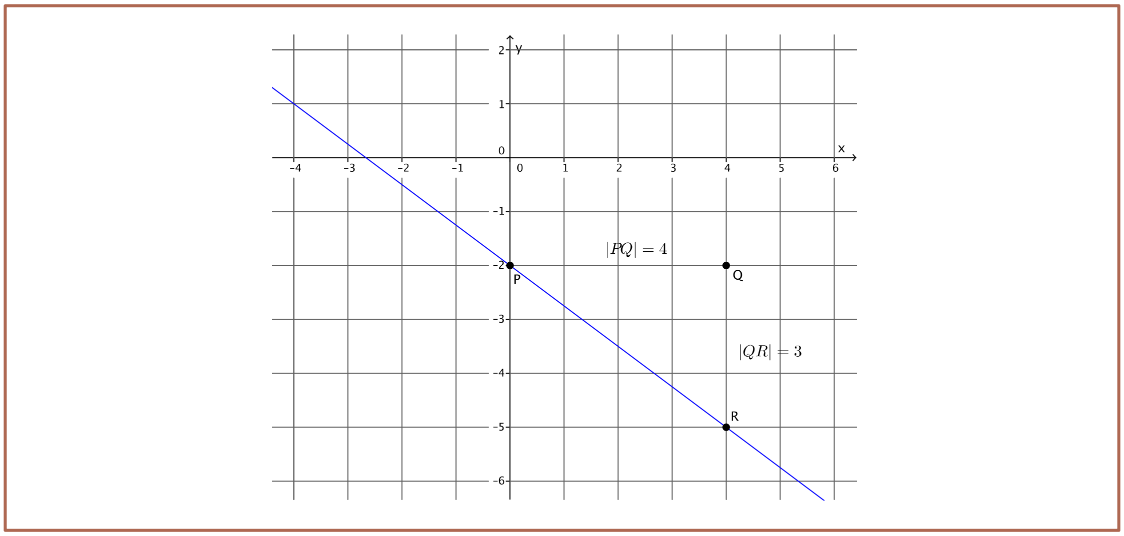
- How many units will we go to the right in order to find point $Q$? How do you know?
  - *We need to go $4$ units to the right of point $P$ to find $Q$. We go $4$ units because $|PQ| = 4$.*




- How many units from point $Q$ must we go to find point $R$? How do you know?
  - *We need to go $3$ units from point $Q$ to find $R$. We go $3$ units because $|QR| = 3$.*
- Will we go up from point $Q$ or down from point $Q$ to find $R$? How do you know?
  - *We need to go down from point $Q$ to point $R$ because the slope is negative. That means that the line will be left-to-right declining.*



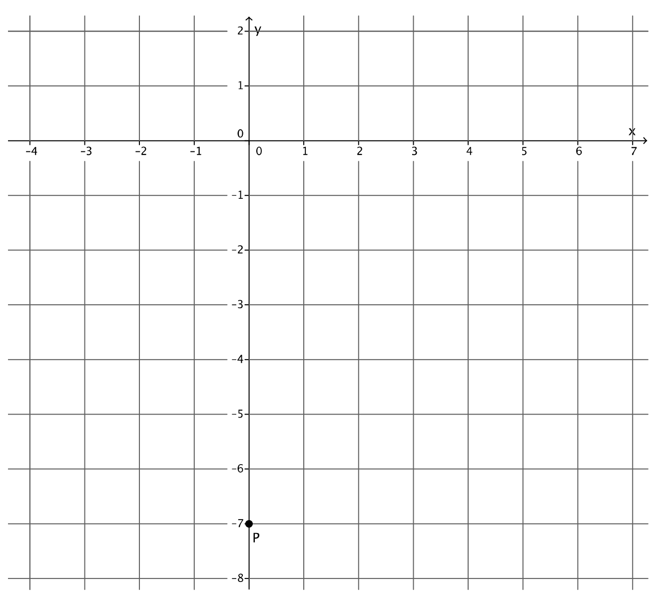
- Now we draw the line through the points $P$ and $R$.



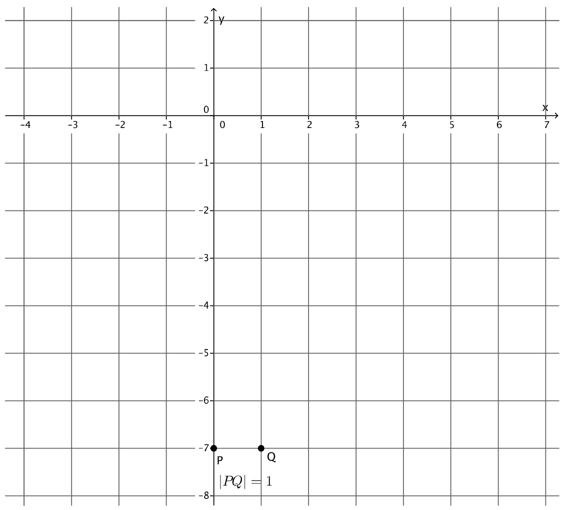

## Example 3 (4 minutes)

Graph equation in the form of $y = mx + b$.

