



## Lesson 29: Word Problems

### Student Outcomes

- Students write word problems into systems of linear equations.
- Students solve systems of linear equations using elimination and substitution methods.

### Lesson Notes

MP.4

In this lesson, students will use many of the skills learned in this module. For example, they begin defining the variables as described in a word problem. Next, students must write a system of linear equations to represent the situation. Students will then have to decide which method for solving the system is most efficient. Finally, they solve the system and check to make sure their answer is correct. For each of the examples in the lesson, pose the questions and provide students time to answer them on their own. Then select students to share their thoughts and solutions with the class.

### Classwork

#### Example 1 (5 minutes)

##### Example 1

The sum of two numbers is 361 and the difference between the two numbers is 173. What are the two numbers?

- Together we will read a word problem and work towards finding the solution.
- The sum of two numbers is 361 and the difference between the two numbers is 173. What are the two numbers?

Provide students time to work independently or in pairs to solve this problem. Have students share their solutions and explain how they arrived at their answers. Then show how the problem can be solved using a system of linear equations.

- What do we need to do first?
  - *We need to define our variables.*
- If we define our variables we can better represent the situation we have been given. What should the variables be for this problem?
  - *Let  $x$  represent one number and let  $y$  represent the other number.*
- Now that we know the numbers are  $x$  and  $y$ , what do we need to do now?
  - *We need to write equations to represent the information in the word problem.*
- Using  $x$  and  $y$ , write equations to represent the information we are provided.
  - *The sum of two numbers is 361 can be written as  $x + y = 361$ . The difference between the two numbers is 173 can be written as  $x - y = 173$ .*



- We have two equations to represent this problem. What is it called when we have more than one linear equation for a problem, and how is it represented symbolically?

- *We have a system of linear equations:*

$$\begin{cases} x + y = 361 \\ x - y = 173 \end{cases}$$

- We know several methods for solving systems of linear equations. Which method do you think will be the most efficient and why?
  - *We should add the equations together to eliminate the variable  $y$  because we can do that in one step.*

- Solve the system:  $\begin{cases} x + y = 361 \\ x - y = 173 \end{cases}$

- *Sample student work:*

$$\begin{aligned} \begin{cases} x + y = 361 \\ x - y = 173 \end{cases} \\ x + x + y - y = 361 + 173 \\ 2x = 534 \\ x = 267 \end{aligned}$$

$$\begin{aligned} 267 + y &= 361 \\ y &= 94 \end{aligned}$$

*The solution is (267, 94).*

- Based on our work, we believe the two numbers are 267 and 94. Check to make sure your answer is correct by substituting the numbers in to both equations. If it makes a true statement, then we know we are correct. If it makes a false statement, then we need to go back and check our work.

- *Sample student work:*

$$\begin{aligned} 267 + 94 &= 361 \\ 361 &= 361 \end{aligned}$$

and

$$\begin{aligned} 267 - 94 &= 173 \\ 173 &= 173 \end{aligned}$$

- Now we are sure that the numbers are 267 and 94. Does it matter which number is  $x$  and which number is  $y$ ?
  - *Not necessarily, but we need their difference to be positive, so  $x$  must be the larger of the two numbers to make sense of our equation  $x - y = 173$ .*

### Example 2 (7 minutes)

#### Example 2

There are 356 Grade 8 students at Euclid's Middle School. Thirty-four more than four times the number of girls is equal to half the number of boys. How many boys are in Grade 8 at Euclid's Middle School? How many girls?

- Again we will work together to solve the following word problem.
- There are 356 Grade 8 students at Euclid’s Middle School. Thirty-four more than four times the number of girls is equal to half the number of boys. How many boys are in Grade 8 at Euclid’s Middle School? How many girls? What do we need to do first?
  - *We need to define our variables.*
- If we define our variables we can better represent the situation we have been given. What should the variables be for this problem?
  - *Let  $x$  represent the number of girls and let  $y$  represent the number of boys.*

Whichever way students define the variables, ask them if it could be done the opposite way. For example, if the students respond as stated above, ask them if we could let  $x$  represent the number of boys and  $y$  represent the number of girls. They should say that at this stage it doesn’t matter if  $x$  represents girls or boys, but once the variable is defined, it does matter.

