



Lesson 20: Every Line is a Graph of a Linear Equation

Student Outcomes

- Students know that any non-vertical line is the graph of a linear equation in the form of $y = mx + b$, where b is a constant.
- Students write the equation that represents the graph of a line.

Lesson Notes

The proof that every line is the graph of a linear equation in the first discussion below is optional. If you choose to use the Discussion below, you will need to skip the Opening Exercise that begins on page 302 and resume the lesson with Example 1 on page 303. Complete all other examples and exercises that follow. As an alternative to the discussion below, you may choose to complete the opening activity by showing a graph of a line on the coordinate plane and having students attempt to name the equation of the line. Two graphs are provided for your use beginning on page 302. Have students write their equations and strategies for determining the equation of the line; then lead the discussion described on page 302. Once the Opening Exercise is complete, work through Example 1 on page 303 and the remaining examples and exercises in the lesson. Revisit the equations and strategies students developed by having them critique their reasoning in comparison to the work in the example; then continuing with the remainder of the lesson.

Classwork

Discussion (10 minutes)

- Now that we are confident that the graph of every linear equation is a line, can we say that every line is the graph of a linear equation? We can say yes with respect to vertical and horizontal lines, recall $x = c$ and $y = c$. But what about other non-vertical lines?
- We must prove that any non-vertical (and non-horizontal) line is a graph of a linear equation in the form of $y = mx + b$, where $m \neq 0$ and b are constants.
- Let l be any non-vertical (and non-horizontal) line. Suppose the slope of the line is m and that the line intersects the y -axis at point $Q = (0, b)$.
- First, we show that any point on the line l is a point on the graph of the linear equation $y = mx + b$.
- Let $P = (x, y)$ be any point on line l . We need to show that P is a solution to $y = mx + b$. Think about how we did this in the last lesson. What should we do?
 - Use the points P and Q in the slope formula:

$$m = \frac{y - b}{x - 0}$$

$$mx = y - b$$

$$mx + b = y - b + b$$

$$mx + b = y$$

That shows that point P is a point on the graph of $y = mx + b$. Point Q is also on the graph of $y = mx + b$ because $b = m \cdot 0 + b$. Therefore, any point on the line l is a point on the graph of the linear equation $y = mx + b$.

MP.3

MP.3

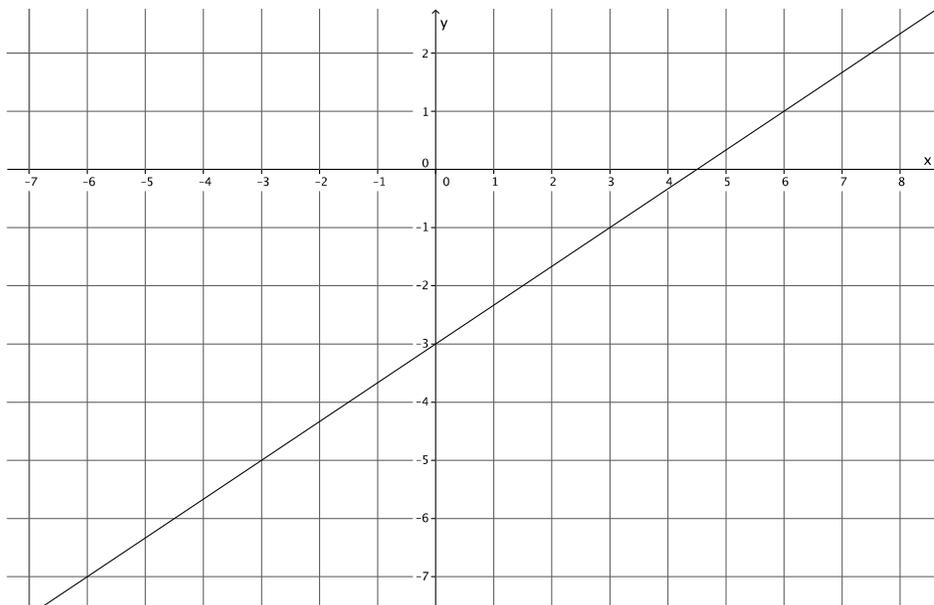
- Now, we want to show that any point on the graph of $y = mx + b$ is on l .
- Let R be any point on the graph of the linear equation $y = mx + b$. We know that the graph of $y = mx + b$ is a line with slope m . Let's call this line l' . We know that Q is on l' because $b = m \cdot 0 + b$. Therefore, l' is a line with slope m that passes through point Q . But l is a line with slope m that passes through point Q . What does that mean about lines l and l' ?
 - *The lines l and l' are the same line because there is only one line with a given slope that can go through a given point.*
- Therefore, R is a point on l .
- Now we can be certain that every line is a graph of a linear equation.

Opening (10 minutes)

Show students Graph 1 below and challenge them to write the equation for the line. Provide students time to work independently, then in pairs. Lead a discussion where students share their strategies for developing the equation of the line. Ask students how they knew their equations were correct, i.e., did they verify that the points with integer coordinates were solutions to the equation they wrote? Ask students what kind of equation they wrote: linear or non-linear. Ask students if they were given another line, could they write an equation for it using their strategy. Show them Graph 2 and, again, ask them to write the equation of the line. Verify that they wrote the correct equation, and conclude the discussion by stating that every line is the graph of a linear equation.