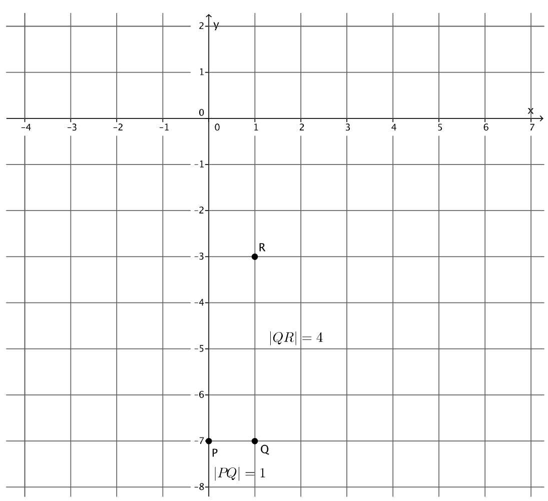
- Graph the equation $y = 4x - 7$. Name the slope and $y$-intercept.
  - *The slope,* $m = 4$ *and the* $y$-intercept is $(0, -7)$.
- How do we begin?
  - *We must begin by putting a known point on the graph,* $(0, -7)$.



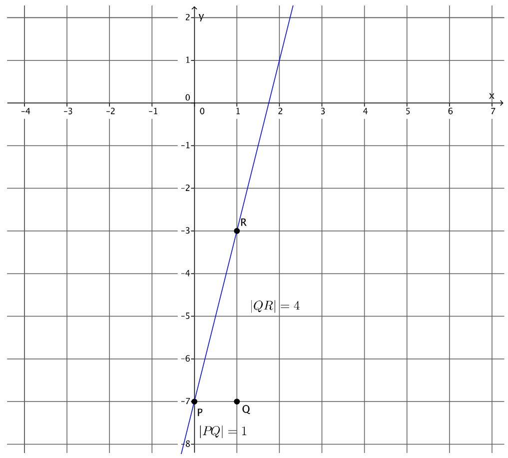
- Notice this time that the slope is the integer $4$. In the last two examples, our slopes have been in the form of a fraction so that we can use the information in the numerator and denominator to determine the lengths of $|PQ|$ and $|QR|$. If $m = \frac{|QR|}{|PQ|} = 4$, then what fraction can we use to represent slope to help us graph?
    - *The number $4$ is equivalent to the fraction $\frac{4}{1}$.*
- How many units will we go to the right in order to find point $Q$? How do you know?
    - *We need to go $1$ unit to the right of point $P$ to find $Q$. We go $1$ unit because $|PQ| = 1$.*




- How many vertical units from point $Q$ must we go to find point $R$? How do you know?
    - *We need to go $4$ units from point $Q$ to find $R$. We go $4$ units because $|QR| = 4$.*
- Will we go up from point $Q$ or down from point $Q$ to find $R$? How do you know?
    - *We need to go up from point $Q$ to point $R$ because the slope is positive. That means that the line will be left-to-right inclining.*



- Now we join the points $P$ and $R$ to make the line.




## Exercises 1–4 (9 minutes)

Students complete Exercises 1–4 individually or in pairs.

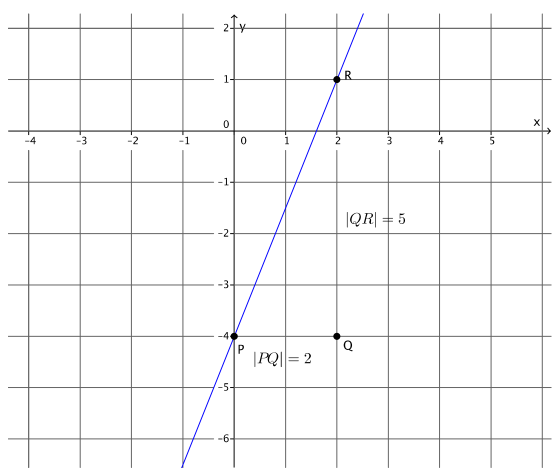
**Exercises 1–4**

1. **Graph the equation $y = \frac{5}{2}x - 4$.**

   a. **Name the slope and the $y$-intercept.**

   *The slope is $m = \frac{5}{2}$ and the $y$-intercept is $(0, -4)$.*

   b. **Graph the known point, then use the slope to find a second point before drawing the line.**

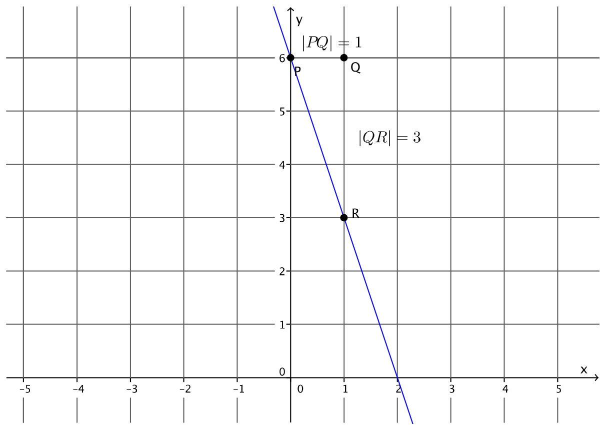


2. Graph the equation $y = -3x + 6$.

   a. Name the slope and the $y$-intercept.