- Now that we know that  $x$  is the number of girls and  $y$  is the number of boys, what do we need to do now?
  - *We need to write equations to represent the information in the word problem.*
- Using  $x$  and  $y$ , write equations to represent the information we are provided.
  - *There are 356 Grade 8 students can be represented as  $x + y = 356$ . Thirty-four more than four times the number of girls is equal to half the number of boys can be represented as  $4x + 34 = \frac{1}{2}y$ .*
- We have two equations to represent this problem. What is it called when we have more than one linear equation for a problem, and how is it represented symbolically?
  - *We have a system of linear equations:*

$$\begin{cases} x + y = 356 \\ 4x + 34 = \frac{1}{2}y \end{cases}$$

- We know several methods for solving systems of linear equations. Which method do you think will be the most efficient and why?
  - *Answers will vary. There is no obvious “most efficient” method. Accept any reasonable responses as long as they are justified.*
- Solve the system:  $\begin{cases} x + y = 356 \\ 4x + 34 = \frac{1}{2}y \end{cases}$ .
  - *Sample student work:*

$$\begin{cases} x + y = 356 \\ 4x + 34 = \frac{1}{2}y \end{cases}$$

$$2\left(4x + 34 = \frac{1}{2}y\right)$$

$$8x + 68 = y$$

$$\begin{cases} x + y = 356 \\ 8x + 68 = y \end{cases}$$

$$\begin{aligned}x + 8x + 68 &= 356 \\9x + 68 &= 356 \\9x &= 288 \\x &= 32\end{aligned}$$

$$\begin{aligned}32 + y &= 356 \\y &= 324\end{aligned}$$

The solution is (32, 324).

- What does the solution mean in context?
  - Since we let  $x$  represent the number of girls and  $y$  represent the number of boys, it means that there are 32 girls and 324 boys at Euclid's Middle School in Grade 8.
- Based on our work, we believe there are 32 girls and 324 boys. How can we be sure we are correct?
  - We need to substitute the values into both equations of the system to see if it makes a true statement. Then

$$\begin{aligned}32 + 324 &= 356 \\356 &= 356\end{aligned}$$

and

$$\begin{aligned}4(32) + 34 &= \frac{1}{2}(324) \\128 + 34 &= 162 \\162 &= 162\end{aligned}$$

### Example 3 (5 minutes)

#### Example 3

A family member has some five-dollar bills and one-dollar bills in their wallet. Altogether she has 18 bills and a total of \$62. How many of each bill does she have?

- Again we will work together to solve the following word problem.
- A family member has some five-dollar bills and one-dollar bills in their wallet. Altogether she has 18 bills and a total of \$62. How many of each bill does she have? What do we do first?
  - We need to define our variables.
- If we define our variables we can better represent the situation we have been given. What should the variables be for this problem?
  - Let  $x$  represent the number of \$5 bills and let  $y$  represent the number of \$1 bills.

Again, whichever way students define the variables, ask them if it could be done the opposite way.

- Now that we know that  $x$  is the number of \$5 bills and  $y$  is the number of \$1 bills, what do we need to do now?
  - We need to write equations to represent the information in the word problem.



- Using  $x$  and  $y$ , write equations to represent the information we are provided.
  - *Altogether she has 18 bills and a total of \$62 must be represented with two equations. The first being  $x + y = 18$  to represent the total of 18 bills and the second being  $5x + y = 62$  to represent the total amount of money she has.*
- We have two equations to represent this problem. What is it called when we have more than one linear equation for a problem, and how is it represented symbolically?
  - *We have a system of linear equations:*

