



Lesson 22: Constant Rates Revisited

Student Outcomes

- Students know that any constant rate problem can be described by a linear equation in two variables where the slope of the graph is the constant rate.
- Students compare two different proportional relationships represented by graphs, equations, and tables to determine which has a greater rate of change.

Classwork

Example 1 (8 minutes)

- Recall our definition of constant rate: If the average speed of motion over any time interval is equal to the same constant, then we say the motion has constant speed.
- Erika set her stopwatch to zero and switched it on at the beginning of her walk. She walks at a constant speed of 3 miles per hour. Work in pairs to express this situation as an equation, table of values, and a graph.

Scaffolding:

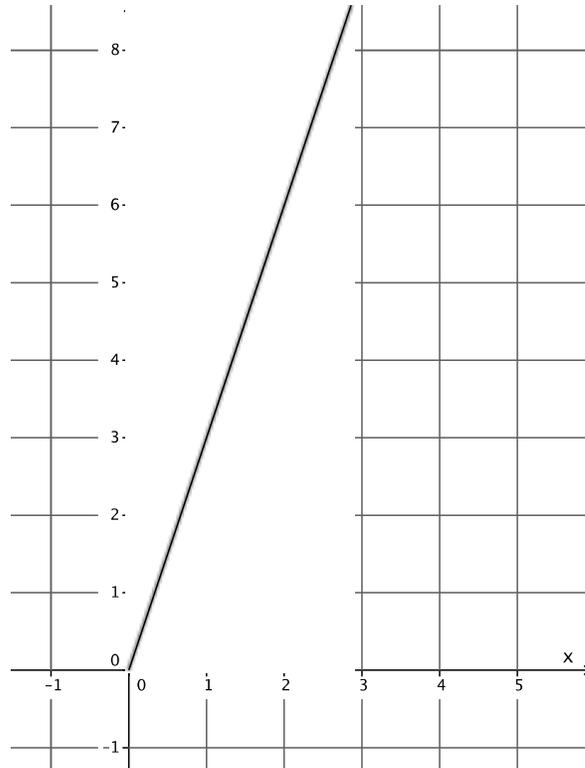
You may need to review the definition of average speed as well: Average speed is the distance (or area, pages typed, etc.) divided by the time interval spent moving that distance (or painting the given area, or typing a specific number of pages, etc.)

Provide students time to analyze the situation and represent it in the requested forms. If necessary, provide support to students with the next few bullet points. Once students have finished working resume the work on the example (bullets below the graph).

- Suppose Erika walked a distance of y miles in x hours. Then her average speed in the time interval from 0 to x hours is $\frac{y}{x}$. Since Erika walks at a constant speed of 3 miles per hour, we can express her motion as $\frac{y}{x} = 3$.
- Solve the equation for y .
 - $\frac{y}{x} = 3$
 - $y = 3x$
- We can use the equation to develop a table of values where x is the number of hours Erika walks and y is the distance she walks in miles. Complete the table for the given x -values.

x	y
0	0
1	3
2	6
3	9

- In terms of x and y , $y = 3x$ is a linear equation in two variables. Its graph is a line.



MP.7

- What is the meaning of the 3 in the equation $y = 3x$?
 - The 3 is the slope of the line. It represents the rate at which Erika walks.
- We can prove that her motion is constant, i.e., she walks at a constant speed, by showing that the slope is 3 over any time interval that she walks:
Assume Erika walks y_1 miles in the time interval from 0 to x_1 , and she walks y_2 miles in the time interval from 0 to x_2 .

