Lesson 5: Solving Radical Equations

Student Outcomes

- Students find the positive solutions for equations of the form $x^2 = p$ and $x^3 = p$.

Classwork

Discussion (15 minutes)

- Just recently, we began solving equations that required us to find the square root or cube root of a number. All of those equations were in the form of $x^2 = p$ or $x^3 = p$, where p was a positive rational number.

Example 1

Example 1

$$\boldsymbol{x^3 + 9x = \frac{1}{2}(18x + 54)}$$

MP.1

- Now that we know about a little more about square roots and cube roots, we can begin solving non-linear equations like $x^3 + 9x = \frac{1}{2}(18x + 54)$. Transform the equation using our properties of equality until you can determine the positive value of x that makes the equation true.

Challenge students to solve the equation independently or in pairs. Have students share their strategy for solving the equation. Ask them to explain each step.

- *Sample response:*

$$\begin{aligned} x^3 + 9x &= \frac{1}{2}(18x + 54) \\ x^3 + 9x &= 9x + 27 \\ x^3 + 9x - 9x &= 9x - 9x + 27 \\ x^3 &= 27 \\ \sqrt[3]{x^3} &= \sqrt[3]{27} \\ x &= \sqrt[3]{3^3} \\ x &= 3 \end{aligned}$$

Scaffolding:

Consider using a simpler version of the equation, line 2 for example:

$$x^3 + 9x = 9x + 27$$

- Now we verify our solution is correct:

$$\begin{aligned} 3^3 + 9(3) &= \frac{1}{2}(18(3) + 54) \\ 27 + 27 &= \frac{1}{2}(56 + 54) \\ 54 &= \frac{1}{2}(108) \\ 54 &= 54 \end{aligned}$$

- Since the left side is the same as the right side, our solution is correct.

Example 2

Example 2

$$\boldsymbol{x(x-3)-51=-3x+13}$$

- Let's look at another non-linear equation. Find the positive value of x that makes the equation true: $x(x-3)-51=-3x+13$.

Provide students with time to solve the equation independently or in pairs. Have students share their strategy for solving the equation. Ask them to explain each step.

- *Sample response:*

$$\begin{aligned} x(x-3)-51 &= -3x+13 \\ x^2-3x-51 &= -3x+13 \\ x^2-3x+3x-51 &= -3x+3x+13 \\ x^2-51 &= 13 \\ x^2-51+51 &= 13+51 \\ x^2 &= 64 \\ \sqrt{x^2} &= \pm\sqrt{64} \\ x &= \pm\sqrt{64} \\ x &= \pm 8 \end{aligned}$$

- Now we verify our solution is correct:

Provide students time to check their work.

Let $x = 8$

$$\begin{aligned} 8(8-3)-51 &= -3(8)+13 \\ 8(5)-51 &= -24+13 \\ 40-51 &= -11 \\ -11 &= -11 \end{aligned}$$

Let $x = -8$

$$\begin{aligned} -8(-8-3)-51 &= -3(-8)+13 \\ -8(-11)-51 &= 24+13 \\ 88-51 &= 37 \\ 37 &= 37 \end{aligned}$$

- Now it is clear that the left side is exactly the same as the right side and our solution is correct.

Exercises 1–8 (20 minutes)

Students complete Exercises 1–8 independently or in pairs.

Exercises 1–8

Find the positive value of x that makes each equation true, and then verify your solution is correct.

1. **Solve $x^2 - 14 = 5x + 67 - 5x$.**

$$x^2 - 14 = 5x + 67 - 5x$$
$$x^2 - 14 = 67$$
$$x^2 - 14 + 14 = 67 + 14$$
$$x^2 = 81$$
$$\sqrt{x^2} = \pm\sqrt{81}$$
$$x = \pm\sqrt{81}$$
$$x = \pm 9$$

Check:

$$9^2 - 14 = 5(9) + 67 - 5(9)$$
$$81 - 14 = 45 + 67 - 45$$
$$67 = 67$$

$$(-9)^2 - 14 = 5(-9) + 67 - 5(-9)$$
$$81 - 14 = -45 + 67 + 45$$
$$67 = 67$$

Explain how you solved the equation.

To solve the equation, I had to first use the properties of equality to transform the equation into the form of $x^2 = 81$. Then, I had to take the square root of both sides of the equation to determine that $x = 9$ since the number x is being squared.