$$\begin{cases} x + y = 18 \\ 5x + y = 62 \end{cases}$$

- We know several methods for solving systems of linear equations. Which method do you think will be the most efficient and why?
  - *Answers will vary. Students will either say they could multiply one of the equations by  $-1$ , then they would be able to eliminate the variable  $y$  when they add the equations together. Other students may say it would be easiest to solve for  $y$  in the first equation and then substitute the value of  $y$  into the second equation. After they have justified their method, allow them to solve the system in any manner they choose.*
- Solve the system:  $\begin{cases} x + y = 18 \\ 5x + y = 62 \end{cases}$ .
  - *Sample student work:*

$$\begin{aligned} &\begin{cases} x + y = 18 \\ 5x + y = 62 \end{cases} \\ &x + y = 18 \\ &\quad y = -x + 18 \\ &\begin{cases} y = -x + 18 \\ 5x + y = 62 \end{cases} \\ &5x + (-x) + 18 = 62 \\ &\quad 4x + 18 = 62 \\ &\quad\quad 4x = 44 \\ &\quad\quad\quad x = 11 \\ &11 + y = 18 \\ &\quad\quad y = 7 \end{aligned}$$

*The solution is (11, 7).*

- What does the solution mean in context?
  - *Since we let  $x$  represent the number of \$5 bills and  $y$  represent the number of \$1 bills, it means that our family member has 11 \$5 bills, and 7 \$1 bills.*
- The next step is to check our work.
  - *It is obvious that  $11 + 7 = 18$ , so we know our family member has 18 bills. It makes more sense to check our work again the actual value of those 18 bills in this case. Now we check the second equation*

$$\begin{aligned} 5(11) + 1(7) &= 62 \\ 55 + 7 &= 62 \\ 62 &= 62 \end{aligned}$$

## Example 4 (9 minutes)

## Example 4

A friend bought 2 boxes of pencils and 8 notebooks for school and it cost him \$11. He went back to the store the same day to buy school supplies for his younger brother. He spent \$11.25 on 3 boxes of pencils and 5 notebooks. How much would 7 notebooks cost?

- Again we will work together to solve the following word problem.
- A friend bought 2 boxes of pencils and 8 notebooks for school and it cost him \$11. He went back to the store the same day to buy school supplies for his younger brother. He spent \$11.25 on 3 boxes of pencils and 5 notebooks. How much would 7 notebooks cost? What do we do first?
  - *We need to define our variables.*
- If we define our variables we can better represent the situation we have been given. What should the variables be for this problem?
  - *Let  $x$  represent the cost of a box of pencils and let  $y$  represent the cost of a notebook.*

Again, whichever way students define the variables, ask them if it could be done the opposite way.

- Now that we know that  $x$  is the cost of a box of pencils and  $y$  is the cost of a notebook, what do we need to do now?
  - *We need to write equations to represent the information in the word problem.*
- Using  $x$  and  $y$ , write equations to represent the information we are provided.
  - *A friend bought 2 boxes of pencils and 8 notebooks for school and it cost him \$11 is represented by the equation  $2x + 8y = 11$ . He spent \$11.25 on 3 boxes of pencils and 5 notebooks is represented by the equation  $3x + 5y = 11.25$ .*
- We have two equations to represent this problem. What is it called when we have more than one linear equation for a problem, and how is it represented symbolically?
  - *We have a system of linear equations:*

$$\begin{cases} 2x + 8y = 11 \\ 3x + 5y = 11.25 \end{cases}$$
- We know several methods for solving systems of linear equations. Which method do you think will be the most efficient and why?
  - *Answers will vary. Ask several students how what they believe the most efficient method is and have them share with the class a brief description of their plan. For example, a student may decide to multiply the first equation by 3 and the second equation by  $-2$  to eliminate  $x$  from the system after adding the equations together. After several students have shared their plan, allow students to solve in any manner they choose.*