Her average distance between time interval x_1 and $x_2 = \frac{y_2 - y_1}{x_2 - x_1}$

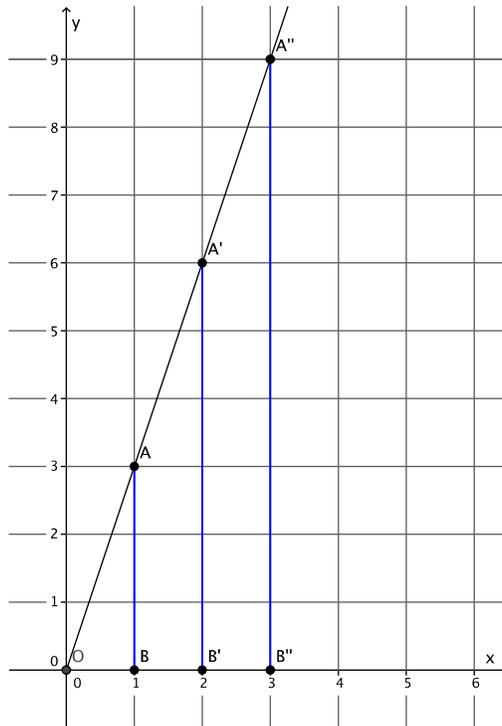
We know that $y = 3x$; therefore, $y_2 = 3x_2$ and $y_1 = 3x_1$. Then by substitution we have

$$\begin{aligned} &= \frac{3x_2 - 3x_1}{x_2 - x_1} \\ &= \frac{3(x_2 - x_1)}{x_2 - x_1} \\ &= 3 \end{aligned}$$

Scaffolding:
Explain that we are looking at the distance traveled between times x_1 and x_2 ; therefore, we must subtract the distances traveled to represent the total distance traveled in $x_2 - x_1$. Concrete numbers and a diagram like the following may be useful as well:

Average speed of walking over interval from 1 hour to 4 hours = $\frac{\text{distance traveled in time interval}}{\text{time of interval}} = \frac{12 - 3}{4 - 1} = \frac{9}{3} = 3$

- Therefore, Erika’s average speed in the time interval between x_1 and x_2 is 3 mph. Since x_1 and x_2 can be any two times, then the average speed she walks over any time interval is 3 mph and we have confirmed that she walks at a constant speed of 3 mph. We can also show that her motion is constant by looking at the slope between different points on the graph:



- Are triangles $\triangle AOB$, $\triangle A'OB'$, and $\triangle A''OB''$ similar?
 - Yes, because each triangle has a common angle, $\angle AOB$ and a right angle. By the AA criterion the triangles are similar.
- We used similar triangles before to show that the slope of a line would be the same between any two points. Since we have similar triangles, we can conclude that the rate at which Erika walks is constant because the ratio of the corresponding sides will be equal to a constant, i.e., the scale factor of dilation. For example, when we compare the ratio of corresponding sides for triangles $\triangle A'OB'$ and $\triangle A''OB''$ we see that

$$\frac{9}{6} = \frac{3}{2}$$

Make clear to students that the ratio of corresponding sides will not be equal to the slope of the line; rather, the ratios will be equal to one another and the scale factor of dilation.

- Or we could compare the ratio of corresponding side lengths for triangles $\triangle AOB$ and $\triangle A'OB'$:

$$\frac{6}{3} = \frac{2}{1}$$

- Again, because the ratio of corresponding sides are equal, we know that the rate at which Erika walks is constant.

Example 2 (5 minutes)

MP.4

- A faucet leaks at a constant rate of 7 gallons per hour. Suppose y gallons leak in x hours from 6:00 a.m. Express the situation as a linear equation in two variables.
- If we say the leak began at 6:00 a.m. and the total number of gallons leaked over the time interval from 6:00 a.m. to x hours is y , then we know the average rate of the leak is $\frac{y}{x}$.
- Since we know that the faucet leaks at a constant rate, then we can write the linear equation in two variables as $\frac{y}{x} = 7$, which can then be transformed to $y = 7x$.
- Again, take note of the fact that the number that represents slope in the equation is the same as the constant rate of water leaking.
- Another faucet leaks at a constant rate and the table below shows the number of gallons y , that leak in x hours.

