Lesson 3: Existence and Uniqueness of Square and Cube Roots

Student Outcomes

- Students know that the positive square root and cube root exists for all positive numbers and is unique.
- Students solve simple equations that require them to find the square or cube root of a number.

Lesson Notes

This lesson has two options for showing the existence and uniqueness of positive square roots and cube roots. Each option has an Opening Exercise and Discussion that follows. The first option has students explore facts about numbers on a number line, leading to an understanding of the Trichotomy Law, followed by a discussion of how the law applies to squares of numbers, which should give students a better understanding of what square and cube roots are and how they are unique. The second option explores numbers and their squares via a "Find the Rule" exercise, followed by a discussion that explores how square and cube roots are unique. The first option includes a discussion of the *Basic Inequality,* a property referred to in subsequent lessons. The Basic Inequality states that if x, y, w, and z are positive numbers so that $x < y$ and $w < z$, then $xw < yz$. Further, if $x = w$ and $y = z$, when $x < y$, then $x^2 < y^2$. Once the first or second option is completed, the lesson continues with a discussion of how to solve equations using square roots.

Throughout this and subsequent lessons we ask students to find only the positive values of x that satisfy a radical equation. The reason is that in Algebra 1 students will solve radical equations by setting the equation equal to zero, then factoring the quadratic to find the solutions:

$$\begin{aligned} x^2 &= 25 \\ x^2 - 25 &= 0 \\ (x+5)(x-5) &= 0 \\ x &= \pm 5 \end{aligned}$$

At this point, students have not learned how to factor quadratics and will solve all equations using the square root symbol, which means students are only responsible for finding the positive solution(s) to an equation.

Classwork

Opening (5 minutes): Option 1

Ask students the following to prepare for the discussion that follows.

- Considering only the positive integers, if $x^2 = 4$, what must x equal? Could x be any other number?
  - *The number $x = 2$ and no other number.*
- If $c = 3$ and $d = 4$, compare the numbers c^2 and d^2 and the numbers c and d.
  - $c^2 < d^2$ and $c < d$.

- If $c < d$, could $c = d$? Explain.
  - *By definition if $c < d$ then $c \neq d$. Because $c < d$, c will be to the left of d on the number line, which means c and d are not at the same point on the number line. Therefore, $c \neq d$.*
- If $c < d$, could $c > d$?
  - *By definition if $c < d$, c will be to the left of d on the number line. The inequality $c > d$ means that c would be to the right of d on the number line. If $c < d$, then $c > d$ cannot also be true because c cannot simultaneously be to the right and to the left of d.*

Discussion (12 minutes): Option 1

(An alternative discussion is provided below.) Once this discussion is complete, continue with the discussion on page 39.

MP.3

- We will soon be solving equations that include roots. For this reason, we want to be sure that the answer we get when we simplify is correct. Specifically, we want to be sure that we can get an answer, that it exists, that the answer we get is correct, and that it is unique to the given situation.
- To this end, existence requires us to show that given a positive number b and a positive integer n, there is one and only one positive number c, so that $c^n = b$, "c is the positive n^{th} root of b." When $n = 2$ we say "c is the positive square root of b" and when $n = 3$ we say "c is the positive cube root of b." Uniqueness requires us to show that given two positive numbers c and d and $n = 2$: if $c^2 = d^2$, then $c = d$. This statement implies uniqueness because both c and d are the positive square root of b, i.e., $c^2 = b$ and $d^2 = b$, since $c^2 = d^2$, then $c = d$. Similarly, when $n = 3$, if $c^3 = d^3$, then $c = d$. The reasoning is the same, since both c and d are the positive cube root of b, i.e., $c^3 = b$ and $d^3 = b$, since $c^3 = d^3$, then $c = d$. Showing uniqueness will also show existence, so we will focus on proving the uniqueness of square and cube roots.
- To show that $c = d$, we will use the Trichotomy Law. The Trichotomy Law states that given two numbers c and d, one and only one of the following three possibilities is true:

  (i) $c = d$

  (ii) $c < d$

  (iii) $c > d$

  We will show $c = d$ by showing that $c < d$ and $c > d$ cannot be true.