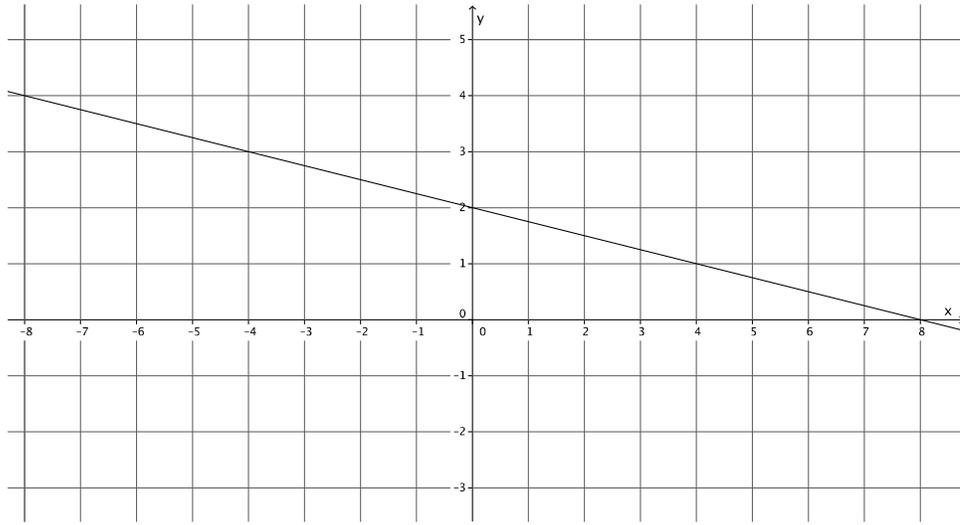
Opening Exercise

Graph 1



The equation for the line in Graph 1 is $y = \frac{2}{3}x - 3$.

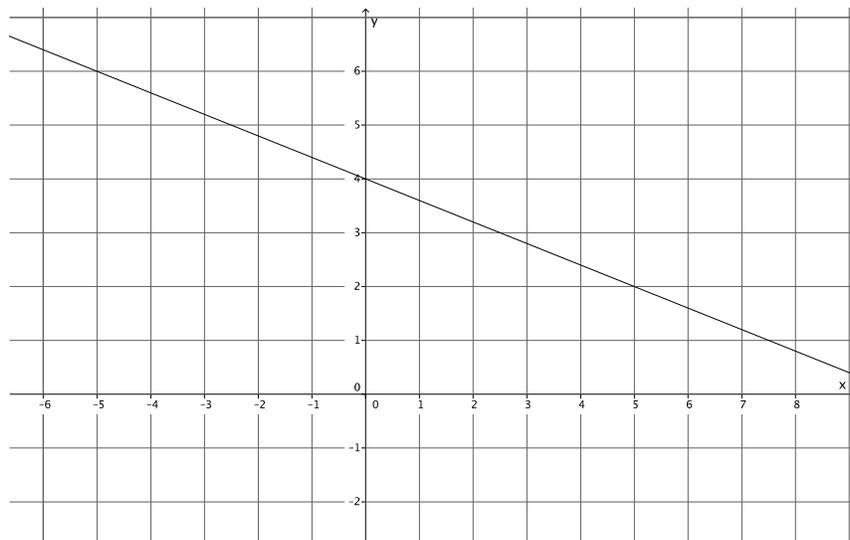
Graph 2



The equation for the line in Graph 2 is $y = -\frac{1}{4}x + 2$.

Example 1 (5 minutes)

- Given the graph of a line, we want to be able to write the equation that represents it.
- Which form of a linear equation do you think will be most valuable for this task, the standard form $ax + by = c$ or slope-intercept form $y = mx + b$?
 - *The slope-intercept form because we can easily identify the slope and y-intercept from both the equation and the graph.*
- Write the equation that represents the graph of the line shown below:

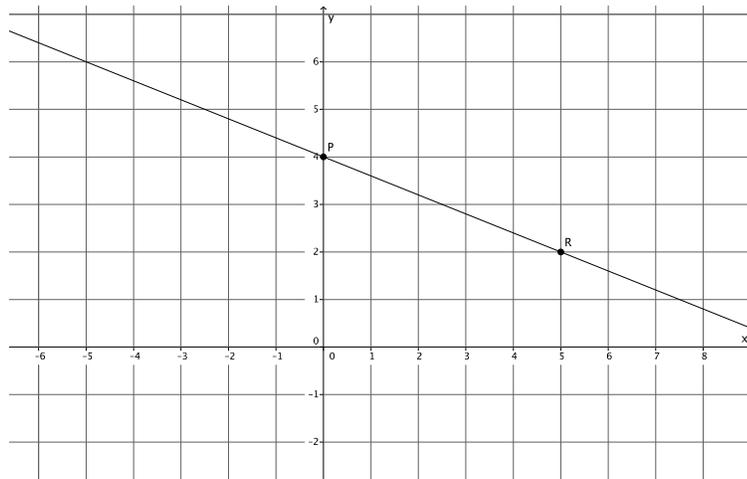


- First, identify the y-intercept.
 - *The line intersects the y-axis at (0, 4).*
- Now we must use what we know about slope to determine the slope of the line. Recall the following:

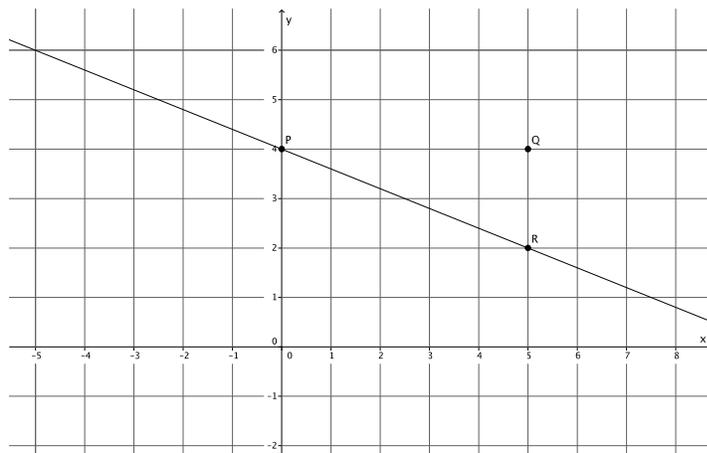
$$m = \frac{|QR|}{|PQ|}$$

The point P represents our y-intercept. Let's locate a point R on the line with integer coordinates.