2. **Solve and simplify: $x(x - 1) = 121 - x$.**

$$x(x - 1) = 121 - x$$
$$x^2 - x = 121 - x$$
$$x^2 - x + x = 121 - x + x$$
$$x^2 = 121$$
$$\sqrt{x^2} = \pm\sqrt{121}$$
$$x = \pm\sqrt{121}$$
$$x = \pm 11$$

Check:

$$11(11 - 1) = 121 - 11$$
$$11(10) = 110$$
$$110 = 110$$

$$-11(-11 - 1) = 121 - (-11)$$
$$-11(-12) = 121 + 11$$
$$132 = 132$$

3. **A square has a side length of $3x$ and an area of 324 in^2. What is the value of x?**

$$(3x)^2 = 324$$
$$3^2x^2 = 324$$
$$9x^2 = 324$$
$$\frac{9x^2}{9} = \frac{324}{9}$$
$$x^2 = 36$$
$$\sqrt{x^2} = \pm\sqrt{36}$$
$$x = \pm 6$$

Check:

$$(3 \times 6)^2 = 324$$
$$18^2 = 324$$
$$324 = 324$$

$$(3 \times (-6))^2 = 324$$
$$(-18)^2 = 324$$
$$324 = 324$$

4. $-3x^3 + 14 = -67$

$$-3x^3 + 14 = -67$$
$$-3x^3 + 14 - 14 = -67 - 14$$
$$-3x^3 = -81$$
$$\frac{-3x^3}{-3} = \frac{-81}{-3}$$
$$x^3 = 27$$
$$\sqrt[3]{x^3} = \sqrt[3]{27}$$
$$x = 3$$

Check:

$$-3(3)^3 + 14 = -67$$
$$-3(27) + 14 = -67$$
$$-81 + 14 = -67$$
$$-67 = -67$$

5. $x(x + 4) - 3 = 4(x + 19.5)$

$$x(x + 4) - 3 = 4(x + 19.5)$$
$$x^2 + 4x - 3 = 4x + 78$$
$$x^2 + 4x - 4x - 3 = 4x - 4x + 78$$
$$x^2 - 3 = 78$$
$$x^2 - 3 + 3 = 78 + 3$$
$$x^2 = 81$$
$$\sqrt{x^2} = \pm\sqrt{81}$$
$$x = \pm 9$$

Check:

$$9(9 + 4) - 3 = 4(9 + 19.5)$$
$$9(13) - 3 = 4(28.5)$$
$$117 - 3 = 114$$
$$114 = 114$$

$$-9(-9 + 4) - 3 = 4(-9 + 19.5)$$
$$-9(-5) - 3 = 4(10.5)$$
$$45 - 3 = 42$$
$$42 = 42$$

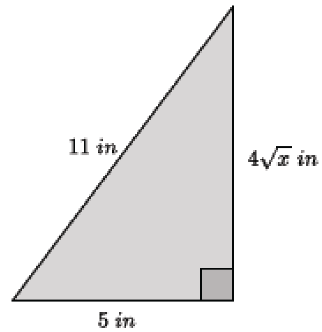
6. $216 + x = x(x^2 - 5) + 6x$

$$216 + x = x(x^2 - 5) + 6x$$
$$216 + x = x^3 - 5x + 6x$$
$$216 + x = x^3 + x$$
$$216 + x - x = x^3 + x - x$$
$$216 = x^3$$
$$\sqrt[3]{216} = \sqrt[3]{x^3}$$
$$6 = x$$

Check:

$$216 + 6 = 6(6^2 - 5) + 6(6)$$
$$222 = 6(31) + 36$$
$$222 = 186 + 36$$
$$222 = 222$$

7. What are we trying to determine in the diagram below?



We need to determine the value of x so that its square root, multiplied by 4 satisfies the equation $5^2 + (4\sqrt{x})^2 = 11^2$.

Determine the value of x and check your answer.

$$5^2 + (4\sqrt{x})^2 = 11^2$$
$$25 + 4^2(\sqrt{x})^2 = 121$$
$$25 - 25 + 4^2(\sqrt{x})^2 = 121 - 25$$
$$16x = 96$$
$$\frac{16x}{16} = \frac{96}{16}$$
$$x = 6$$

Check:

$$5^2 + (4\sqrt{6})^2 = 11^2$$
$$25 + 16(6) = 121$$
$$25 + 96 = 121$$
$$121 = 121$$

Closing (5 minutes)

Summarize, or ask students to summarize, the main points from the lesson:

- We know how to solve equations with squared and cubed variables and verify that our solutions are correct.