> *Scaffolding:*
>
> The number line can help students make sense of the Trichotomy Law. Give students two numbers and ask them for which of the three possibilities those numbers are true. In each case, only one of three is true for any pair of numbers. For example, given the numbers $c = 2$ and $d = 3$, which of the following are true: $c = d$, $c < d$, $c > d$?
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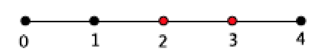
> 
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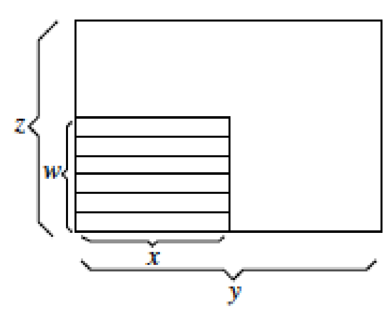
- If x, y, w, and z are positive numbers so that $x < y$ and $w < z$, is it true that $xw < yz$? Explain.
  - *Yes, it is true that $xw < yz$. Since all of the numbers are positive and both x and w are less than y and z, respectively, then their product must also be less. For example, since $3 < 4$, and $5 < 6$, then $3 \times 5 < 4 \times 6$.*
- This Basic Inequality can also be explained in terms of areas of a rectangle. The picture below clearly shows that when $x < y$ and $w < z$, then $xw < yz$.



MP.3

- We will use this fact to show that $c < d$ and $c > d$ cannot be true. We begin with $c^n < d^n$ when $n = 2$. By the Basic Inequality, $c^2 < d^2$. Now we look at the case where $n = 3$. We will use the fact that $c^2 < d^2$ to show $c^3 < d^3$. What can we do to show $c^3 < d^3$?
  - *We can multiply c^2 by c and d^2 by d. The Basic Inequality guarantees that since $c < d$ and $c^2 < d^2$, that $c^2 \times c < d^2 \times d$, which is the same as $c^3 < d^3$.*
- Using $c^3 < d^3$, how can we show $c^4 < d^4$?
  - *We can multiply c^3 by c and d^3 by d. The Basic Inequality guarantees that since $c < d$ and $c^3 < d^3$, $c^3 \times c < d^3 \times d$, which is the same as $c^4 < d^4$.*
- We can use the same reasoning for any positive integer n. We can use similar reasoning to show that if $c > d$, then $c^n > d^n$ for any positive integer n.
- Recall that we are trying to show that if $c^n = d^n$, then $c = d$ for $n = 2$ or $n = 3$. If we assume that $c < d$, then we know that $c^n < d^n$, which contradicts our hypothesis of $c^n = d^n$. By the same reasoning, if $c > d$, then $c^n > d^n$, which is also a contradiction of the hypothesis. By the Trichotomy Law, the only possibility left is that $c = d$. Therefore, we have shown that the square root or cube root of a number is unique and also exists.

Opening (8 minutes): Option 2

Begin by having students "find the rule" given numbers in two columns. The goal is for students to see the relationship between the square of a number and its square root and the cube of a number and its cube root. Students have to figure out the rule, then find missing values in the columns and explain their reasoning. Provide time for students to do this independently. If necessary, allow students to work in pairs.

MP.8

- The numbers in each column are related. Your goal is to determine how they are related, determine which numbers belong in the blank parts of the columns, and write an explanation for how you know the numbers belong there.

Opening

Find the Rule Part 1

1	1
2	4
3	9
9	81
11	121
15	225
7	49
10	100
12	144
13	169
m	m^2
$\sqrt{n}$	n

Note: Students will not know how to write the cube root of a number using the proper notation, but it will be a good way to launch into the discussion below.

MP.8

Find the Rule Part 2

1	1
2	8
3	27
5	125
6	216
11	1,331
4	64
10	1,000
7	343
14	2,744
p	p^3
$\sqrt[3]{q}$	q

Discussion (9 minutes): Option 2

Once the "Find the Rule" exercise is finished, use the discussion points below, then continue with the Discussion that follows on page 39.