Hours (x)	Gallons (y)
2	13
4	26
7	45.5
10	65

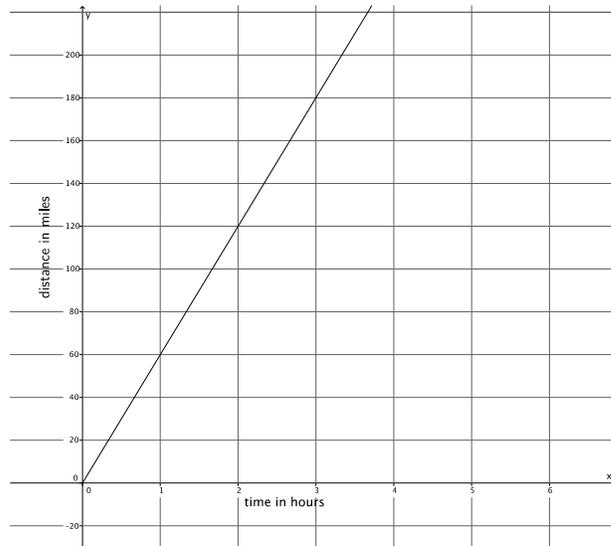
- How can we determine the rate at which this faucet leaks?
 - *If these were points on a graph, we could find the slope between them.*
- Using what you know about slope, determine the rate the faucet leaks.
 - *Sample student work:*

$$\begin{aligned}
 m &= \frac{26 - 13}{4 - 2} \\
 &= \frac{13}{2} \\
 &= 6.5
 \end{aligned}$$

- The number of gallons that leak from each faucet is dependent on the amount of time the faucet leaks. The information provided for each faucet's rate of change, i.e., slope allows us to answer a question, such as the following: Which faucet has the worse leak? That is, which faucet leaks more water over a given time interval?
 - *The first faucet has the worse leak because the rate is greater, 7 compared to 6.5.*

Example 3 (4 minutes)

- The graph below represents the constant rate at which Train A travels.



- What is the constant rate of travel for Train A?
 - We know that the constant rate of travel is the same as the slope. On the graph, we can see that the train is traveling 60 miles every hour. That means that the slope of the line is 60.
- Train B travels at a constant rate. The train travels at an average rate of 95 miles every one and a half hours. We want to know which train is traveling at a greater speed. Let's begin by writing a linear equation that represents the constant rate of Train B. If y represents the total distance traveled over any time period x , then

$$\frac{95}{1.5} = \frac{y}{x}$$

$$y = \frac{95}{1.5}x$$

$$y = 63\frac{1}{3}x$$

- Which train is traveling at a greater speed? Explain.

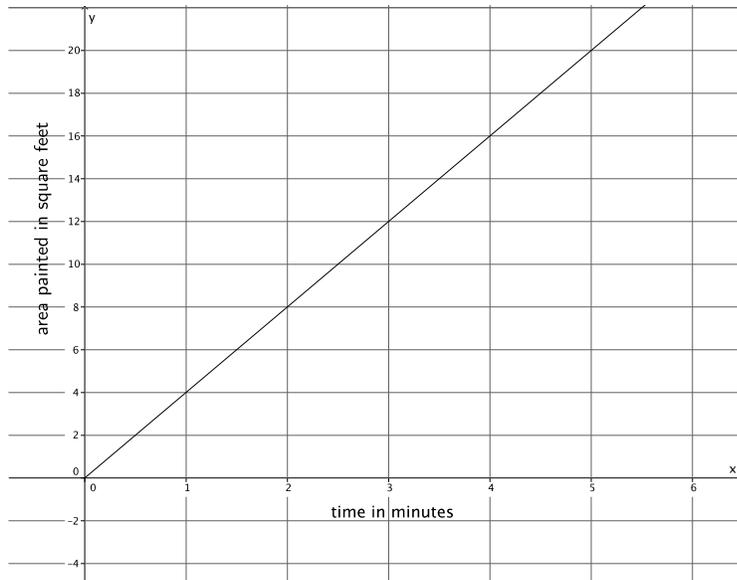
MP.4

Provide students time to talk to their partners about how to determine which train is traveling at a greater speed.

- Train B is traveling at a greater speed. The graph provided the information about the constant rate of Train A, that is, Train A travels at a constant rate of 60 miles per hour. The given rate for Train B, the slope of the graph for Train B, is equal to $63\frac{1}{3}$. Since $63\frac{1}{3} > 60$, Train B is traveling at a greater speed.
- Why do you think the strategy of comparing each rate of change allows us to determine which train is travelling at a greater speed?
 - The distance that each train travels is proportional to the amount of time the train travels. Therefore, if we can describe the rate of change as a number, i.e. slope, and compare the numbers, we can determine which train travels at a greater speed.