- *We can use the point (5, 2) or (-5, 6).*
- We can use either point. For this example, let's use (5, 2).

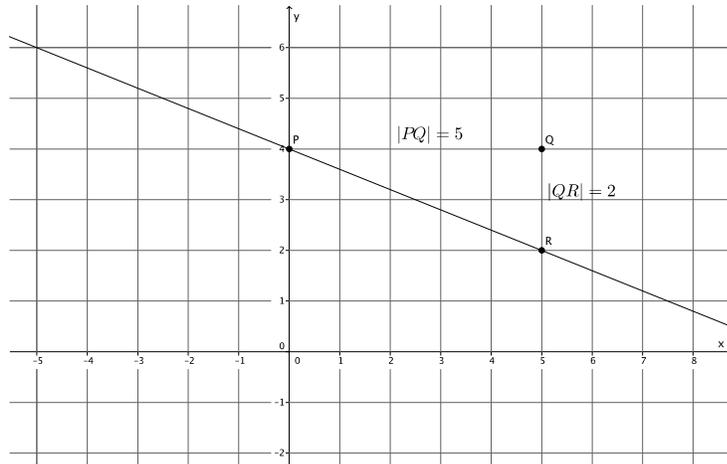


- Now we can locate point Q . It must be to the right of point P and be on a line parallel to the y-axis that goes through point R . What is the location of Q ?
 - *Point Q must be (5, 4).*
- What fraction represents the slope of this line?



▫ *The slope $m = -\frac{2}{5}$.*

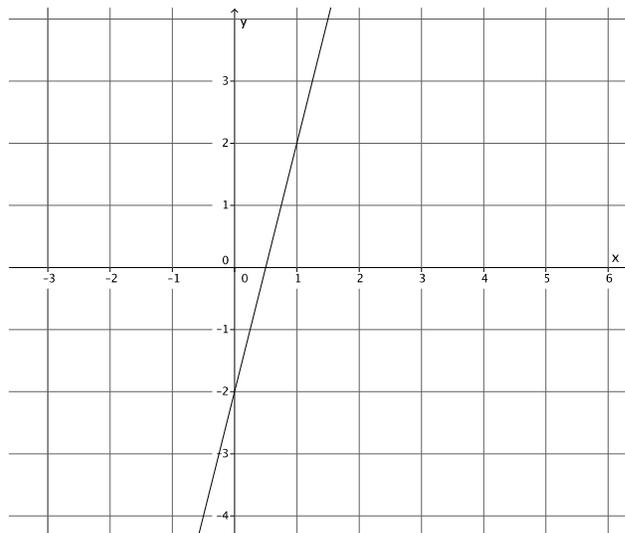
- The slope of the line is $m = -\frac{2}{5}$ and the y -intercept is $(0, 4)$. What must the equation of the line be?



- The line is the graph of $y = -\frac{2}{5}x + 4$.

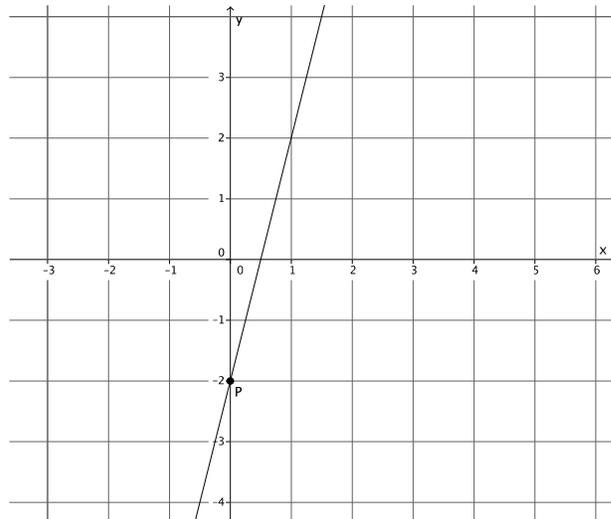
Example 2 (5 minutes)

- What is the y -intercept of the line?



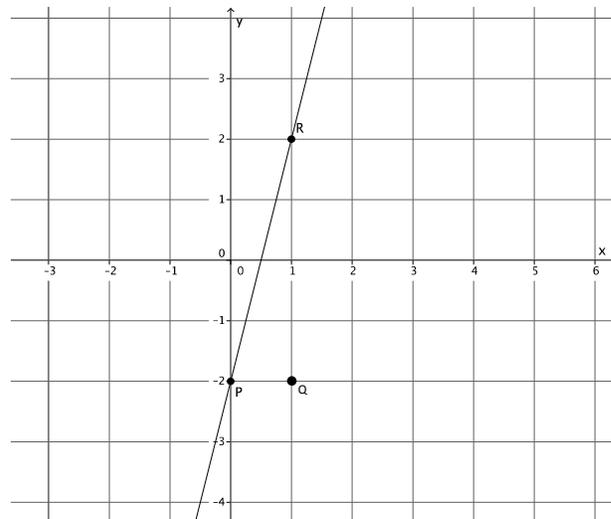
- The y -intercept is $(0, -2)$.

- Select another point, R , on the line with integer coordinates.