- For Find the Rule Part 1, how were you able to determine which number belonged in the blank?
  - *To find the numbers that belonged in the blanks in the right column, I had to square the number in the left column. To find the numbers that belonged in the left column, I had to take the square root of the number in the right column.*
- When given the number m in the left column, how did we know the number that belonged to the right?
  - *Given m on the left, the number that belonged on the right was m^2.*
- When given the number n in the right column, how did we know the number that belonged to the left?
  - *Given n on the right, the number that belonged on the left was $\sqrt{n}$.*
- For Find the Rule Part 2, how were you able to determine which number belonged in the blank?
  - *To find the number that belonged in the blank in the right column, I had to multiply the number in the left column by itself 3 times. To find the number that belonged in the left column, I had to figure out which number multiplied by itself 3 times, equaled the number that was in the right column.*
- When given the number p in the left column, how did we note the number that belonged to the right?
  - *Given p on the left, the number that belonged on the right was p^3.*
- When given the number q in the right column, the notation we use to denote the number that belongs to the left is similar to the notation we used to denote the square root. Given the number q in the right column, we write $\sqrt[3]{q}$ on the left. The 3 in the notation shows that we must find the number that multiplied by itself 3 times is equal to q.

- Were you able to write more than one number in any of the blanks?
  - *No, there was only one number that worked.*
- Were there any blanks that could not be filled?
  - *No, in each case there was a number that worked.*
- For Find the Rule Part 1, you were working with squared numbers and square roots. For Find the Rule Part 2, you were working with cubed numbers and cube roots. Just like we have perfect squares there are also perfect cubes. For example, 27 is a perfect cube because it is the product of 3^3. For Find the Rule Part 2 you cubed the number on the left to fill the blank on the right and took the cube root of the number on the right to fill the blank on the left.
- We could extend the "Find the Rule" exercise to include an infinite number of rows and in each case we would be able to fill the blanks. Therefore, we can say that positive square roots and cube roots exist. Because only one number worked in each of the blanks, we can say that the positive roots are unique.
- We must learn about square roots and cube roots to solve equations. The properties of equality allow us to add, subtract, multiply, and divide the same number to both sides of an equal sign. We want to extend the properties of equality to include taking the square root and taking the cube root of both sides of an equation.
- Consider the equality $25 = 25$. What happens when we take the square root of both sides of the equal sign? Do we get a true number sentence?
  - *When we take the square root of both sides of the equal sign we get* $5 = 5$. *Yes, we get a true number sentence.*
- Consider the equality $27 = 27$. What happens when we take the cube root of both sides of the equal sign? Do we get a true number sentence?
  - *When we take the cube root of both sides of the equal sign we get* $3 = 3$. *Yes, we get a true number sentence.*
- At this point we only know the properties of equality can extend to those numbers that are perfect squares and perfect cubes, but it is enough to allow us to begin solving equations using square and cube roots.

Discussion (8 minutes)

The properties of equality have been proven for rational numbers, which are central in school mathematics. As we begin to solve equations that require roots, we are confronted with the fact that we may be working with irrational numbers (which have not yet been defined for students). Therefore, we make the assumption that all of the properties of equality for rational numbers are also true for irrational numbers, i.e., the real numbers, as far as computations are concerned. This is sometimes called the Fundamental Assumption of School Mathematics (FASM). In the discussion below, we reference n^{th} roots. You may choose to discuss square and cube roots only.

- In the past, we have determined the length of the missing side of a right triangle, x, when $x^2 = 25$. What is that value and how did you get the answer?
  - *The value of* x *is* 5 *because* x^2 *means* $x \cdot x$. *Since* $5 \times 5 = 25$, x *must be* 5.
- If we didn't know that we were trying to find the length of the side of a triangle, then the answer could also be -5 because $-5 \times -5 = 25$. However, because we were trying to determine the length of the side of a triangle, the answer must be positive because a length of -5 does not make sense.
- Now that we know that positive square roots exist and are unique, we can begin solving equations that require roots.