Lesson Summary

Equations that contain variables that are squared or cubed can be solved using the properties of equality and the definition of square and cube roots.

Simplify an equation until it is in the form of $x^2 = p$ or $x^3 = p$ where p is a positive rational number, then take the square or cube root to determine the positive value of x.

Example:

Solve for x.

$$\frac{1}{2}(2x^2 + 10) = 30$$
$$x^2 + 5 = 30$$
$$x^2 + 5 - 5 = 30 - 5$$
$$x^2 = 25$$
$$\sqrt{x^2} = \sqrt{25}$$
$$x = 5$$

Check:

$$\frac{1}{2}(2(5)^2 + 10) = 30$$
$$\frac{1}{2}(2(25) + 10) = 30$$
$$\frac{1}{2}(50 + 10) = 30$$
$$\frac{1}{2}(60) = 30$$
$$30 = 30$$

Exit Ticket (5 minutes)

Name ____ Date ____________________

Lesson 5: Solving Radical Equations

Exit Ticket

1. Find the positive value of x that makes the equation true, and then verify your solution is correct.

$$x^2 + 4x = 4(x + 16)$$

2. Find the positive value of x that makes the equation true, and then verify your solution is correct.

$$(4x)^3 = 1728$$

Exit Ticket Sample Solutions

1. Find the positive value of x that makes the equation true, and then verify your solution is correct.

$$x^2 + 4x = 4(x + 16)$$

$$x^2 + 4x = 4(x + 16)$$
$$x^2 + 4x = 4x + 64$$
$$x^2 + 4x - 4x = 4x - 4x + 64$$
$$x^2 = 64$$
$$\sqrt{x^2} = \sqrt{64}$$
$$x = 8$$

Check:

$$8^2 + 4(8) = 4(8 + 16)$$
$$64 + 32 = 4(24)$$
$$96 = 96$$

2. Find the positive value of x that makes the equation true, and then verify your solution is correct.

$$(4x)^3 = 1728$$

$$(4x)^3 = 1{,}728$$
$$64x^3 = 1{,}728$$
$$\frac{1}{64}(64x^3) = (1{,}728)\frac{1}{64}$$
$$x^3 = 27$$
$$\sqrt[3]{x^3} = \sqrt[3]{27}$$
$$x = 3$$

Check:

$$(4 \times 3)^3 = 1{,}728$$
$$12^3 = 1{,}728$$
$$1{,}728 = 1{,}728$$

Problem Set Sample Solutions

Find the positive value of x that makes each equation true, and then verify your solution is correct.

1. $x^2(x + 7) = \frac{1}{2}(14x^2 + 16)$

$$x^2(x + 7) = \frac{1}{2}(14x^2 + 16)$$
$$x^3 + 7x^2 = 7x^2 + 8$$
$$x^3 + 7x^2 - 7x^2 = 7x^2 - 7x^2 + 8$$
$$x^3 = 8$$
$$\sqrt[3]{x^3} = \sqrt[3]{8}$$
$$x = 2$$

Check:

$$2^2(2 + 7) = \frac{1}{2}(14(2^2) + 16)$$
$$4(9) = \frac{1}{2}(56 + 16)$$
$$36 = \frac{1}{2}(72)$$
$$36 = 36$$

2. $x^3 = 1,331^{-1}$

$$x^3 = 1,331^{-1}$$
$$\sqrt[3]{x^3} = \sqrt[3]{1,331^{-1}}$$
$$x = \sqrt[3]{\frac{1}{1,331}}$$
$$x = \sqrt[3]{\frac{1}{11^3}}$$
$$x = \frac{1}{11}$$

Check:

$$\left(\frac{1}{11}\right)^3 = 1,331^{-1}$$
$$\frac{1}{11^3} = 1,331^{-1}$$
$$\frac{1}{1,331} = 1,331^{-1}$$
$$1,331^{-1} = 1,331^{-1}$$

3. $\frac{x^9}{x^7} - 49 = 0$. Determine the positive value of x that makes the equation true, and then explain how you solved the equation.

$$\frac{x^9}{x^7} - 49 = 0$$
$$x^2 - 49 = 0$$
$$x^2 - 49 + 49 = 0 + 49$$
$$x^2 = 49$$
$$\sqrt{x^2} = \sqrt{49}$$
$$x = 7$$

Check:

$$7^2 - 49 = 0$$
$$49 - 49 = 0$$
$$0 = 0$$

To solve the equation I first had to simplify the expression $\frac{x^9}{x^7}$ to x^2. Next, I used the properties of equality to transform the equation into $x^2 = 49$. Finally, I had to take the square root of both sides of the equation to solve for x.