- Solve the system:  $\begin{cases} 2x + 8y = 11 \\ 3x + 5y = 11.25 \end{cases}$

- *Sample student work:*

$$\begin{aligned} &\begin{cases} 2x + 8y = 11 \\ 3x + 5y = 11.25 \end{cases} \\ &3(2x + 8y = 11) \\ &6x + 24y = 33 \\ &-2(3x + 5y = 11.25) \\ &-6x - 10y = -22.50 \end{aligned}$$

$$\begin{cases} 6x + 24y = 33 \\ -6x - 10y = -22.50 \end{cases}$$

$$6x + 24y - 6x - 10y = 33 - 22.50$$

$$24y - 10y = 10.50$$

$$14y = 10.50$$

$$y = \frac{10.50}{14}$$

$$y = 0.75$$

$$2x + 8(0.75) = 11$$

$$2x + 6 = 11$$

$$2x = 5$$

$$x = 2.50$$

*The solution is (2.50, 0.75).*

- What does the solution mean in context?
  - *It means that a box of pencils costs \$2.50 and a notebook costs \$0.75.*
- Before we answer the question that this word problem asked, check to make sure the solution is correct.

- *Sample student work:*

$$2(2.50) + 8(0.75) = 11$$

$$5 + 6 = 11$$

$$11 = 11$$

*and*

$$3(2.50) + 5(0.75) = 11.25$$

$$7.50 + 3.75 = 11.25$$

$$11.25 = 11.25$$

- Now that we know we have the correct costs for the box of pencils and notebooks we can answer the original question: How much would 7 notebooks cost?
  - *The cost of 7 notebooks is  $7(0.75) = 5.25$ . Therefore, 7 notebooks cost \$7.25.*
- Keep in mind that some word problems require us to solve the system in order to answer a specific question, like this example about the cost of 7 notebooks. Other problems may just require the solution to the system to answer the word problem, like the first example about the two numbers and their sums and difference. It is always a good practice to reread the word problem to make sure you know what you are being asked to do.

**Exercises 1–3 (9 minutes)**

Students complete Exercises 1–3 independently or in pairs.

**Exercises**

1. A farm raises cows and chickens. The farmer has a total of 42 animals. One day he counts the legs of all of his animals and realizes he has a total of 114. How many cows does the farmer have? How many chickens?

*Let  $x$  represent the number of cows and  $y$  represent the number of chickens. Then,*

$$\begin{aligned} &\begin{cases} x + y = 42 \\ 4x + 2y = 114 \end{cases} \\ &-2(x + y = 42) \\ &-2x - 2y = -84 \\ &\begin{cases} -2x - 2y = -84 \\ 4x + 2y = 114 \end{cases} \\ &-2x - 2y + 4x + 2y = -84 + 114 \\ &-2x + 4x = 30 \\ &2x = 30 \\ &x = 15 \\ &15 + y = 42 \\ &y = 27 \end{aligned}$$

*The solution is (15, 27).*

$$\begin{aligned} 4(15) + 2(27) &= 114 \\ 60 + 54 &= 114 \\ 114 &= 114 \end{aligned}$$

*The farmer has 15 cows and 27 chickens.*

2. The length of a rectangle is 4 times the width. The perimeter of the rectangle is 45 inches. What is the area of the rectangle?

*Let  $x$  represent the length and  $y$  represent the width. Then,*

$$\begin{aligned} &\begin{cases} x = 4y \\ 2x + 2y = 45 \end{cases} \\ &2(4y) + 2y = 45 \\ &8y + 2y = 45 \\ &10y = 45 \\ &y = 4.5 \\ &x = 4(4.5) \\ &x = 18 \end{aligned}$$

*The solution is (18, 4.5).*

$$\begin{aligned} 2(18) + 2(4.5) &= 45 \\ 36 + 9 &= 45 \\ 45 &= 45 \end{aligned}$$

*The area of the rectangle is  $18 \times 4.5 = 81 \text{ in}^2$ .*





3. The sum of the measures of angles  $x$  and  $y$  is  $127^\circ$ . If the measure of  $\angle x$  is  $34^\circ$  more than half the measure of  $\angle y$ , what is the measure of each angle?