- When we solve equations that contain roots, we do what we do for all properties of equality, that is, we apply the operation to both sides of the equal sign. In terms of solving a radical equation, if we assume x is positive, then:

$$x^2 = 25$$
$$\sqrt{x^2} = \sqrt{25}$$
$$x = \sqrt{25}$$
$$x = 5$$

  Explain the first step in solving this equation.

  - *The first step is to take the square root of both sides of the equation.*

- It is by definition that when we use the symbol $\sqrt{\ }$, it automatically denotes a positive number; therefore, the solution to this equation is 5. In Algebra 1 you will learn how to solve equations of this form without using the square root symbol, which means the possible values for x can be both 5 and -5 because $5^2 = 25$ and $(-5)^2 = 25$, but for now we will only look for the positive solution(s) to our equations.

Note to teacher: In Algebra 1 students will solve equations of this form by setting the equation equal to zero, then factoring the quadratic to find the solutions:

$$x^2 = 25$$
$$x^2 - 25 = 0$$
$$(x+5)(x-5) = 0$$
$$x = \pm 5$$

At this point, students have not learned how to factor quadratics and will solve all equations using the square root symbol, which means students are only responsible for finding the positive solution(s) to an equation. Make it clear to students that *for now* we need only find the positive solutions, as they continue to learn more about non-linear equations, they will need to find all of the possible solutions.

- Consider the equation $x^2 = 25^{-1}$. What is another way to write 25^{-1}?

  - *The number 25^{-1} is the same as $\frac{1}{25}$.*

- Again, assuming that x is positive, we can solve the equation as before:

$$x^2 = 25^{-1}$$
$$x^2 = \frac{1}{25}$$
$$\sqrt{x^2} = \sqrt{\frac{1}{25}}$$
$$x = \sqrt{\frac{1}{25}}$$
$$x = \frac{1}{5}$$

  We know we are correct because $\left(\frac{1}{5}\right)^2 = \frac{1}{25} = 25^{-1}$.

- The symbol $\sqrt[n]{\ }$ is called a radical. Then an equation that contains that symbol is referred to as a radical equation. So far we have only worked with square roots ($n = 2$). Technically, we would denote a square root as $\sqrt[2]{\ }$, but it is understood that the symbol $\sqrt{\ }$ alone represents a square root.

- When $n = 3$, then the symbol $\sqrt[3]{}$ is used to denote the cube root of a number. Since $x^3 = x \cdot x \cdot x$, then the cube root of x^3 is x, i.e., $\sqrt[3]{x^3} = x$.
- For what value of x is the equation $x^3 = 8$ true?
  - $$x^3 = 8$$ $$\sqrt[3]{x^3} = \sqrt[3]{8}$$ $$x = \sqrt[3]{8}$$ $$x = 2$$
- The n^{th} root of a number is denoted by $\sqrt[n]{}$. In the context of our learning, we will limit our work with radicals to square and cube roots.

Exercises 1–9 (10 minutes)

Students complete Exercises 1–9 independently. Allow them to use a calculator to check their answers. Also consider showing students how to use the calculator to find the square root of a number.

Exercises 1–9

Find the positive value of x that makes each equation true. Check your solution.

1. $x^2 = 169$

a. **Explain the first step in solving this equation.**

The first step is to take the square root of both sides of the equation.

b. **Solve the equation and check your answer.**

$$x^2 = 169$$ $$\sqrt{x^2} = \sqrt{169}$$ $$x = \sqrt{169}$$ $$x = 13$$

Check:

$$13^2 = 169$$ $$169 = 169$$

2. **A square-shaped park has an area of 324 ft². What are the dimensions of the park? Write and solve an equation.**

$$x^2 = 324$$ $$\sqrt{x^2} = \sqrt{324}$$ $$x = \sqrt{324}$$ $$x = 18$$

Check:

$$18^2 = 324$$ $$324 = 324$$

The square park is 18 ft. in length and 18 ft. in width.