   *The slope is $m = -3$ and the $y$-intercept is $(0, 6)$.*

   b. Graph the known point, then use the slope to find a second point before drawing the line.

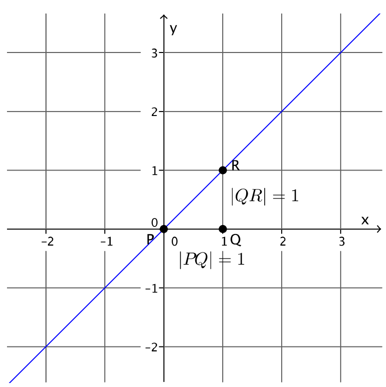



3. The equation $y = 1x + 0$ can be simplified to $y = x$. Graph the equation $y = x$.

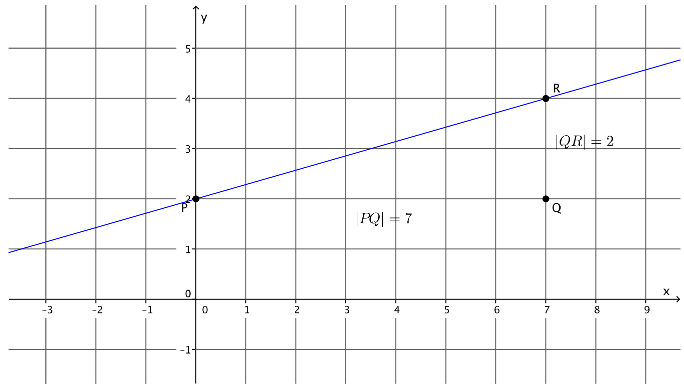
   a. Name the slope and the $y$-intercept.

   *The slope is $m = 1$ and the $y$-intercept is $(0, 0)$.*

   b. Graph the known point, then use the slope to find a second point before drawing the line.



4. **Graph the point $(0, 2)$.**




a. **Find another point on the graph using the slope, $m = \frac{2}{7}$.**

b. **Connect the points to make the line.**

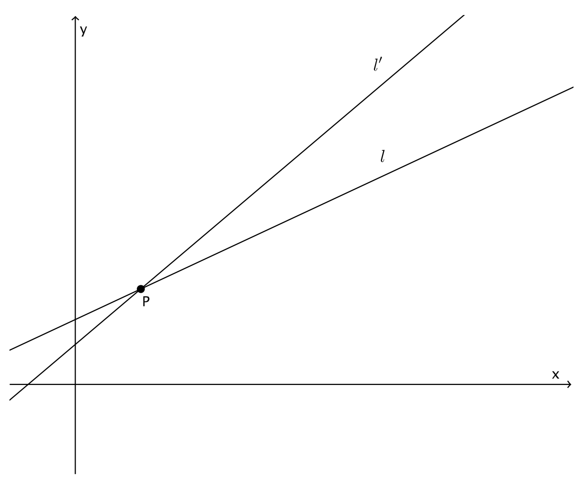
c. **Draw a different line that goes through the point $(0, 2)$ with slope $m = \frac{2}{7}$. What do you notice?**

*Only one line can be drawn through the given point with the given slope.*

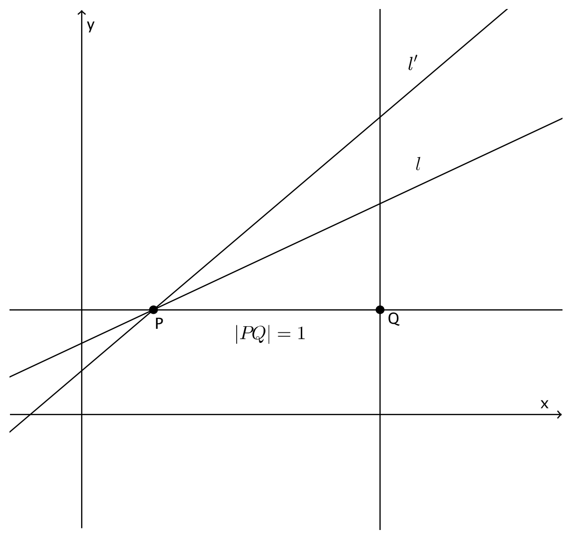
## Discussion (8 minutes)

The following proof is optional. An exercise, below the discussion, can be used as an alternative.