4. $(8x)^2 = 1$. Determine the positive value of x that makes the equation true.

$$(8x)^2 = 1$$
$$64x^2 = 1$$
$$\sqrt{64x^2} = \sqrt{1}$$
$$8x = 1$$
$$\frac{8x}{8} = \frac{1}{8}$$
$$x = \frac{1}{8}$$

Check:

$$\left(8\left(\frac{1}{8}\right)\right)^2 = 1$$
$$1^2 = 1$$
$$1 = 1$$

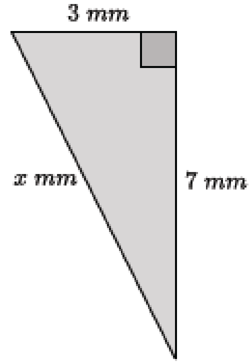
5. $\left(9\sqrt{x}\right)^2 - 43x = 76$.

$$\left(9\sqrt{x}\right)^2 - 43x = 76$$
$$9^2\left(\sqrt{x}\right)^2 - 43x = 76$$
$$81x - 43x = 76$$
$$38x = 76$$
$$\frac{38x}{38} = \frac{76}{38}$$
$$x = 2$$

Check:

$$(9(\sqrt{2}))^2 - 43(2) = 76$$
$$9^2\left(\sqrt{2}\right)^2 - 86 = 76$$
$$81(2) - 86 = 76$$
$$162 - 86 = 76$$
$$76 = 76$$

6. **Determine the length of the hypotenuse of the right triangle below.**



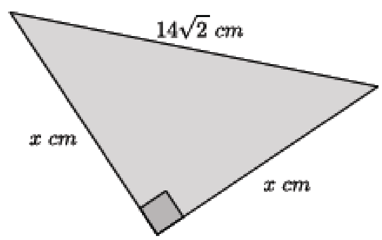
$$3^2 + 7^2 = x^2$$
$$9 + 49 = x^2$$
$$58 = x^2$$
$$\pm\sqrt{58} = \sqrt{x^2}$$
$$\pm\sqrt{58} = x$$
$$\sqrt{58} = x$$

Check:

$$3^2 + 7^2 = \sqrt{58}^2$$
$$9 + 49 = 58$$
$$58 = 58$$

A negative number would not make sense as a length, so $x = \sqrt{58}$.

7. **Determine the length of the legs in the right triangle below.**



$$x^2 + x^2 = \left(14\sqrt{2}\right)^2$$
$$2x^2 = 14^2\left(\sqrt{2}\right)^2$$
$$2x^2 = 196(2)$$
$$\frac{2x^2}{2} = \frac{196(2)}{2}$$
$$x^2 = 196$$
$$\sqrt{x^2} = \pm\sqrt{196}$$
$$x = \pm\sqrt{14^2}$$
$$x = \pm 14$$
$$x = 14$$

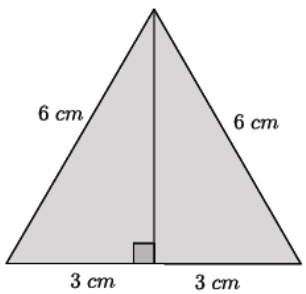
Check:

$$14^2 + 14^2 = \left(14\sqrt{2}\right)^2$$
$$196 + 196 = 14^2\left(\sqrt{2}\right)^2$$
$$392 = 196(2)$$
$$392 = 392$$

A negative number would not make sense as a length, so $x = 14$.

8. An equilateral triangle has side lengths of 6 cm. What is the height of the triangle? What is the area of the triangle?

Note: This problem has two solutions, one with a simplified root and one without. Choose the appropriate solution for your classes based on how much simplifying you have taught them.