3. $625 = x^2$

$$625 = x^2$$ $$\sqrt{625} = \sqrt{x^2}$$ $$\sqrt{625} = x$$ $$25 = x$$

Check:

$$625 = 25^2$$ $$625 = 625$$

4. A cube has a volume of 27 in³. What is the measure of one of its sides? Write and solve an equation.

$$27 = x^3$$
$$\sqrt[3]{27} = \sqrt[3]{x^3}$$
$$\sqrt[3]{27} = x$$
$$3 = x$$

Check:

$$27 = 3^3$$
$$27 = 27$$

The cube has side lengths of 3 in.

5. What positive value of x makes the following equation true: $x^2 = 64$? Explain.

$$x^2 = 64$$
$$\sqrt{x^2} = \sqrt{64}$$
$$x = \sqrt{64}$$
$$x = 8$$

Check:

$$8^2 = 64$$
$$64 = 64$$

To solve the equation, I need to find the positive value of x so that when it is squared, it is equal to 64. Therefore, I can take the square root of both sides of the equation. The square root of x^2, $\sqrt{x^2}$ is x because $x^2 = x \cdot x$. The square root of 64, $\sqrt{64}$, is 8 because $64 = 8 \cdot 8$. Therefore, $x = 8$.

6. What positive value of x makes the following equation true: $x^3 = 64$? Explain.

$$x^3 = 64$$
$$\sqrt[3]{x^3} = \sqrt[3]{64}$$
$$x = \sqrt[3]{64}$$
$$x = 4$$

Check:

$$4^3 = 64$$
$$64 = 64$$

To solve the equation, I need to find the positive value of x so that when it is cubed, it is equal to 64. Therefore, I can take the cube root of both sides of the equation. The cube root of x^3, $\sqrt[3]{x^3}$, is x because $x^3 = x \cdot x \cdot x$. The cube root of 64, $\sqrt[3]{64}$, is 4 because $64 = 4 \cdot 4 \cdot 4$. Therefore, $x = 4$.

7. $x^2 = 256^{-1}$ Find the positive value of x that makes the equation true.

$$x^2 = 256^{-1}$$
$$\sqrt{x^2} = \sqrt{256^{-1}}$$
$$x = \sqrt{256^{-1}}$$
$$x = \sqrt{\frac{1}{256}}$$
$$x = \frac{1}{16}$$
$$x = 16^{-1}$$

Check:

$$(16^{-1})^2 = 256^{-1}$$
$$16^{-2} = 256^{-1}$$
$$\frac{1}{16^2} = 256^{-1}$$
$$\frac{1}{256} = 256^{-1}$$
$$256^{-1} = 256^{-1}$$

8. $x^3 = 343^{-1}$ Find the positive value of x that makes the equation true.

$$x^3 = 343^{-1}$$
$$\sqrt[3]{x^3} = \sqrt[3]{343^{-1}}$$
$$x = \sqrt[3]{343^{-1}}$$
$$x = \sqrt[3]{\frac{1}{343}}$$
$$x = \frac{1}{7}$$
$$x = 7^{-1}$$

Check:

$$(7^{-1})^3 = 343^{-1}$$
$$7^{-3} = 343^{-1}$$
$$\frac{1}{7^3} = 343^{-1}$$
$$\frac{1}{343} = 343^{-1}$$
$$343^{-1} = 343^{-1}$$

MP.6

9. Is 6 a solution to the equation $x^2 - 4 = 5x$? Explain why or why not.

$$6^2 - 4 = 5(6)$$
$$36 - 4 = 30$$
$$32 \neq 30$$

No, 6 is not a solution to the equation $x^2 - 4 = 5x$. When the number is substituted into the equation and simplified, the left side of the equation and the right side of the equation are not equal, i.e., it is not a true number sentence. Since the number 6 does not satisfy the equation, then it is not a solution to the equation.

Closing (5 minutes)

Summarize, or ask students to summarize, the main points from the lesson:

- We know that the positive n^{th} root of a number exists and is unique.
- We know how to solve equations that contain exponents of 2 and 3; we must use square roots and cube roots.