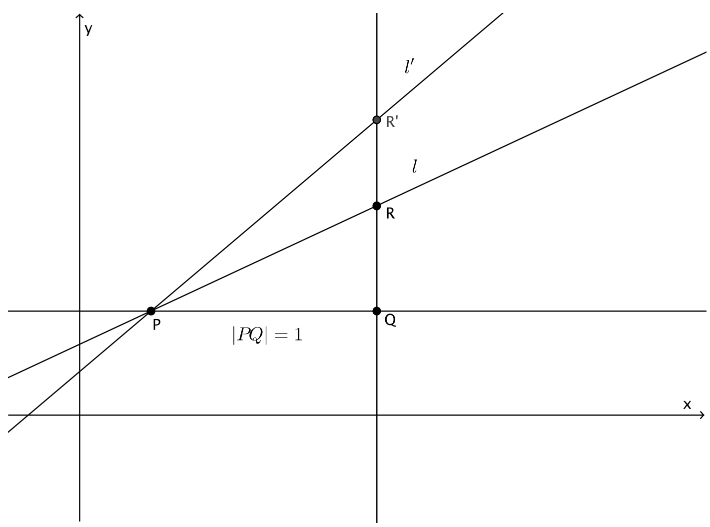
- Now we must show that if two straight lines have the same slope and pass through the same point, then they are the same line, which is what you observed in Exercise 4.
- We let $l$ and $l'$ be two lines with the same slope $m$ passing through the same point $P$. Since $m$ is a number, $m$ could be positive, negative, or equal to zero. For this proof, we will let $m > 0$. Since $m > 0$, both lines $l$ and $l'$ are left-to-right inclining.
- Since we are trying to show that $l$ and $l'$ are the same line, let's assume that they are not and prove that our assumption is false.



- Using our first understanding of slope, we will pick a point $Q$ one unit to the right of point $P$. Then draw a line parallel to the $y$-axis going through point $Q$, as shown.




- Now we label the point of intersection of the line we just drew and line $l$ and $l'$ as $R$ and $R'$, respectively.




- By definition of slope, the length $|QR|$ is the slope of line $l$ and the length of $|QR'|$ is the slope of line $l'$. What do we know about the slopes of lines $l$ and $l'$?
  - *The slopes are the same.*
- Then what do we know about the lengths $|QR|$ and $|QR'|$?
  - *They must be equal.*
- For that reason points $R$ and $R'$ must coincide. This means lines $l$ and $l'$ are the same line, not as we originally drew them. Therefore, there is just one line passing through a given point with a given slope.

## Exercises 5–6 (8 minutes)

Students complete Exercises 5–6 individually or in pairs.

**Exercises 5–6**

5. **A bank put $\$10$ into a savings account when you opened the account. Eight weeks later you have a total of $\$24$. Assume you saved the same amount every week.**

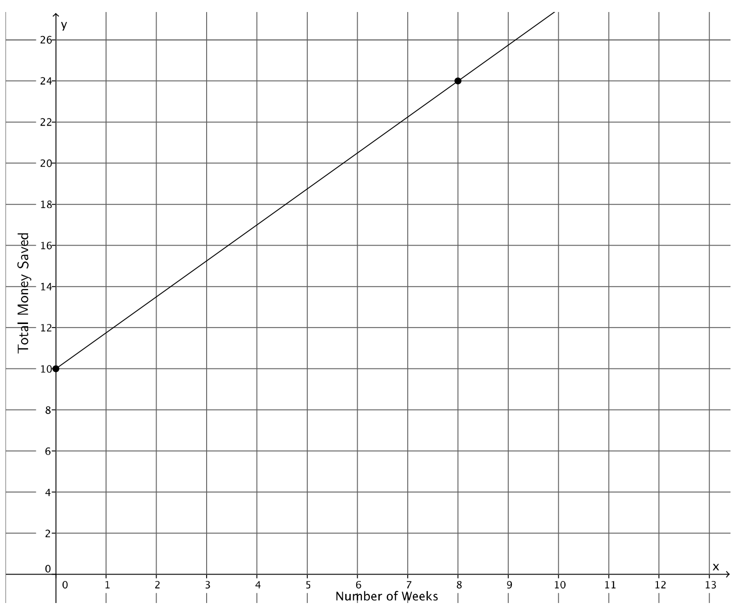
   a. **If $y$ is the total amount of money in the savings account and $x$ represents the number of weeks, write an equation in the form $y = mx + b$ that describes the situation.**

   $$24 = m(8) + 10$$
   $$14 = 8m$$
   $$\frac{14}{8} = m$$
   $$\frac{7}{4} = m$$
   $$y = \frac{7}{4}x + 10$$

   b. **Identify the slope and the $y$-intercept. What do these numbers represent?**

   *The slope is $\frac{7}{4}$ and the $y$-intercept is $(0, 10)$. The $y$-intercept represents the amount of money the bank gave me, in the amount of $\$10$. The slope represents the amount of money I save each week, $\frac{7}{4} = \$1.75$.*

   c. **Graph the equation on a coordinate plane.**




   d. **Could any other line represent this situation? For example, could a line through point $(0, 10)$ with slope $\frac{7}{5}$ represent the amount of money you save each week? Explain.**

   *No, a line through point $(0, 10)$ with slope $\frac{7}{5}$ cannot represent this situation. That line would show that at the end of the $8$ weeks I would have $\$21.20$, but I was told that I would have $\$24$ by the end of the $8$ weeks.*

6. A group of friends are on a road trip. So far they have driven $120$ miles. They continue their trip and drive at a constant rate of $50$ miles per hour.