Let h represent the height of the triangle.

$$\begin{aligned} 3^2 + h^2 &= 6^2 \\ 9 + h^2 &= 36 \\ 9 - 9 + h^2 &= 36 - 9 \\ h^2 &= 27 \\ \sqrt{h^2} &= \sqrt{27} \\ h &= \sqrt{27} \\ h &= \sqrt{3^3} \\ h &= \sqrt{3^2} \times \sqrt{3} \\ h &= 3\sqrt{3} \end{aligned}$$

Let A represent the area of the triangle.

$$\begin{aligned} A &= \frac{6(3\sqrt{3})}{2} \\ A &= 3(3\sqrt{3}) \\ A &= 9\sqrt{3} \end{aligned}$$

The height of the triangle is $3\sqrt{3}$ cm and the area is $9\sqrt{3}$ cm^2.

The height of the triangle is $\sqrt{27}$ cm and the area is $3\sqrt{27}$ cm^2.

9. Challenge: $\left(\frac{1}{2}x\right)^2 - 3x = 7x + 8 - 10x$. Find the positive value of x that makes the equation true.

$$\begin{aligned} \left(\frac{1}{2}x\right)^2 - 3x &= 7x + 8 - 10x \\ \frac{1}{4}x^2 - 3x &= -3x + 8 \\ \frac{1}{4}x^2 - 3x + 3x &= -3x + 3x + 8 \\ \frac{1}{4}x^2 &= 8 \\ 4\left(\frac{1}{4}\right)x^2 &= 8(4) \\ x^2 &= 32 \\ \sqrt{x^2} &= \sqrt{32} \\ x &= \sqrt{2^5} \\ x &= \sqrt{2^2} \times \sqrt{2^2} \times \sqrt{2} \\ x &= 4\sqrt{2} \end{aligned}$$

Check:

$$\begin{aligned} \left(\frac{1}{2}(4\sqrt{2})\right)^2 - 3(4\sqrt{2}) &= 7(4\sqrt{2}) + 8 - 10(4\sqrt{2}) \\ \frac{1}{4}(16)(2) - 3(4\sqrt{2}) &= 7(4\sqrt{2}) - 10(4\sqrt{2}) + 8 \\ \frac{32}{4} - 3(4\sqrt{2}) &= 7(4\sqrt{2}) - 10(4\sqrt{2}) + 8 \\ 8 - 3(4\sqrt{2}) &= (7 - 10)(4\sqrt{2}) + 8 \\ 8 - 3(4\sqrt{2}) &= -3(4\sqrt{2}) + 8 \\ 8 - 8 - 3(4\sqrt{2}) &= -3(4\sqrt{2}) + 8 - 8 \\ -3(4\sqrt{2}) &= -3(4\sqrt{2}) \end{aligned}$$

10. Challenge: $11x + x(x - 4) = 7(x + 9)$. Find the positive value of x that makes the equation true.

$$\begin{aligned} 11x + x(x - 4) &= 7(x + 9) \\ 11x + x^2 - 4x &= 7x + 63 \\ 7x + x^2 &= 7x + 63 \\ 7x - 7x + x^2 &= 7x - 7x + 63 \\ x^2 &= 63 \\ \sqrt{x^2} &= \sqrt{63} \\ x &= \sqrt{3^2 \times 7} \\ x &= \sqrt{3^2} \times \sqrt{7} \\ x &= 3\sqrt{7} \end{aligned}$$

Check:

$$\begin{aligned} 11(3\sqrt{7}) + 3\sqrt{7}(3\sqrt{7} - 4) &= 7(3\sqrt{7} + 9) \\ 33\sqrt{7} + 3^2(\sqrt{7})^2 - 4(3\sqrt{7}) &= 21\sqrt{7} + 63 \\ 33\sqrt{7} - 4(3\sqrt{7}) + 9(7) &= 21\sqrt{7} + 63 \\ 33\sqrt{7} - 12\sqrt{7} + 63 &= 21\sqrt{7} + 63 \\ (33 - 12)\sqrt{7} + 63 &= 21\sqrt{7} + 63 \\ 21\sqrt{7} + 63 &= 21\sqrt{7} + 63 \\ 21\sqrt{7} + 63 - 63 &= 21\sqrt{7} + 63 - 63 \\ 21\sqrt{7} &= 21\sqrt{7} \end{aligned}$$