Lesson Summary

The symbol $\sqrt[n]{\ }$ is called a radical. Then an equation that contains that symbol is referred to as a radical equation. So far we have only worked with square roots ($n = 2$). Technically, we would denote a positive square root as $\sqrt[2]{\ }$, but it is understood that the symbol $\sqrt{\ }$ alone represents a positive square root.

When $n = 3$, then the symbol $\sqrt[3]{\ }$ is used to denote the cube root of a number. Since $x^3 = x \cdot x \cdot x$, then the cube root of x^3 is x, i.e., $\sqrt[3]{x^3} = x$.

The square or cube root of a positive number exists, and there can be only one positive square root or one cube root of the number.

Exit Ticket (5 minutes)

Name ____ Date ____________________

Lesson 3: Existence and Uniqueness of Square and Cube Roots

Exit Ticket

Find the positive value of x that makes each equation true. Check your solution.

1. $x^2 = 225$
   a. Explain the first step in solving this equation.

   b. Solve and check your solution.

2. $x^3 = 512$

3. $x^2 = 361^{-1}$

4. $x^3 = 1000^{-1}$

Exit Ticket Sample Solutions

Find the positive value of x that makes each equation true. Check your solution.

1. $x^2 = 225$

   a. Explain the first step in solving this equation.

   The first step is to take the square root of both sides of the equation.

   b. Solve and check your solution.

$$x^2 = 225$$
$$\sqrt{x^2} = \sqrt{225}$$
$$x = \sqrt{225}$$
$$x = 15$$

Check:

$$15^2 = 225$$
$$225 = 225$$

2. $x^3 = 512$

$$x^3 = 512$$
$$\sqrt[3]{x^3} = \sqrt[3]{512}$$
$$x = \sqrt[3]{512}$$
$$x = 8$$

Check:

$$8^3 = 512$$
$$512 = 512$$

3. $x^2 = 361^{-1}$

$$x^2 = 361^{-1}$$
$$\sqrt{x^2} = \sqrt{361^{-1}}$$
$$x = \sqrt{361^{-1}}$$
$$x = \sqrt{\frac{1}{361}}$$
$$x = \frac{1}{19}$$
$$x = 19^{-1}$$

Check:

$$(19^{-1})^2 = 361^{-1}$$
$$19^{-2} = 2536$$
$$\frac{1}{19^2} = 361^{-1}$$
$$\frac{1}{361} = 361^{-1}$$
$$361^{-1} = 361^{-1}$$

4. $x^3 = 1,000^{-1}$

$$x^3 = 1,000^{-1}$$
$$\sqrt[3]{x^3} = \sqrt[3]{1,000^{-1}}$$
$$x = \sqrt[3]{1,000^{-1}}$$
$$x = \sqrt[3]{\frac{1}{1,000}}$$
$$x = \frac{1}{10}$$
$$x = 10^{-1}$$

Check:

$$(10^{-1})^3 = 1,000^{-1}$$
$$10^{-3} = 1,000^{-1}$$
$$\frac{1}{10^3} = 1,000^{-1}$$
$$\frac{1}{1,000} = 1,000^{-1}$$
$$1,000^{-1} = 1,000^{-1}$$

Problem Set Sample Solutions

Find the positive value of x that makes each equation true. Check your solution.

1. **What positive value of x makes the following equation true: $x^2 = 289$? Explain.**

$$x^2 = 289$$
$$\sqrt{x^2} = \sqrt{289}$$
$$x = \sqrt{289}$$
$$x = 17$$

Check:

$$17^2 = 289$$
$$289 = 289$$

To solve the equation, I need to find the positive value of x so that when it is squared, it is equal to 289. Therefore, I can take the square root of both sides of the equation. The square root of x^2, $\sqrt{x^2}$, is x because $x^2 = x \cdot x$. The square root of 289, $\sqrt{289}$, is 17 because $289 = 17 \cdot 17$. Therefore, $x = 17$.