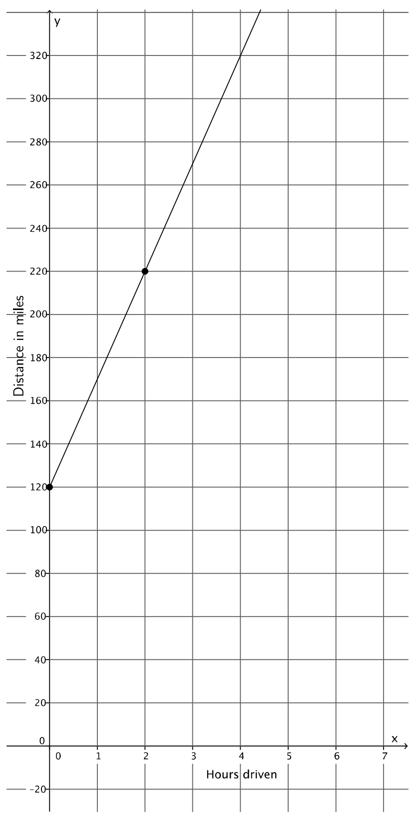
   a. Let $y$ represent the total distance traveled in $x$ hours. Write an equation to represent the total number of miles driven in $x$ hours.

      $y = 50x + 120$

   b. Identify the slope and the $y$-intercept. What do these numbers represent?

      *The slope is $50$ and represents the rate of driving. The $y$-intercept is $120$ and represents the number of miles they had already driven before driving at the given constant rate.*

   c. Graph the equation on a coordinate plane.




   d. Could any other line represent this situation? For example, could a line through point $(0, 120)$ with slope 75 represent the total distance the friends drive? Explain.

      *No, a line through point $(0, 120)$ with a slope of $75$ could not represent this situation. That line would show that after an hour the friends traveled a total distance of $195$ miles. According to the information given, the friends would only have traveled $170$ miles after one hour.*

### Closing (4 minutes)

Summarize, or ask students to summarize, the main points from the lesson:

- We know that in the equation $y = mx + b$, $(0, b)$ is the location where the graph of the line intersects the $y$-axis.
- We know how to graph a line using a point, namely the $y$-intercept, and the slope.
- We know that there is only one line with a given slope passing through a given point.

> **Lesson Summary**
>
> **The equation $y = mx + b$ is in slope-intercept form. The number $m$ represents the slope of the graph and the point $(0, b)$ is the location where the graph of the line intersects the $y$-axis.**
>
> **To graph a line from the slope-intercept form of a linear equation, begin with the known point, $(0, b)$, then use the slope to find a second point. Connect the points to graph the equation.**
>
> **There is only one line passing through a given point with a given slope.**

### Exit Ticket (4 minutes)

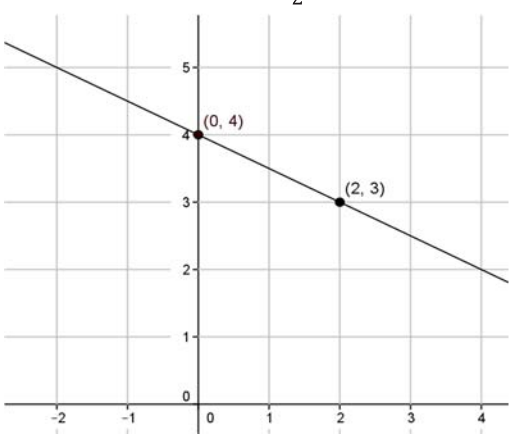
Name ______________________________________________ Date ____________________

# Lesson 18: There is Only One Line Passing Through a Given Point with a Given Slope

## Exit Ticket

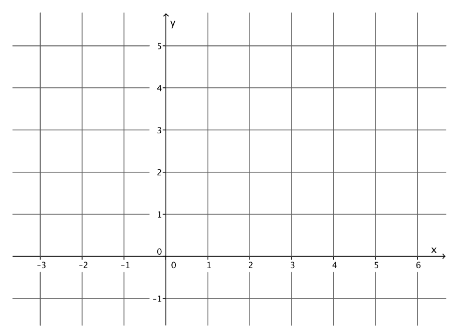
Mrs. Hodson said that the graphs of the equations below are incorrect. Find the student's errors and correctly graph the equations.

1. Student graph of $y = \frac{1}{2}x + 4$.

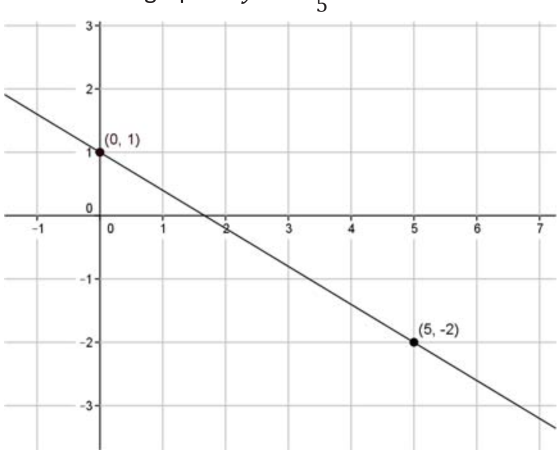



Error:

Correct graph of equation:

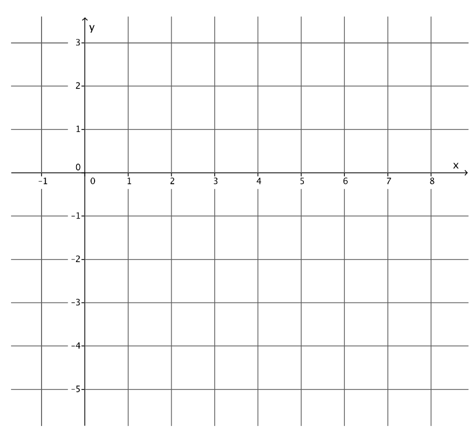


2. Student graph of $y = -\frac{3}{5}x - 1$.




Error:

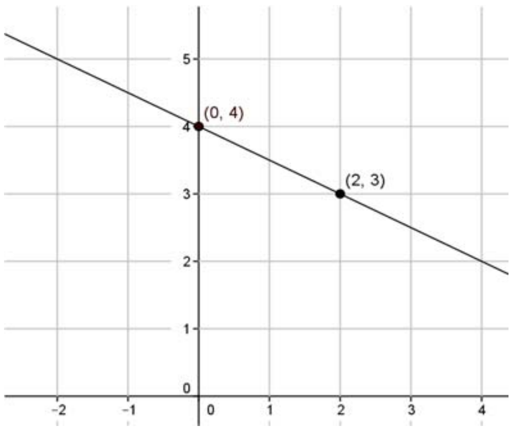
Correct graph of equation:



## Exit Ticket Sample Solutions

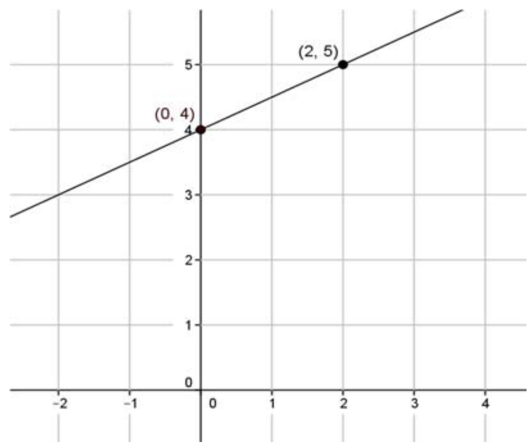
**Mrs. Hodson said that the graphs of the equations below are incorrect. Find the student's errors and correctly graph the equations.**

**1.** **Student graph of the equation $y = \frac{1}{2}x + 4$.**

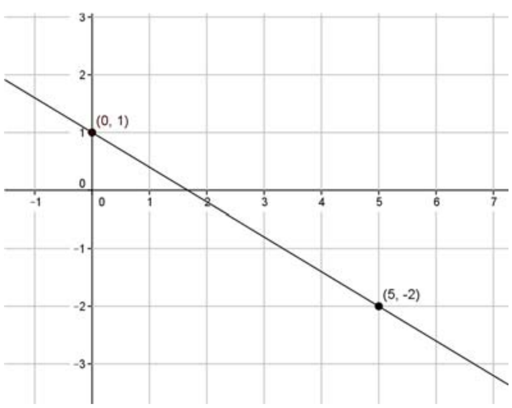



**Error:** ***The student should have gone up $1$ unit when finding $|QR|$ since the slope is positive.***

**Correct graph of equation:**

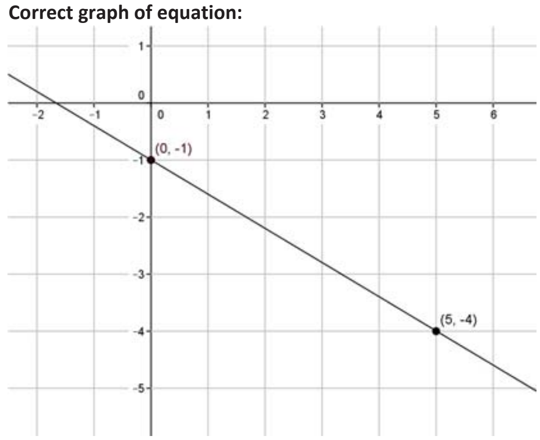



**2.** **Student graph of the equation $y = -\frac{3}{5}x - 1$.**




**Error:** ***The student did not find the y intercept correctly. It should be the point $(0, -1)$.***

**Correct graph of equation:**



## Problem Set Sample Solutions

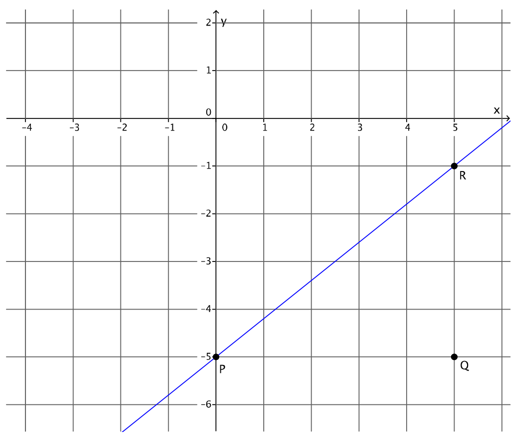
Students practice graphing equations using $y$-intercept and slope. Students need graph paper to complete the problem set. Optional problem 11 has students show that there is only one line passing through a point with a given negative slope.