Example 4 (5 minutes)

- The graph below represents the constant rate at which Kristina can paint.



- Her sister Tracee paints at an average rate of 45 square feet in 12 minutes. Assuming Tracee paints at a constant rate, determine which sister paints faster.

Provide students time to determine the linear equation that represents Tracee’s constant rate and then discuss with their partner who is the faster painter.

- *If we let y represent the total area Tracee paints in x minutes, then her constant rate is*

$$\frac{45}{12} = \frac{y}{x}$$

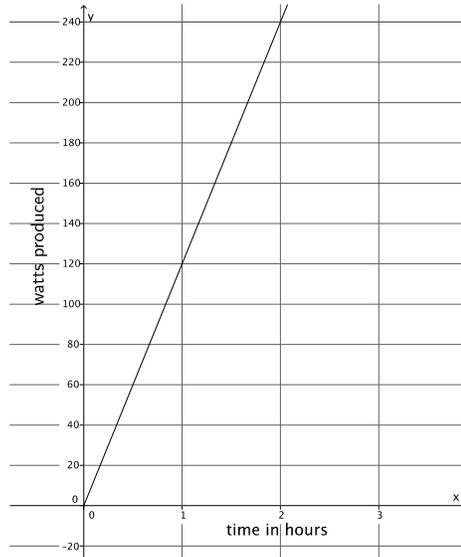
$$\frac{45}{12}x = y$$

We need to compare the slope of the line for Kristina with the slope of the equation for Tracee. The slope of the line is 4 and the slope in the equation that represents Tracee’s rate is $\frac{45}{12}$. Since $4 > \frac{45}{12}$, Kristina paints at a faster rate.

- How does the slope provide the information we need to answer the question about which sister paints faster?
 - *The slope describes the rate of change for proportional relationships. If we know which rate of change is greater, then we can determine which sister paints faster.*

Example 5 (5 minutes)

- The graph below represents the constant rate of watts of energy produced from a single solar panel produced by Company A.



- Company B offers a solar panel that produces energy at an average rate of 325 watts in 2.6 hours. Assuming solar panels produce energy at a constant rate, determine which company produces more efficient solar panels (solar panels that produce more energy per hour).

Provide students time to work with their partners to answer the question.

- *If we let y represent the energy produced by a solar panel made by Company B in x minutes, then the constant rate is*

$$\frac{325}{2.6} = \frac{y}{x}$$

$$\frac{325}{2.6}x = y$$

$$125x = y$$

We need to compare the slope of the line for Company A with the slope in the equation that represents the rate for Company B. The slope of the line representing Company A is 120, and the slope of the line representing Company B is equal to 125. Since $125 > 120$, Company B produces the more efficient solar panel.

Exercises 1–5 (10 minutes)

Students work in pairs or small groups to complete Exercises 1–5. Discuss Exercise 5 as it generalizes a strategy for comparing proportional relationships.

Exercises

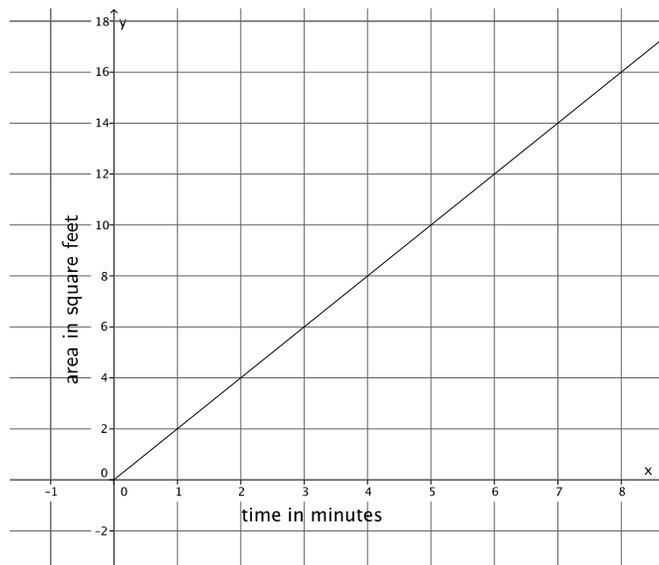
1. Peter paints a wall at a constant rate of 2 square-feet per minute. Assume he paints an area y , in square feet after x minutes.

