Lesson 23: The Defining Equation of a Line

Classwork

Exploratory Challenge/Exercises 1–3

1. Graph the equation $9x + 3y = 18$ using intercepts. Then answer parts (a)–(f) that follow.

   a. Graph the equation $y = -3x + 6$ on the same coordinate plane.

   b. What do you notice about the graphs of $9x + 3y = 18$ and $y = -3x + 6$? Why do you think this is so?

   c. Rewrite $y = -3x + 6$ in standard form.

   d. Identify the constants, a, b, c of the equation in standard form from part (c).

   e. Identify the constants of the equation $9x + 3y = 18$. Note them as a', b', and c'.

   f. What do you notice about $\frac{a'}{a}$, $\frac{b'}{b}$, and $\frac{c'}{c}$?

2. Graph the equation $y = \frac{1}{2}x + 3$ using the y-intercept and the slope. Then answer parts (a)–(f) that follow.

   a. Graph the equation $4x - 8y = -24$ using intercepts on the same coordinate plane.

   b. What do you notice about the graphs of $y = \frac{1}{2}x + 3$ and $4x - 8y = -24$? Why do you think this is so?

   c. Rewrite $y = \frac{1}{2}x + 3$ in standard form.

   d. Identify the constants, a, b, c of the equation in standard form from part (c).

   e. Identify the constants of the equation $-4x - 8y = -24$. Note them as a', b', and c'.

   f. What do you notice about $\frac{a'}{a}$, $\frac{b'}{b}$, and $\frac{c'}{c}$?

3. The equations $y = \frac{2}{3}x - 4$ and $6x - 9y = 36$ graph as the same line.

   a. Rewrite $y = \frac{2}{3}x - 4$ in standard form.

   b. Identify the constants, a, b, c, of the equation in standard form from part (a).

   c. Identify the constants of the equation $6x - 9y = 36$. Note them as a', b', and c'.

   d. What do you notice about $\frac{a'}{a}$, $\frac{b'}{b}$, and $\frac{c'}{c}$?

   e. You should have noticed that each fraction was equal to the same constant. Multiply that constant by the standard form of the equation from part (a). What do you notice?

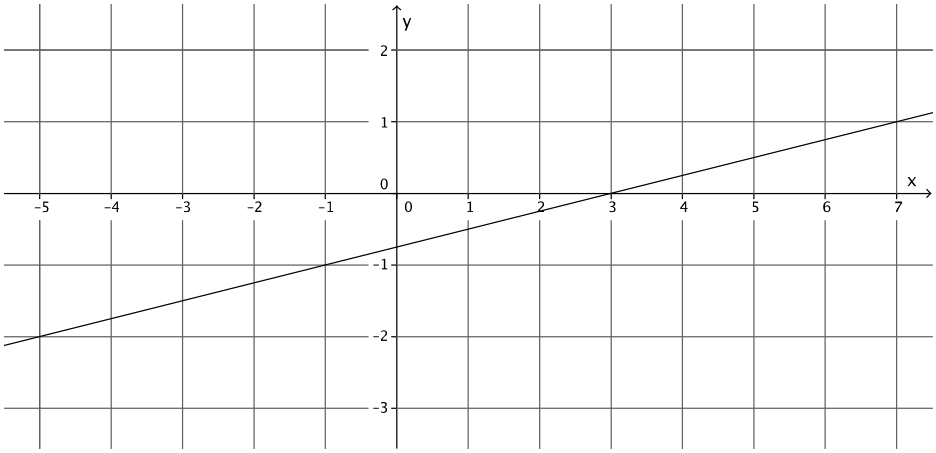
Exercises 4–8

4. Write three equations that would graph as the same line as the equation $3x + 2y = 7$.

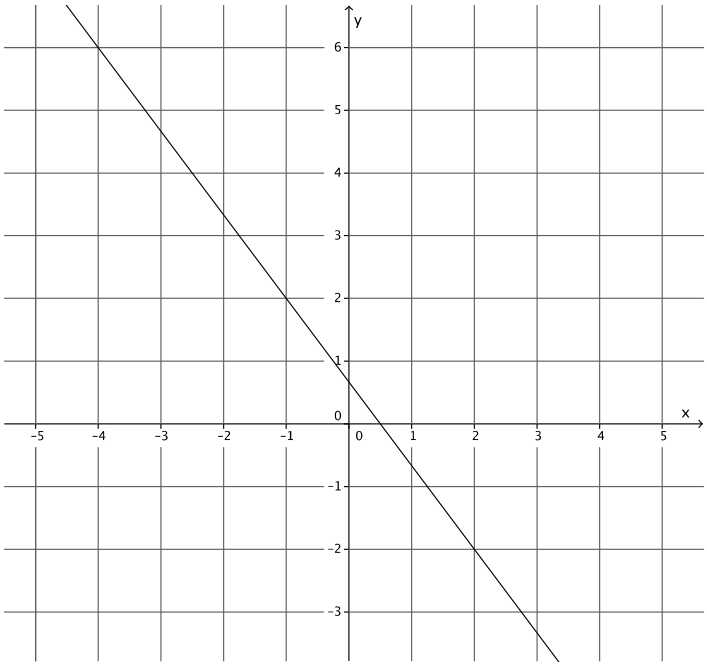
5. Write three equations that would graph as the same line as the equation $x - 9y = \frac{3}{4}$.

6. Write three equations that would graph as the same line as the equation $-9x + 5y = -4$.

7. Write at least two equations in the form $ax + by = c$ that would graph as the line shown below



8. Write at least two equations in the form $ax + by = c$ that would graph as the line shown below.



Lesson Summary

Two equations that graph as the same line are said to define the same line. Two equations that define the same line are the same equation, just in different forms. The equations may look different (different constants, different coefficients, or different forms).

When two equations are written in standard form, $ax + by = c$ and $a'x + b'y = c'$, they define the same line when $\frac{a'}{a} = \frac{b'}{b} = \frac{c'}{c}$ is true.

Problem Set

1. Do the equations $x + y = -2$ and $3x + 3y = -6$ define the same line? Explain.

2. Do the equations $y = -\frac{5}{4}x + 2$ and $10x + 8y = 16$ define the same line? Explain.

3. Write an equation that would define the same line as $7x - 2y = 5$.

4. Challenge: Show that if the two lines given by $ax + by = c$ and $a'x + b'y = c'$ are the same when $b = 0$ (vertical lines), then there exists a non-zero number s, so that $a' = sa$, $b' = sb$, and $c' = sc$.

5. Challenge: Show that if the two lines given by $ax + by = c$ and $a'x + b'y = c'$ are the same when $a = 0$ (horizontal lines), then there exists a non-zero number s, so that $a' = sa$, $b' = sb$, and $c' = sc$.