Lesson 1: The Concept of a Function

Classwork

Example 1

Suppose a moving object travels 256 feet in 4 seconds. Assume that the object travels at a constant speed, that is, the motion of the object is linear with a constant rate of change. Write a linear equation in two variables to represent the situation, and use it to make predictions about the distance traveled over various intervals of time.

Number of seconds (x)	Distance traveled in feet (y)
1	
2	
3	
4	

Example 2

The object, a stone, is dropped from a height of 256 feet. It takes exactly 4 seconds for the stone to hit the ground. How far does the stone drop in the first 3 seconds? What about the last 3 seconds? Can we assume constant speed in this situation? That is, can this situation be expressed using a linear equation?

Number of seconds (x)	Distance traveled in feet (y)
1	
2	
3	
4	

Exercises

Use the table to answer Exercises 1–5.

Number of seconds (x)	Distance traveled in feet (y)
0.5	4
1	16
1.5	36
2	64
2.5	100
3	144
3.5	196
4	256

1. Name two predictions you can make from this table.

2. Name a prediction that would require more information.

3. What is the average speed of the object between zero and three seconds? How does this compare to the average speed calculated over the same interval in Example 1?

$$\text{Average Speed} = \frac{\text{distance traveled over a given time interval}}{\text{time interval}}$$

4. Take a closer look at the data for the falling stone by answering the questions below.
   a. How many feet did the stone drop between 0 and 1 second?
   b. How many feet did the stone drop between 1 and 2 seconds?
   c. How many feet did the stone drop between 2 and 3 seconds?
   d. How many feet did the stone drop between 3 and 4 seconds?

e. Compare the distances the stone dropped from one time interval to the next. What do you notice?

5. What is the average speed of the stone in each interval 0.5 seconds? For example, the average speed over the interval from 3.5 seconds to 4 seconds is

$$\frac{\text{distance traveled over a given time interval}}{\text{time interval}} = \frac{256-196}{4-3.5} = \frac{60}{0.5} = 120 \text{ feet per second}$$

Repeat this process for every half-second interval. Then answer the question that follows.

a. Interval between 0 and 0.5 seconds:

b. Interval between 0.5 and 1 seconds:

c. Interval between 1 and 1.5 seconds:

d. Interval between 1.5 and 2 seconds:

e. Interval between 2 and 2.5 seconds:

f. Interval between 2.5 and 3 seconds:

g. Interval between 3 and 3.5 seconds:

h. Compare the average speed between each time interval. What do you notice?

6. Is there any pattern to the data of the falling stone? Record your thoughts below.

Time of Interval in seconds (t)	1	2	3	4
Distance Stone Fell in feet (y)	16	64	144	256

Lesson Summary

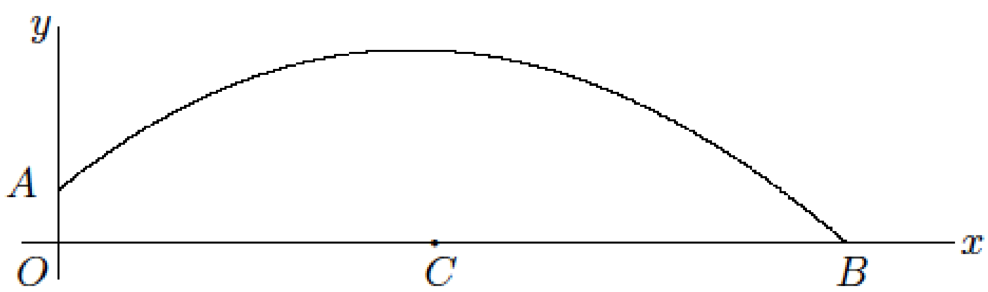
Functions are used to make predictions about real life situations. For example, a function allows you to predict the distance an object has traveled for *any* given time interval.

Constant rate cannot always be assumed. If not stated clearly, you can look at various intervals and inspect the average speed. When the average speed is the same over all time intervals, then you have constant rate. When the average speed is different, you do not have a constant rate.

$$Average\ Speed = \frac{distance\ traveled\ over\ a\ given\ time\ interval}{time\ interval}$$

Problem Set

1. A ball is thrown across the field from point A to point B. It hits the ground at point B. The path of the ball is shown in the diagram below. The x-axis shows the distance the ball travels and the y-axis shows the height of the ball. Use the diagram to complete parts (a)–(g).



   a. Suppose A is approximately 6 feet above ground and that at time $t = 0$ the ball is at point A. Suppose the length of OB is approximately 88 feet. Include this information on the diagram.
   b. Suppose that after 1 second, the ball is at its highest point of 22 feet (above point C) and has traveled a distance of 44 feet. Approximate the coordinates of the ball at the following values of t: 0.25, 0.5, 0.75, 1, 1.25, 1.5, 1.75, and 2.
   c. Use your answer from part (b) to write two predictions.
   d. What is the meaning of the point $(88, 0)$?
   e. Why do you think the ball is at point $(0, 6)$ when $t = 0$? In other words, why isn't the height of the ball zero?
   f. Does the graph allow us to make predictions about the height of the ball at all points?

2. In your own words, explain the purpose of a function and why it is needed.