- a. Express this situation as a linear equation in two variables.

$$\frac{y}{x} = \frac{2}{1}$$

$$y = 2x$$

- b. Graph the linear equation.



- c. Using the graph or the equation, determine the total area he paints after 8 minutes, $1\frac{1}{2}$ hours, and 2 hours. Note that the units are in minutes and hours.

In 8 minutes, he paints 16 square feet.

In $1\frac{1}{2}$ hours, he paints

$$y = 2(90)$$

$$= 180$$

180 square feet.

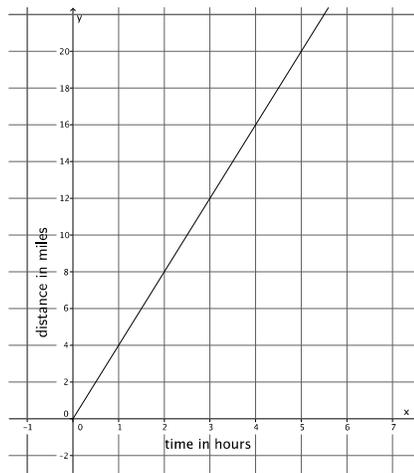
In 2 hours, he paints

$$y = 2(120)$$

$$= 240$$

240 square feet.

2. The graph below represents Nathan’s constant rate of walking.



- a. Nicole just finished a 5 mile walkathon. It took her 1.4 hours. Assume she walks at a constant rate. Let y represent the distance Nicole walks in x hours. Describe Nicole’s walking at a constant rate as a linear equation in two variables.

$$\begin{aligned} \frac{y}{x} &= \frac{5}{1.4} \\ y &= \frac{25}{7}x \end{aligned}$$

- b. Who walks at a greater speed? Explain.

Nathan walks at a greater speed. The slope of the graph for Nathan is 4, and the slope or rate for Nicole is $\frac{25}{7}$. When you compare the slopes, you see that $4 > \frac{25}{7}$.

3. a. Susan can type 4 pages of text in 10 minutes. Assuming she types at a constant rate, write the linear equation that represents the situation.

Let y represent the total number of pages Susan can type in x minutes. We can write $\frac{y}{x} = \frac{4}{10}$ and $y = \frac{2}{5}x$.

- b. The table of values below represents the number of pages that Anne can type, y , in x minutes. Assume she types at a constant rate.

Minutes (x)	Pages Typed (y)
3	2
5	$\frac{10}{3}$
8	$\frac{16}{3}$
10	$\frac{20}{3}$

Who types faster? Explain.

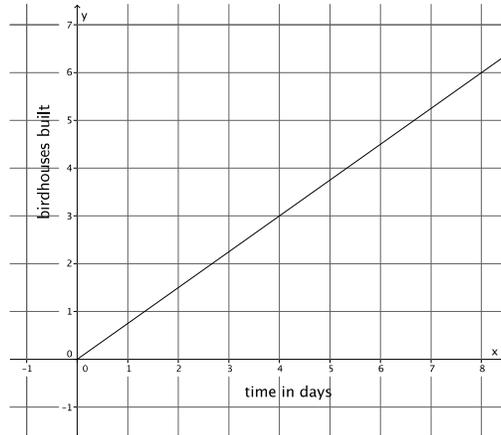
Anne types faster. Using the table we can determine the slope that represents Anne’s constant rate of typing is $\frac{2}{3}$. The slope or rate for Nicole is $\frac{2}{5}$. When you compare the slopes you see that $\frac{2}{3} > \frac{2}{5}$.

4. a. Phil can build 3 birdhouses in 5 days. Assuming he builds birdhouses at a constant rate, write the linear equation that represents the situation.

Let y represent the total number of birdhouses Phil can build in x days. We can write $\frac{y}{x} = \frac{3}{5}$ and $y = \frac{3}{5}x$.