1. **Graph the equation $y = \frac{4}{5}x - 5$.**

   a. **Name the slope and the $y$-intercept.**

   *The slope is $m = \frac{4}{5}$ and the $y$-intercept is $(0, -5)$.*

   b. **Graph the known point, and then use the slope to find a second point before drawing the line.**

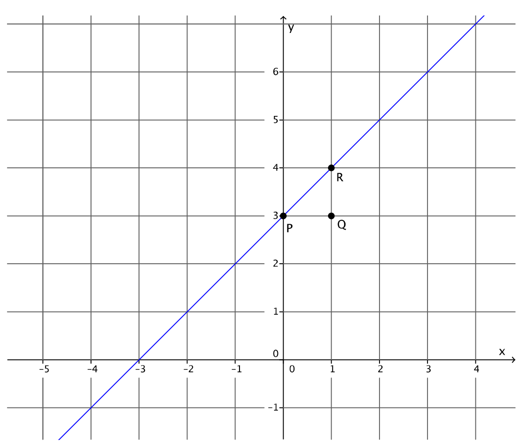



2. **Graph the equation $y = x + 3$.**

   a. **Name the slope and the $y$-intercept.**

   *The slope is $m = 1$ and the $y$-intercept is $(0, 3)$.*

   b. **Graph the known point, and then use the slope to find a second point before drawing the line.**

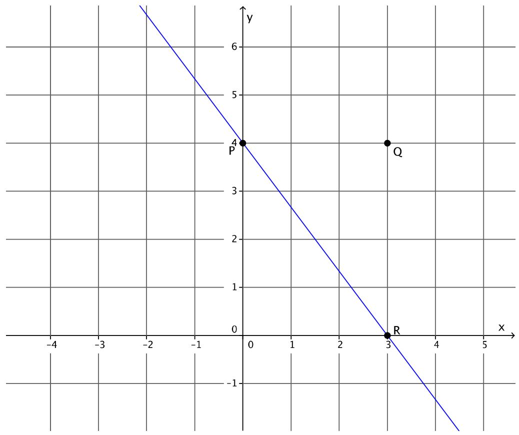


3. Graph the equation $y = -\frac{4}{3}x + 4$.

   a. Name the slope and the $y$-intercept.

   *The slope is $m = -\frac{4}{3}$ and the $y$-intercept is $(0,\ 4)$.*

   b. Graph the known point, and then use the slope to find a second point before drawing the line.

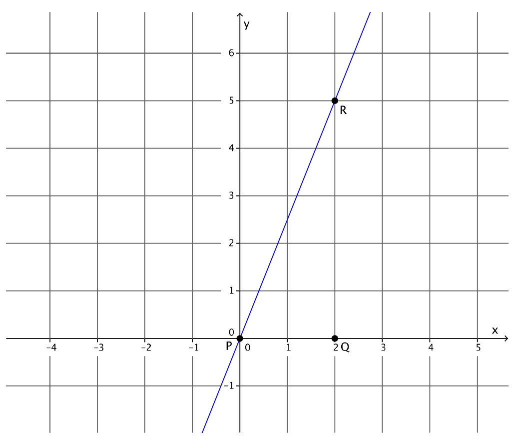



4. Graph the equation $y = \frac{5}{2}x$.

   a. Name the slope and the $y$-intercept.

   *The slope is $m = \frac{5}{2}$ and the $y$-intercept is $(0, 0)$.*

   b. Graph the known point, and then use the slope to find a second point before drawing the line.

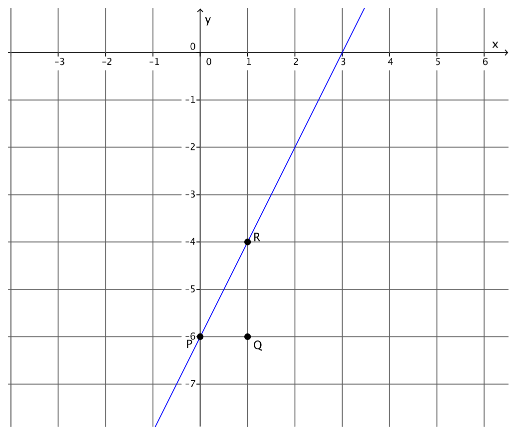


5. **Graph the equation $y = 2x - 6$.**

   a. **Name the slope and the $y$-intercept.**

   *The slope is $m = 2$ and the $y$-intercept is $(0, -6)$.*

   b. **Graph the known point, and then use the slope to find a second point before drawing the line.**

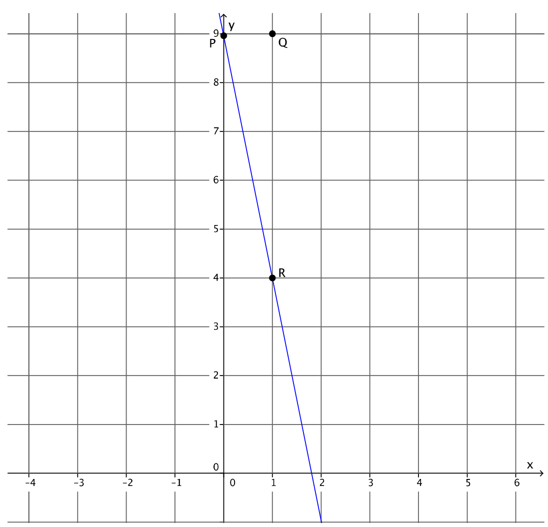



6. **Graph the equation $y = -5x + 9$.**

   a. **Name the slope and the $y$-intercept.**

   *The slope is $m = -5$ and the $y$-intercept is $(0, 9)$.*

   b. **Graph the known point, and then use the slope to find a second point before drawing the line.**