Let  $x$  represent the measure of  $\angle x$  and  $y$  represent the measure of  $\angle y$ . Then,

$$\begin{aligned} \begin{cases} x + y = 127 \\ x = 34 + \frac{1}{2}y \end{cases} \\ 34 + \frac{1}{2}y + y = 127 \\ 34 + \frac{3}{2}y = 127 \\ \frac{3}{2}y = 93 \\ y = 62 \\ x + 62 = 127 \\ x = 65 \end{aligned}$$

The solution is  $(65, 62)$ .

$$\begin{aligned} 65 &= 34 + \frac{1}{2}(62) \\ 65 &= 34 + 31 \\ 65 &= 65 \end{aligned}$$

The measure of  $\angle x = 65^\circ$  and the measure of  $\angle y = 62^\circ$ .

### Closing (5 minutes)

Summarize, or ask students to summarize, the main points from the lesson:

- We know how to write information from word problems into a system of linear equations.
- We can solve systems of linear equations using the elimination and substitution methods.
- When we solve a system we must clearly define the variables we intend to use, consider which method for solving the system would be most efficient, check our answer and think about what it means in context. Finally, we should check to make sure we have answered the question posed in the problem.

### Exit Ticket (5 minutes)



## Exit Ticket Sample Solutions

1. Small boxes contain DVDs and large boxes contain one gaming machine. Three boxes of gaming machines and a box of DVDs weigh 48 pounds. Three boxes of gaming machines and five boxes of DVDs weigh 72 pounds. How much does each box weigh?

Let  $x$  represent the weight of the gaming machine box and let  $y$  represent the weight of the DVD box. Then,

$$\begin{cases} 3x + y = 48 \\ 3x + 5y = 72 \end{cases}$$

$$\begin{aligned} -1(3x + y) &= -48 \\ -3x - y &= -48 \end{aligned}$$

$$\begin{cases} -3x - y = -48 \\ 3x + 5y = 72 \end{cases}$$

$$3x - 3x + 5y - y = 72 - 48$$

$$4y = 24$$

$$y = 6$$

$$3x + 6 = 48$$

$$3x = 42$$

$$x = 14$$

The solution is  $(14, 6)$

$$3(14) + 6 = 48$$

$$48 = 48$$

The box with one gaming machine weighs 14 pounds and the box containing DVDs weighs 6 pounds.

2. A language arts test is worth 100 points. There are a total of 26 questions. There are spelling word questions that are worth 2 points each and vocabulary word questions worth 5 points each. How many of each type of question is there?

Let  $x$  represent the number of spelling word questions and let  $y$  represent the number of vocabulary word questions. Then,

$$\begin{cases} x + y = 26 \\ 2x + 5y = 100 \end{cases}$$

$$-2(x + y) = -2(26)$$

$$-2x - 2y = -52$$

$$\begin{cases} -2x - 2y = -52 \\ 2x + 5y = 100 \end{cases}$$

$$2x - 2x + 5y - 2y = 100 - 52$$

$$3y = 48$$

$$y = 16$$

$$x + 16 = 26$$

$$x = 10$$

The solution is  $(10, 16)$

$$10 + 16 = 26$$

$$26 = 26$$

There are 10 spelling word questions and 16 vocabulary word questions.

## Problem Set Sample Solutions

1. Two numbers have a sum of 1,212 and a difference of 518. What are the two numbers?

Let  $x$  represent one number and  $y$  represent the other number. Then,

$$\begin{cases} x + y = 1212 \\ x - y = 518 \end{cases}$$

$$x + y + x - y = 1212 - 518$$

$$2x = 694$$

$$x = 347$$

$$347 + y = 1212$$

$$y = 865$$

The solution is (347, 865).

$$347 + 865 = 1212$$

$$1212 = 1212$$

The two numbers are 347 and 865.

2. The sum of the ages of two brothers is 46. The younger brother is 10 more than a third of the older brother's age. How old is the younger brother?