2. **A square shaped park has an area of 400 ft². What are the dimensions of the park? Write and solve an equation.**

$$x^2 = 400$$
$$\sqrt{x^2} = \sqrt{400}$$
$$x = \sqrt{400}$$
$$x = 20$$

Check:

$$20^2 = 400$$
$$400 = 400$$

The square park is 20 ft. in length and 20 ft. in width.

3. **A cube has a volume of 64 in³. What is the measure of one of its sides? Write and solve an equation.**

$$x^3 = 64$$
$$\sqrt[3]{x^3} = \sqrt[3]{64}$$
$$x = \sqrt[3]{64}$$
$$x = 4$$

Check:

$$4^3 = 64$$
$$64 = 64$$

The cube has a side length of 4 in.

4. **What positive value of x makes the following equation true: $125 = x^3$? Explain.**

$$125 = x^3$$
$$\sqrt[3]{125} = \sqrt[3]{x^3}$$
$$\sqrt[3]{125} = x$$
$$5 = x$$

Check:

$$125 = 5^3$$
$$125 = 125$$

To solve the equation, I need to find the positive value of x so that when it is cubed, it is equal to 125. Therefore, I can take the cube root of both sides of the equation. The cube root of x^3, $\sqrt[3]{x^3}$, is x because $x^3 = x \cdot x \cdot x$. The cube root of 125, $\sqrt[3]{125}$, is 5 because $125 = 5 \cdot 5 \cdot 5$. Therefore, $x = 5$.

5. $x^2 = 441^{-1}$ Find the positive value of x that makes the equation true.

   a. Explain the first step in solving this equation.

   The first step is to take the square root of both sides of the equation.

   b. Solve and check your solution.

$$x^2 = 441^{-1}$$
$$\sqrt{x^2} = \sqrt{441^{-1}}$$
$$x = \sqrt{441^{-1}}$$
$$x = \sqrt{\frac{1}{441}}$$
$$x = \frac{1}{21}$$
$$x = 21^{-1}$$

Check:

$$(21^{-1})^2 = 441^{-1}$$
$$21^{-2} = 441^{-1}$$
$$\frac{1}{21^2} = 441^{-1}$$
$$\frac{1}{441} = 441^{-1}$$
$$441^{-1} = 441^{-1}$$

6. $x^3 = 125^{-1}$ Find the positive value of x that makes the equation true.

$$x^3 = 125^{-1}$$
$$\sqrt[3]{x^3} = \sqrt[3]{125^{-1}}$$
$$x = \sqrt[3]{125^{-1}}$$
$$x = \sqrt[3]{\frac{1}{125}}$$
$$x = \frac{1}{5}$$
$$x = 5^{-1}$$

Check:

$$(5^{-1})^3 = 125^{-1}$$
$$5^{-3} = 125^{-1}$$
$$\frac{1}{5^3} = 125^{-1}$$
$$\frac{1}{125} = 125^{-1}$$
$$125^{-1} = 125^{-1}$$

7. The area of a square is 196 in^2. What is the length of one side of the square? Write and solve an equation, then check your solution.

Let x represent the length of one side of the square.

$$x^2 = 196$$
$$\sqrt{x^2} = \sqrt{196}$$
$$x = \sqrt{196}$$
$$x = 14$$

Check:

$$14^2 = 196$$
$$196 = 196$$

The length of one side of the square is 14 in.

8. **The volume of a cube is 729 cm^3. What is the length of one side of the cube? Write and solve an equation, then check your solution.**

Let x represent the length of one side of the cube.

$$x^3 = 729$$
$$\sqrt[3]{x^3} = \sqrt[3]{729}$$
$$x = \sqrt[3]{729}$$
$$x = 9$$

Check:

$$9^3 = 729$$
$$729 = 729$$

The length of one side of the cube is 9 cm.

9. **What positive value of x would make the following equation true: $19 + x^2 = 68$?**

$$19 + x^2 = 68$$
$$19 - 19 + x^2 = 68 - 19$$
$$x^2 = 49$$
$$x = 7$$

The positive value for x that makes the equation true is 7.