Lesson 31: System of Equations Leading to Pythagorean Triples

Classwork

Exercises

1. Identify two Pythagorean triples using the known triple $3, 4, 5$ (other than $6, 8, 10$).

2. Identify two Pythagorean triples using the known triple $5, 12, 13$.

3. Identify two triples using either $3, 4, 5$ or $5, 12, 13$.

Use the system $\begin{cases} x + y = \frac{t}{s} \\ x - y = \frac{s}{t} \end{cases}$ to find Pythagorean triples for the given values of s and t. Recall that the solution, in the form of $\left(\frac{c}{b}, \frac{a}{b}\right)$, is the triple, a, b, c.

4. $s = 4,\ t = 5$

5. $s = 7,\ t = 10$

6. $s = 1,\ t = 4$

7. Use a calculator to verify that you found a Pythagorean triple in each of the Exercises 4–6. Show your work below.

Lesson Summary

A Pythagorean triple is a set of three positive integers that satisfies the equation $a^2 + b^2 = c^2$.

An infinite number of Pythagorean triples can be found by multiplying the numbers of a known triple by a whole number. For example, $3, 4, 5$ is a Pythagorean triple. Multiply each number by 7, then you have $21, 28, 35$ which is also a Pythagorean triple.

The system of linear equations, $\begin{cases} x + y = \frac{t}{s} \\ x - y = \frac{s}{t} \end{cases}$ can be used to find Pythagorean triples, just like the Babylonians did $4{,}000$ years ago.

Problem Set

1. Explain in terms of similar triangles why it is that when you multiply the known Pythagorean triple $3, 4, 5$ by 12, it generates a Pythagorean triple.

2. Identify three Pythagorean triples using the known triple $8, 15, 17$.

3. Identify three triples (numbers that satisfy $a^2 + b^2 = c^2$, but a, b, c are not whole numbers) using the triple $8, 15, 17$.

Use the system $\begin{cases} x + y = \frac{t}{s} \\ x - y = \frac{s}{t} \end{cases}$ to find Pythagorean triples for the given values of s and t. Recall that the solution, in the form of $\left(\frac{c}{b}, \frac{a}{b}\right)$, is the triple, a, b, c.

4. $s = 2,\ t = 9$

5. $s = 6,\ t = 7$

6. $s = 3,\ t = 4$

7. Use a calculator to verify that you found a Pythagorean triple in each of the problems 4–6. Show your work below.