Let  $x$  represent the age of the younger brother and  $y$  represent the age of the older brother. Then,

$$\begin{cases} x + y = 46 \\ x = 10 + \frac{1}{3}y \end{cases}$$

$$10 + \frac{1}{3}y + y = 46$$

$$10 + \frac{4}{3}y = 46$$

$$\frac{4}{3}y = 36$$

$$y = 27$$

$$x + 27 = 46$$

$$x = 19$$

The solution is (19, 27).

$$19 = 10 + \frac{1}{3}(27)$$

$$19 = 10 + 9$$

$$19 = 19$$

The younger brother is 19 years old.

3. One angle measures 54 more than 3 times another angle. The angles are supplementary. What are their measures?

Let  $x$  represent the measure of one angle and  $y$  represent the measure of the other angle. Then,

$$\begin{cases} x = 3y + 54 \\ x + y = 180 \end{cases}$$

$$3y + 54 + y = 180$$

$$4y + 54 = 180$$

$$4y = 126$$

$$y = 31.5$$

$$x = 3(31.5) + 54$$

$$x = 94.5 + 54$$

$$x = 148.5$$

The solution is  $(31.5, 148.5)$ .

$$148.5 + 31.5 = 180$$

$$180 = 180$$

One angle measures  $148.5^\circ$  and the other measures  $31.5^\circ$ .

4. Some friends went to the local movie theatre and bought four buckets of large popcorn and six boxes of candy. The total for the snacks was \$46.50. The last time you were at the theatre you bought a large popcorn and a box of candy and the total was \$9.75. How much would 2 large buckets of popcorn and 3 boxes of candy cost?

Let  $x$  represent the cost of a large bucket of popcorn and  $y$  represent the cost of a box of candy. Then,

$$\begin{cases} 4x + 6y = 46.50 \\ x + y = 9.75 \end{cases}$$

$$-4(x + y = 9.75)$$

$$-4x - 4y = -39$$

$$\begin{cases} 4x + 6y = 46.50 \\ -4x - 4y = -39 \end{cases}$$

$$4x + 6y - 4x - 4y = 46.50 - 39$$

$$6y - 4y = 7.50$$

$$2y = 7.50$$

$$y = 3.75$$

$$x + 3.75 = 9.75$$

$$x = 6$$

The solution is  $(6, 3.75)$ .

$$4(6) + 6(3.75) = 46.50$$

$$24 + 22.50 = 46.50$$

$$46.50 = 46.50$$

Two large buckets of popcorn and three boxes of candy will cost  $2(6) + 3(3.75) = 12 + 11.25 = 23.25$  dollars.



5. You have 59 total coins for a total of \$12.05. You only have quarters and dimes. How many of each coin do you have?

Let  $x$  represent the number of quarters and  $y$  represent the number of dimes. Then,

$$\begin{cases} x + y = 59 \\ 0.25x + 0.1y = 12.05 \end{cases}$$

$$\begin{aligned} -4(0.25x + 0.1y &= 12.05) \\ -x - 0.4y &= -48.20 \end{aligned}$$

$$\begin{cases} x + y = 59 \\ -x - 0.4y = -48.20 \end{cases}$$

$$x + y - x - 0.4y = 59 - 48.20$$

$$y - 0.4y = 10.80$$

$$0.6y = 10.80$$

$$y = \frac{10.80}{0.6}$$

$$y = 18$$

$$x + 18 = 59$$

$$x = 41$$

The solution is (41, 18).

$$0.25(41) + 0.1(18) = 12.05$$

$$10.25 + 1.80 = 12.05$$

$$12.05 = 12.05$$

I have 41 quarters and 18 dimes.

6. A piece of string is 112 inches long. Isabel wants to cut it into 2 pieces so that one piece is three times as long as the other. How long is each piece?

Let  $x$  represent one piece and  $y$  represent the other, then

$$\begin{cases} x + y = 112 \\ 3y = x \end{cases}$$

$$3y + y = 112$$

$$4y = 112$$

$$y = 28$$

$$x + 28 = 112$$

$$x = 84$$

The solution is (84, 28).

$$84 + 28 = 112$$

$$112 = 112$$

One piece should be 84 inches long and the other should be 28 inches long.