- b. The graph represents Karl's constant rate of building the same kind of birdhouses. Who builds birdhouses faster? Explain.

Karl can build birdhouses faster. The slope of the graph for Karl is $\frac{3}{4}$, and the slope or rate of change for Phil is $\frac{3}{5}$. When you compare the slopes, $\frac{3}{4} > \frac{3}{5}$.



5. Explain your general strategy for comparing proportional relationships.

When comparing proportional relationships we look specifically at the rate of change for each situation. The relationship with the greater rate of change will end up producing more, painting a greater area, or walking faster when compared to the same amount of time with the other proportional relationship.

Closing (4 minutes)

Summarize, or ask students to summarize, the main points from the lesson:

- We know how to write a constant rate problem as a linear equation in two variables.
- We know that in order to determine who has a greater speed (or is faster at completing a task), we need only to compare the slopes (rates of change) that apply to two situations. Whichever has the greater slope has the greater rate. We can determine rate (slope) from an equation, a graph, or a table.

Lesson Summary

Problems involving constant rate can be expressed as linear equations in two variables.

When given information about two proportional relationships, determine which is less or greater by comparing their slopes (rates of change).

Exit Ticket (4 minutes)



Name _____

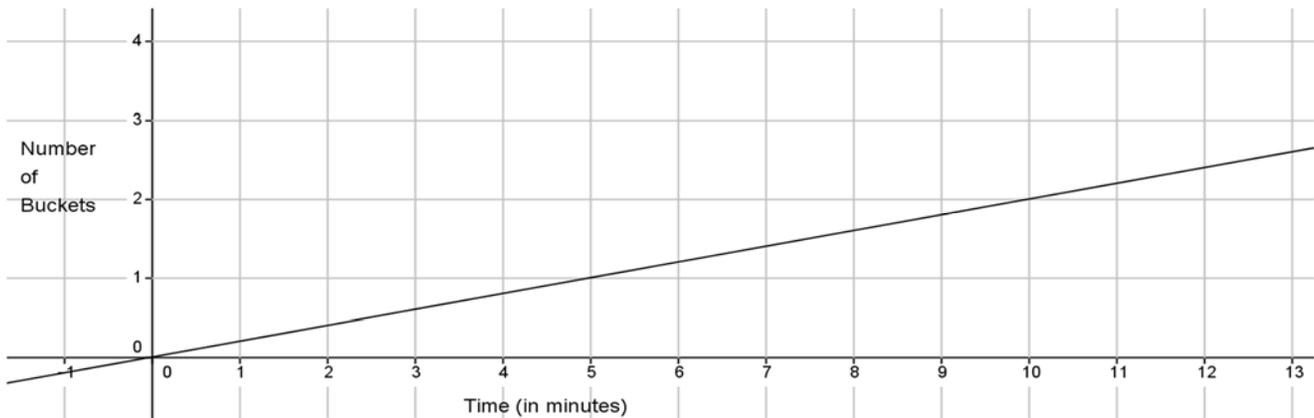
Date _____

Lesson 22: Constant Rates Revisited

Exit Ticket

1. Water flows out of Pipe A at a constant rate. Pipe A can fill 3 buckets of the same size in 14 minutes.
 - a. Write a linear equation that represents the situation.

The graph below represents the rate at which Pipe B can fill the same sized buckets.



- b. Which pipe fills buckets faster? Explain.

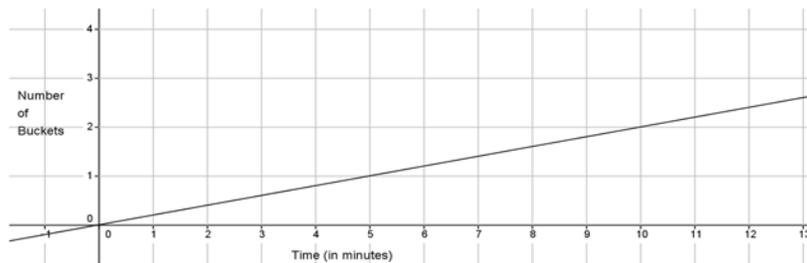
Exit Ticket Sample Solutions

1. Water flows out of Pipe A at a constant rate. Pipe A can fill 3 buckets of the same size in 14 minutes.

a. Write a linear equation that represents the situation.