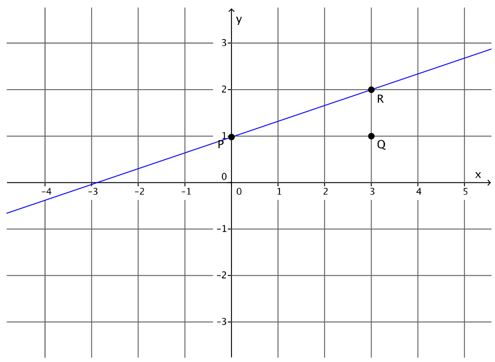


7. Graph the equation $y = \frac{1}{3}x + 1$.

   a. Name the slope and the $y$-intercept.

   *The slope is $m = \frac{1}{3}$ and the $y$-intercept is $(0, 1)$.*

   b. Graph the known point, and then use the slope to find a second point before drawing the line.



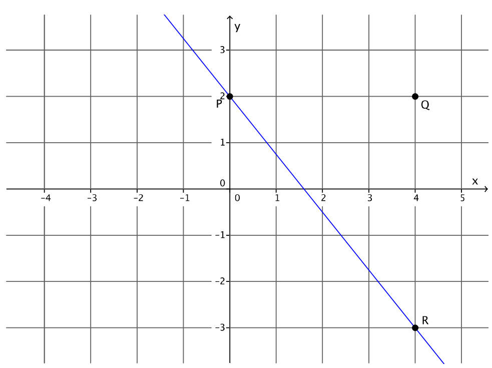

8. Graph the equation $5x + 4y = 8$. (Hint: transform the equation so that it is of the form $y = mx + b$.)

   a. Name the slope and the $y$-intercept.

$$5x + 4y = 8$$
$$5x - 5x + 4y = 8 - 5x$$
$$4y = 8 - 5x$$
$$\frac{4}{4}y = \frac{8}{4} - \frac{5}{4}x$$
$$y = 2 - \frac{5}{4}x$$
$$y = -\frac{5}{4}x + 2$$

   *The slope is $m = -\frac{5}{4}$ and the $y$-intercept is $(0, 2)$.*

   b. Graph the known point, and then use the slope to find a second point before drawing the line.



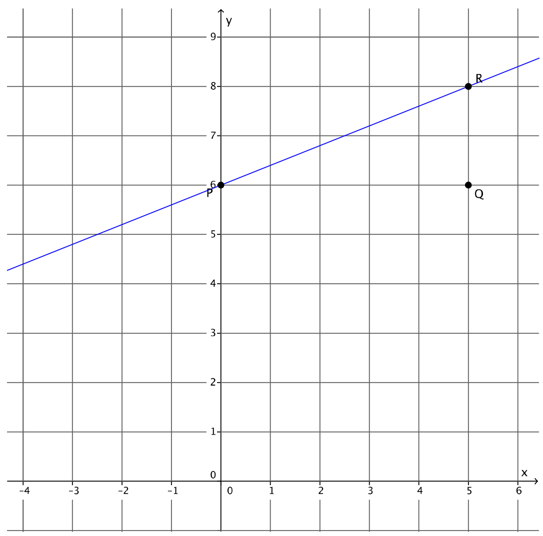
9. **Graph the equation $-2x + 5y = 30$.**

   a. **Name the slope and the $y$-intercept.**

$$-2x + 5y = 30$$
$$2x + 2x + 5y = 30 + 2x$$
$$5y = 30 + 2x$$
$$\frac{5}{5}y = \frac{30}{5} + \frac{2}{5}x$$
$$y = 6 + \frac{2}{5}x$$
$$y = \frac{2}{5}x + 6$$

*The slope is $m = \frac{2}{5}$ and the $y$-intercept is $(0, 6)$.*

   b. **Graph the known point, and then use the slope to find a second point before drawing the line.**



10. Let $l$ and $l'$ be two lines with the same slope $m$ passing through the same point $P$. Show that there is only one line with a slope $m$, where $m < 0$, passing through the given point $P$. Draw a diagram if needed.

*First assume that there are two different lines $l$ and $l'$ with the same negative slope passing through $P$. From point $P$, I mark a point $Q$ one unit to the right. Then I draw a line parallel to the $y$-axis through point $Q$. The intersection of this line and line $l$ and $l'$ are noted with points $R$ and $R'$. By definition of slope, the lengths $|QR|$ and $|QR'|$ represent the slopes of lines $l$ and $l'$, respectively. We are given that the lines have the same slope, which means that lengths $|QR|$ and $|QR'|$ are equal. Since that is true, then points $R$ and $R'$ coincide and so do lines $l$ and $l'$. Therefore, our assumption that they are different lines is false; $l$ and $l'$ must be the same line. Therefore, there is only one line with slope $m$ passing through the given point $P$.*