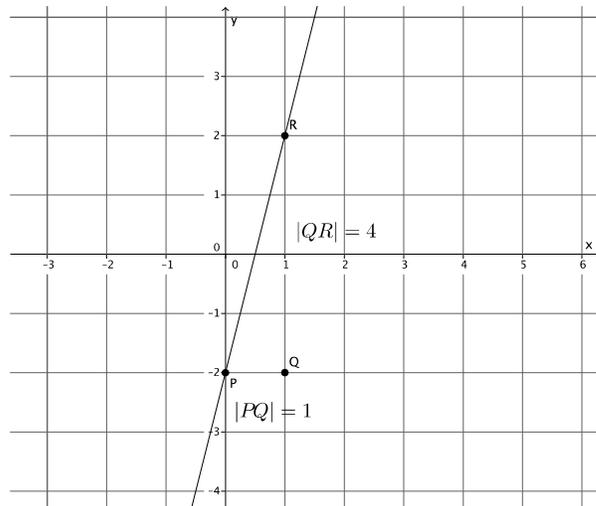
- Let R be the point located at $(1, 2)$.

- Now place point Q and find the slope of the line.



- The slope of the line is $m = 4$.

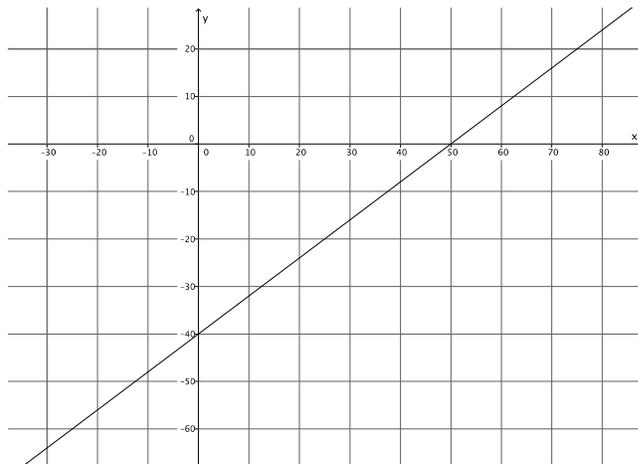
- Write the equation for the line.



- The line is the graph of $y = 4x - 2$.

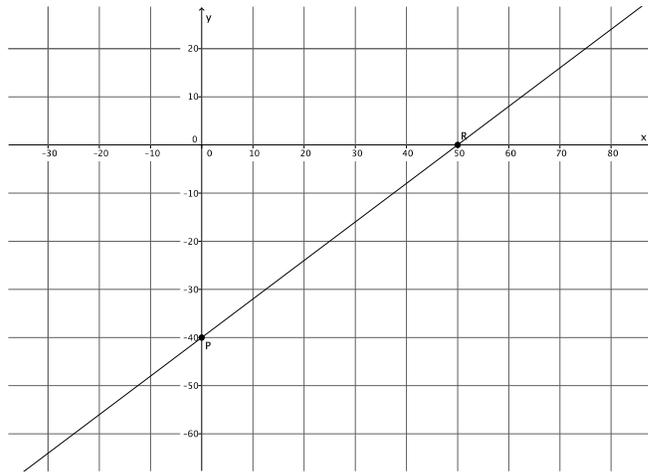
Example 3 (5 minutes)

- What is the y-intercept of the line? Notice the units on the coordinate plane have increased.



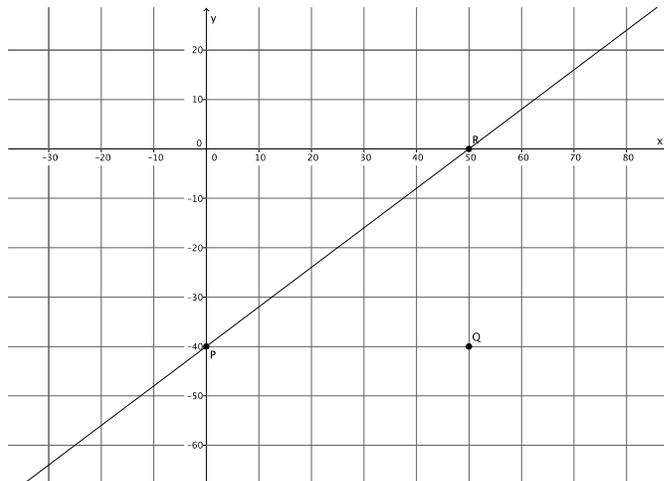
- The y-intercept is $(0, -40)$.

- Select another point, R , on the line with integer coordinates.



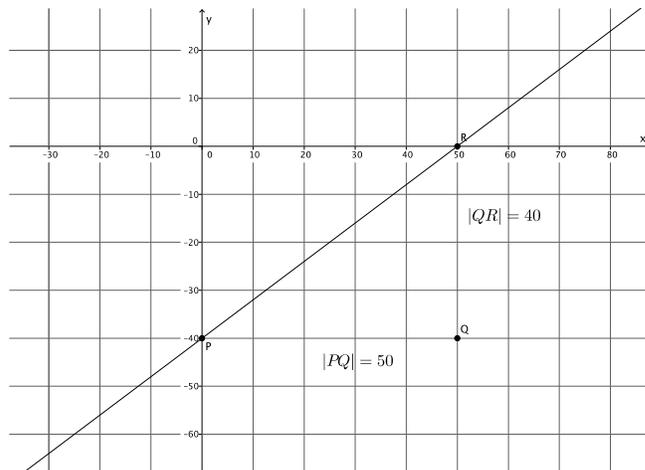
▫ Let $R = (50, 0)$.

- Now place point Q and find the slope of the line.



▫ The slope of the line is $m = \frac{40}{50} = \frac{4}{5}$.

- Write the equation for the line.