Let y represent the total number of buckets that Pipe A can fill in x minutes. We can write $\frac{y}{x} = \frac{3}{14}$ and $y = \frac{3}{14}x$.

The graph below represents the rate at which Pipe B can fill the same sized buckets.



b. Which pipe fills buckets faster? Explain.

Pipe A fills the same sized buckets faster than Pipe B. The slope of the graph for Pipe B is $\frac{1}{5}$, the slope or rate for Pipe A is $\frac{3}{14}$. When you compare the slopes you see that $\frac{3}{14} > \frac{1}{5}$.

Problem Set Sample Solutions

Students practice writing constant rate problems as linear equations in two variables. Students determine which of two proportional relationships is greater.

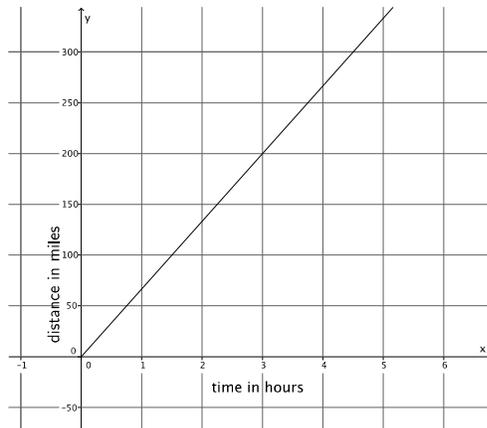
1. a. Train A can travel a distance of 500 miles in 8 hours. Assuming the train travels at a constant rate, write the linear equation that represents the situation.

Let y represent the total number of miles Train A travels in x minutes. We can write $\frac{y}{x} = \frac{500}{8}$ and $y = \frac{125}{2}x$.

b. The graph represents the constant rate of travel for Train B.

Which train is faster? Explain.

Train B is faster than Train A. The slope or rate for Train A is $\frac{125}{2}$, and the slope of the line for Train B is $\frac{200}{3}$. When you compare the slopes, $\frac{200}{3} > \frac{125}{2}$.



2. a. Natalie can paint 40 square feet in 9 minutes. Assuming she paints at a constant rate, write the linear equation that represents the situation.

Let y represent the total square feet Natalie can paint in x minutes. We can write $\frac{y}{x} = \frac{40}{9}$ and $y = \frac{40}{9}x$.

- b. The table of values below represents Steven’s constant rate of painting.

Minutes (x)	Area Painted (y)
3	10
5	$\frac{50}{3}$
6	20
8	$\frac{80}{3}$

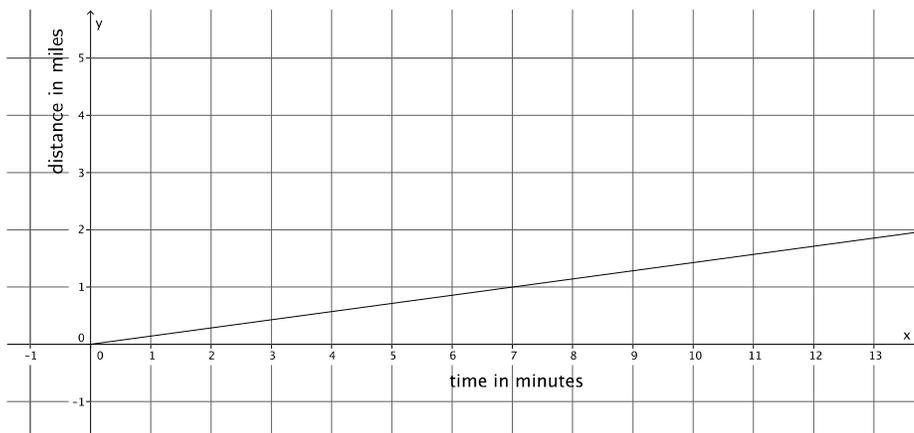
Who paints faster? Explain.