- The line is the graph of $y = \frac{4}{5}x - 40$.
- The last thing we will do to this linear equation is rewrite it in standard form $ax + by = c$ where a , b , and c are integers and a is not negative. That means we must multiply the entire equation by a number that will turn $\frac{4}{5}$ into an integer. What number should we multiply by?
 - $\frac{4}{5}(5) = 4$.
- We multiply the entire equation by 5:

$$\begin{aligned} \left(y = \frac{4}{5}x - 40\right) 5 \\ 5y = 4x - 200 \\ -4x + 5y = 4x - 4x - 200 \\ -4x + 5y = -200 \\ -1(-4x + 5y = -200) \\ 4x - 5y = 200 \end{aligned}$$

The standard form of the linear equation is $4x - 5y = 200$.

Exercises 1–6 (10 minutes)

Students complete Exercises 1–6 independently.

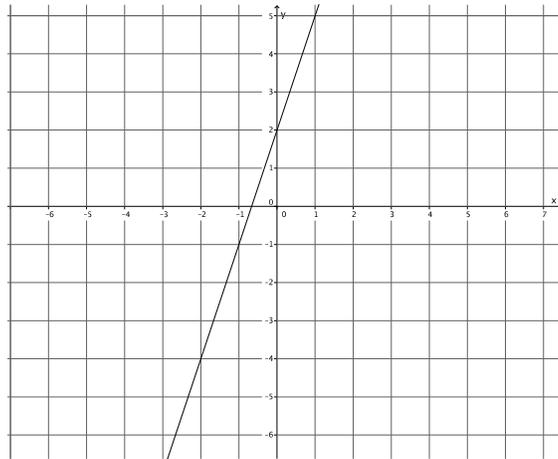
Exercises

1. Write the equation that represents the line shown.

$$y = 3x + 2$$

Use the properties of equality to change the equation from slope intercept form, $y = mx + b$, to standard form, $ax + by = c$, where a , b , and c are integers and a is not negative.

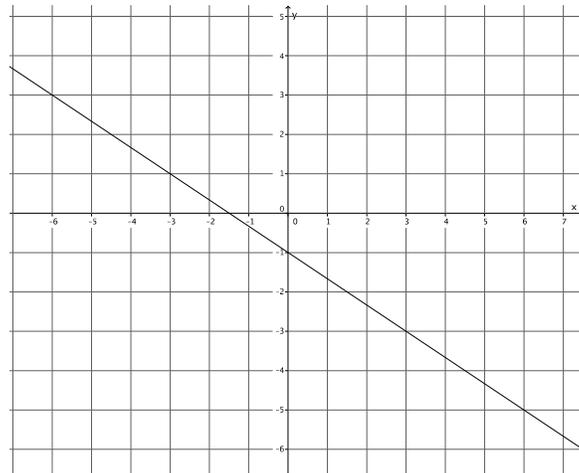
$$\begin{aligned} y &= 3x + 2 \\ -3x + y &= 3x - 3x + 2 \\ -3x + y &= 2 \\ -1(-3x + y = 2) \\ 3x - y &= -2 \end{aligned}$$



2. Write the equation that represents the line shown.

Use the properties of equality to change the equation from slope intercept form, $y = mx + b$, to standard form, $ax + by = c$, where a , b , and c are integers and a is not negative.

$$\begin{aligned} y &= -\frac{2}{3}x - 1 \\ \left(y = -\frac{2}{3}x - 1\right) 3 \\ 3y &= -2x - 3 \\ 2x + 3y &= -2x + 2x - 3 \\ 2x + 3y &= -3 \end{aligned}$$



3. Write the equation that represents the line shown.

Use the properties of equality to change the equation from slope intercept form, $y = mx + b$, to standard form, $ax + by = c$, where a , b , and c are integers and a is not negative.

$$y = -\frac{1}{5}x - 4$$

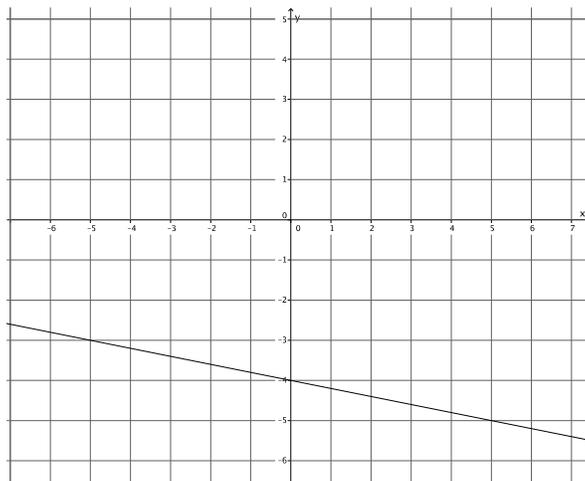
$$\left(y = -\frac{1}{5}x - 4\right) 5$$

$$5y = -x - 20$$

$$x + 5y = -x + x - 20$$

$$x + 5y = -20$$

$$x + 5y = -20$$



4. Write the equation that represents the line shown.

$$y = x$$

Use the properties of equality to change the equation from slope intercept form, $y = mx + b$, to standard form, $ax + by = c$, where a , b , and c are integers and a is not negative.

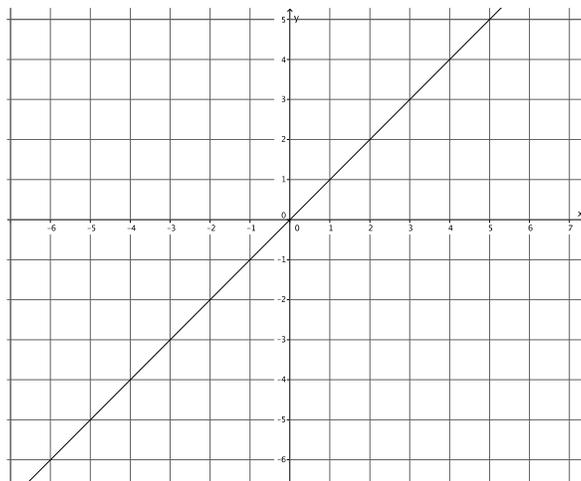
$$y = x$$

$$-x + y = x - x$$

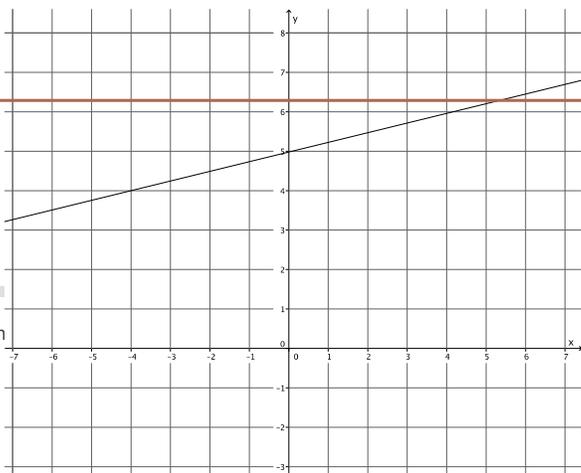
$$-x + y = 0$$

$$-1(-x + y = 0)$$

$$x - y = 0$$



5. Write the equation that represents the line shown.



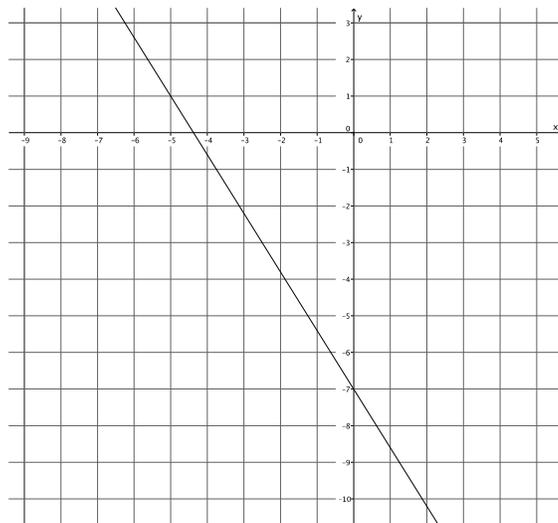
Use the properties of equality to change the equation from slope intercept form, $y = mx + b$, to standard form, $ax + by = c$, where a , b , and c are integers and a is not negative.

$$\begin{aligned}
 y &= \frac{1}{4}x + 5 \\
 \left(y = \frac{1}{4}x + 5\right) 4 \\
 4y &= x + 20 \\
 -x + 4y &= x - x + 20 \\
 -x + 4y &= 20 \\
 -1(-x + 4y = 20) \\
 x - 4y &= -20
 \end{aligned}$$

6. Write the equation that represents the line shown.

Use the properties of equality to change the equation from slope intercept form, $y = mx + b$, to standard form, $ax + by = c$, where a , b , and c are integers and a is not negative.

$$\begin{aligned}
 y &= -\frac{8}{5}x - 7 \\
 \left(y = -\frac{8}{5}x - 7\right) 5 \\
 5y &= -8x - 35 \\
 8x + 5y &= -8x + 8x - 35 \\
 8x + 5y &= -35
 \end{aligned}$$



Closing (5 minutes)

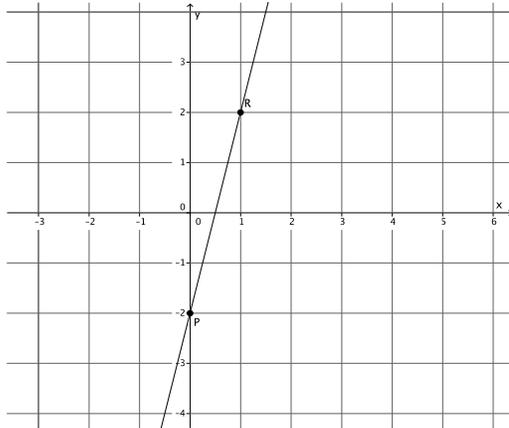
Summarize, or ask students to summarize, the main points from the lesson:

- We know that every line is a graph of a linear equation.
- We know how to use the y -intercept and the slope of a line to write the equation that represents the graph of a line.

Lesson Summary

Write the equation of a line by determining the y -intercept, $(0, b)$ and the slope, m , and replacing the numbers b and m into the equation $y = mx + b$.

Example:



The y -intercept of this graph is $(0, -2)$.

The slope of this graph is $m = \frac{4}{1} = 4$.

The equation that represents the graph of this line is $y = 4x - 2$.

Exit Ticket (5 minutes)