Natalie paints faster. Using the table of values I can find the slope that represents Steven’s constant rate of painting: $\frac{10}{3}$. The slope or rate for Natalie is $\frac{40}{9}$. When you compare the slopes you see that $\frac{40}{9} > \frac{10}{3}$.

3. a. Bianca can run 5 miles in 41 minutes. Assuming she runs at a constant rate, write the linear equation that represents the situation.

Let y represent the total number of miles Bianca can run in x minutes. We can write $\frac{y}{x} = \frac{5}{41}$ and $y = \frac{5}{41}x$.

- b. The graph below represents Cynthia’s constant rate of running.



Who runs faster? Explain.

Cynthia runs faster. The slope of the graph for Cynthia is $\frac{1}{7}$, and the slope or rate for Nicole is $\frac{5}{41}$. When you compare the slopes you see that $\frac{1}{7} > \frac{5}{41}$.

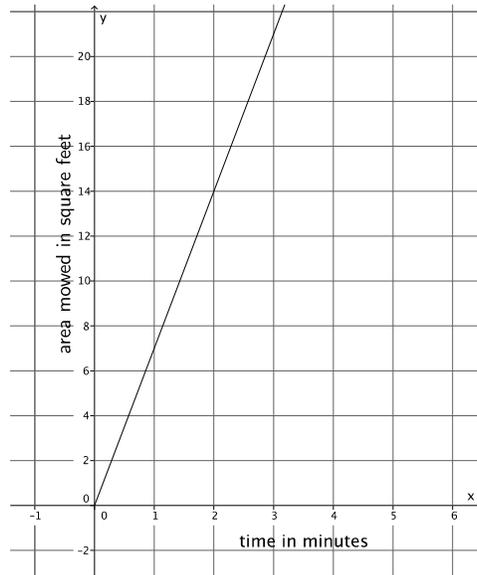
4. a. Geoff can mow an entire lawn of 450 square feet in 30 minutes. Assuming he mows at a constant rate, write the linear equation that represents the situation.

Let y represent the total number of square feet Geoff can mow in x minutes. We can write $\frac{y}{x} = \frac{450}{30}$ and $y = 15x$.

- b. The graph represents Mark's constant rate of mowing a lawn.

Who mows faster? Explain.

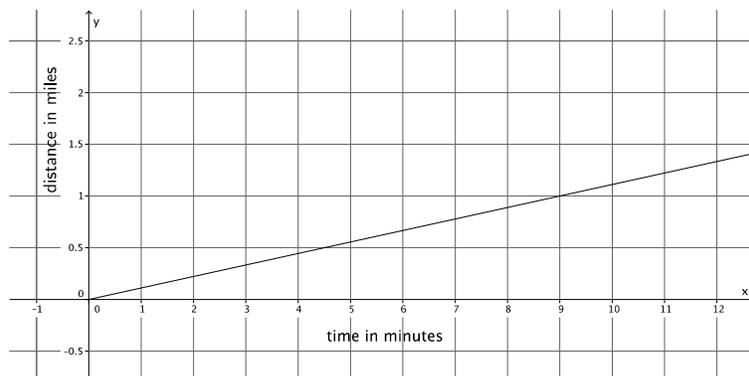
Geoff mows faster. The slope of the graph for Mark is $\frac{14}{2} = 7$, and the slope or rate for Geoff is $\frac{450}{30} = 15$. When you compare the slopes you see that $15 > 7$.



5. a. Juan can walk to school, a distance of 0.75 miles, in 8 minutes. Assuming he walks at a constant rate, write the linear equation that represents the situation.

Let y represent the total distance in miles that Juan can walk in x minutes. We can write $\frac{y}{x} = \frac{0.75}{8}$ and $y = \frac{3}{32}x$.

- b. The graph below represents Lena's constant rate of walking.



Who walks faster? Explain.

Lena walks faster. The slope of the graph for Lena is $\frac{1}{9}$ and the slope of the equation for Juan is $\frac{0.75}{8}$ or $\frac{3}{32}$. When you compare the slopes you see that $\frac{1}{9} > \frac{3}{32}$.