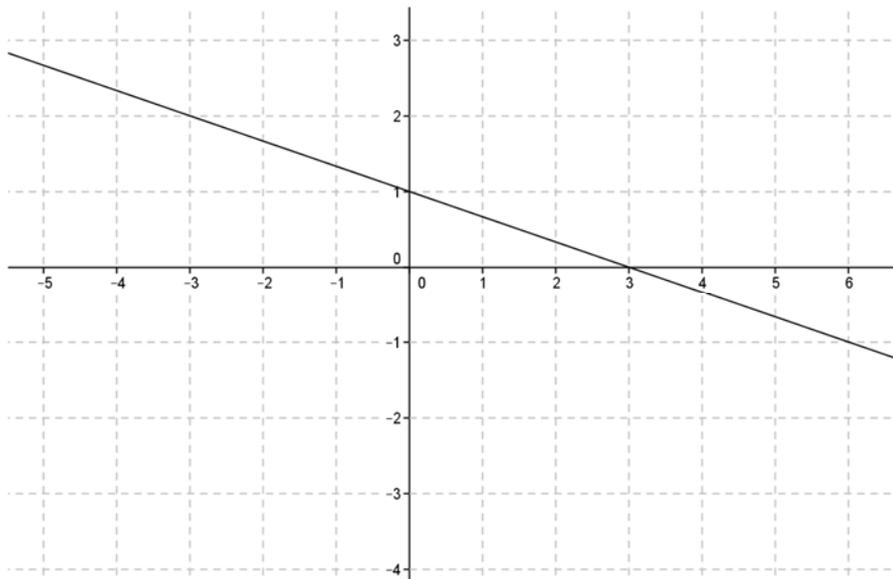
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Lesson 20: Every Line is a Graph of a Linear Equation

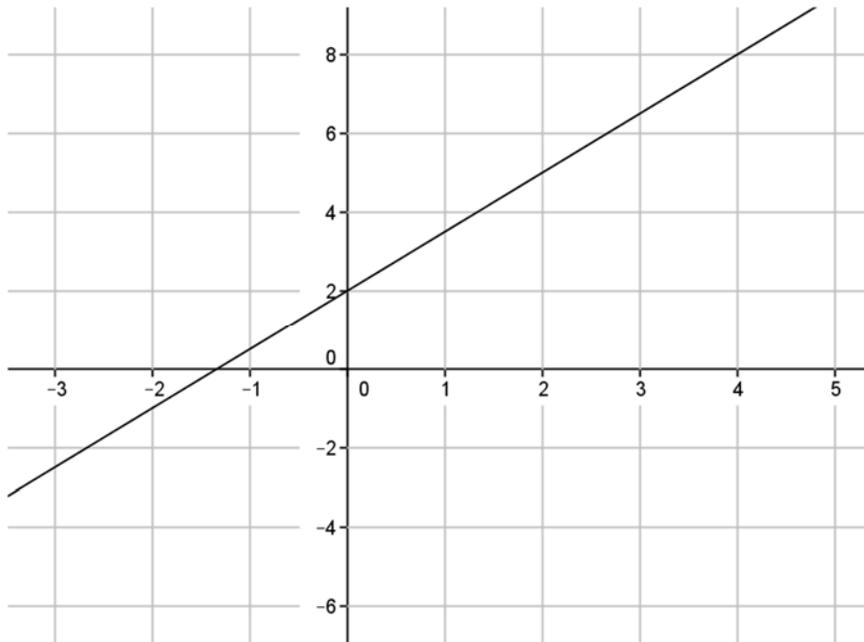
Exit Ticket

1. Write an equation in slope intercept form that represents the line shown.



2. Use the properties of equality to change the equation you wrote for problem 1 from slope intercept form, $y = mx + b$, to standard form, $ax + by = c$, where a , b , and c are integers and a is not negative.

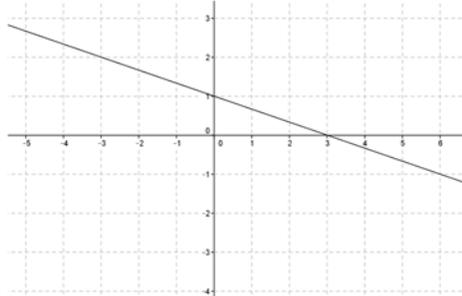
3. Write an equation in slope intercept form that represents the line shown.



4. Use the properties of equality to change the equation you wrote for problem 3 from slope intercept form, $y = mx + b$, to standard form, $ax + by = c$, where a , b , and c are integers and a is not negative.

Exit Ticket Sample Solutions

1. Write an equation in slope intercept form that represents the line shown.

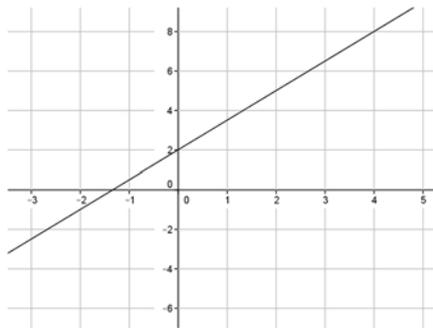


$$y = -\frac{1}{3}x + 1$$

2. Use the properties of equality to change the equation you wrote for problem 1 from slope intercept form, $y = mx + b$, to standard form, $ax + by = c$, where a , b , and c are integers and a is not negative.

$$\begin{aligned} y &= -\frac{1}{3}x + 1 \\ \left(y = -\frac{1}{3}x + 1\right) 3 \\ 3y &= -x + 3 \\ x + 3y &= -x + x + 3 \\ x + 3y &= 3 \end{aligned}$$

3. Write an equation in slope intercept form that represents the line shown.



$$y = \frac{3}{2}x + 2$$

4. Use the properties of equality to change the equation you wrote for problem 3 from slope intercept form, $y = mx + b$, to standard form, $ax + by = c$, where a , b , and c are integers and a is not negative.

$$\begin{aligned} y &= \frac{3}{2}x + 2 \\ \left(y = \frac{3}{2}x + 2\right) 2 \\ 2y &= 3x + 4 \\ -3x + 2y &= 3x - 3x + 4 \\ -3x + 2y &= 4 \\ -1(-3x + 2y = 4) \\ 3x - 2y &= -4 \end{aligned}$$

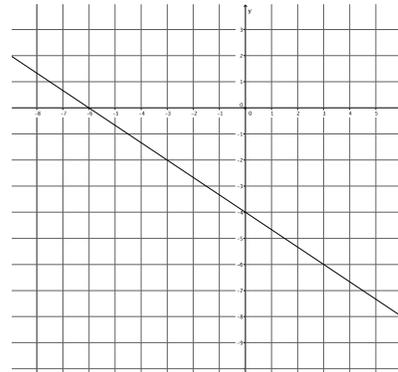
Problem Set Sample Solutions

Students practice writing equations for lines.

1. Write the equation that represents the line shown.

$$y = -\frac{2}{3}x - 4$$

Use the properties of equality to change the equation from slope intercept form, $y = mx + b$, to standard form, $ax + by = c$, where a , b , and c are integers and a is not negative.

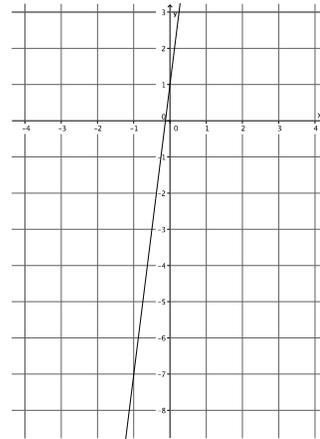


2. Write the equation that represents the line shown.

$$y = 8x + 1$$

Use the properties of equality to change the equation from slope intercept form, $y = mx + b$, to standard form, $ax + by = c$, where a , b , and c are integers and a is not negative.

$$\begin{aligned} y &= 8x + 1 \\ -8x + y &= 8x - 8x + 1 \\ -8x + y &= 1 \\ -1(-8x + y = 1) \\ 8x - y &= -1 \end{aligned}$$

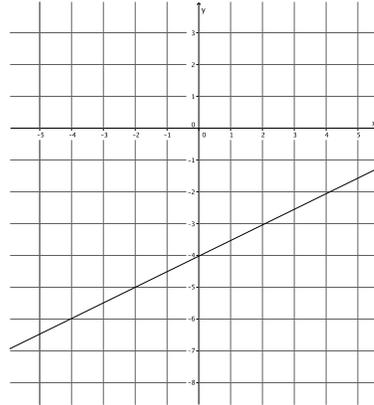


3. Write the equation that represents the line shown.

$$y = \frac{1}{2}x - 4$$

Use the properties of equality to change the equation from slope intercept form, $y = mx + b$, to standard form, $ax + by = c$, where a , b , and c are integers and a is not negative.

$$\begin{aligned} y &= \frac{1}{2}x - 4 \\ \left(y = \frac{1}{2}x - 4\right) 2 \\ 2y &= x - 8 \\ -x + 2y &= x - x - 8 \\ -x + 2y &= -8 \\ -1(-x + 2y = -8) \\ x - 2y &= 8 \end{aligned}$$

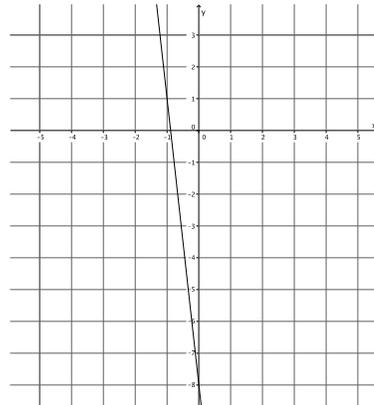


4. Write the equation that represents the line shown.

$$y = -9x - 8$$

Use the properties of equality to change the equation from slope intercept form, $y = mx + b$, to standard form, $ax + by = c$, where a , b , and c are integers and a is not negative.

$$\begin{aligned} y &= -9x - 8 \\ 9x + y &= -9x + 9x - 8 \\ 9x + y &= -8 \end{aligned}$$

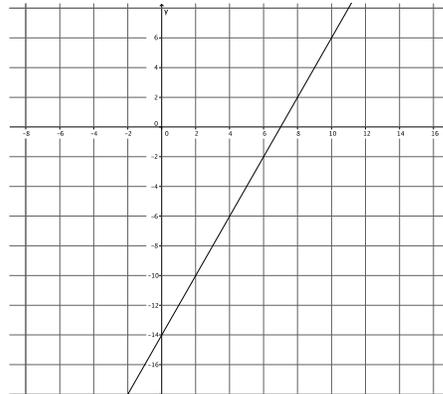


5. Write the equation that represents the line shown.

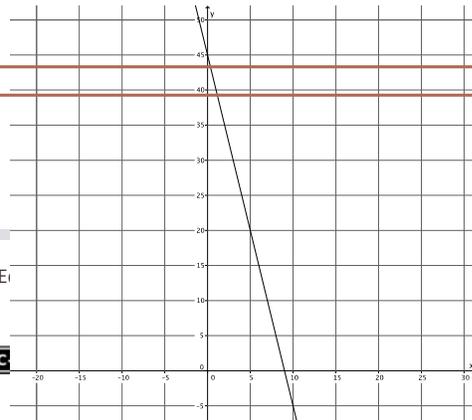
$$y = 2x - 14$$

Use the properties of equality to change the equation from slope intercept form, $y = mx + b$, to standard form, $ax + by = c$, where a , b , and c are integers and a is not negative.

$$\begin{aligned} y &= 2x - 14 \\ -2x + y &= 2x - 2x - 14 \\ -2x + y &= -14 \\ -1(-2x + y = -14) \\ 2x - y &= 14 \end{aligned}$$



6. Write the equation that represents the line shown.





$$y = -5x + 45$$

Use the properties of equality to change the equation from slope intercept form, $y = mx + b$, to standard form, $ax + by = c$, where a , b , and c are integers and a is not negative.

$$\begin{aligned}y &= -5x + 45 \\5x + y &= -5x + 5x + 45 \\5x + y &= 45\end{aligned}$$