Lesson 8: The Long Division Algorithm

Classwork

Example 1

Show that the decimal expansion of $\frac{26}{4}$ is 6.5.

Exercises 1–5

1. Use long division to determine the decimal expansion of $\frac{142}{2}$.

   a. Fill in the blanks to show another way to determine the decimal expansion of $\frac{142}{2}$.

$$142 = \_\_\_\_ \times 2 + \_\_\_\_$$

$$\frac{142}{2} = \frac{\_\_\_\_ \times 2 + \_\_\_\_}{2}$$

$$\frac{142}{2} = \frac{\_\_\_\_ \times 2}{2} + \frac{\_\_\_\_}{2}$$

$$\frac{142}{2} = \_\_\_\_ + \frac{\_\_\_\_}{2}$$

$$\frac{142}{2} = \_\_\_\_\_\_$$

b. Does the number $\frac{142}{2}$ have a finite or infinite decimal expansion? Explain how you know.

2. Use long division to determine the decimal expansion of $\frac{142}{4}$.

a. Fill in the blanks to show another way to determine the decimal expansion of $\frac{142}{4}$.

$$142 = \_\_\_\_ \times 4 + \_\_\_\_$$

$$\frac{142}{4} = \frac{\_\_\_\_ \times 4 + \_\_\_\_}{4}$$

$$\frac{142}{4} = \frac{\_\_\_\_ \times 4}{4} + \frac{\_\_\_\_}{4}$$

$$\frac{142}{4} = \_\_\_\_ + \frac{\_\_\_\_}{4}$$

$$\frac{142}{4} = \_\_\_\_\_\_\_\_$$

b. Does the number $\frac{142}{4}$ have a finite or infinite decimal expansion? Explain how you know.

3. Use long division to determine the decimal expansion of $\frac{142}{6}$.

a. Fill in the blanks to show another way to determine the decimal expansion of $\frac{142}{6}$.

$$142 = \_\_\_\_ \times 6 + \_\_\_\_$$

$$\frac{142}{6} = \frac{\_\_\_\_ \times 6 + \_\_\_\_}{6}$$

$$\frac{142}{6} = \frac{\_\_\_\_ \times 6}{6} + \frac{\_\_\_\_}{6}$$

$$\frac{142}{6} = \_\_\_\_ + \frac{\_\_\_\_}{6}$$

$$\frac{142}{6} = \_\_\_\_\_\_\_\_$$

b. Does the number $\frac{142}{6}$ have a finite or infinite decimal expansion? Explain how you know.

4. Use long division to determine the decimal expansion of $\frac{142}{11}$.

a. Fill in the blanks to show another way to determine the decimal expansion of $\frac{142}{11}$.

$$142 = \_\_\_\_ \times 11 + \_\_\_\_$$

$$\frac{142}{11} = \frac{\_\_\_\_ \times 11 + \_\_\_\_}{11}$$

$$\frac{142}{11} = \frac{\_\_\_\_ \times 11}{11} + \frac{\_\_\_\_}{11}$$

$$\frac{142}{11} = \_\_\_\_ + \frac{\_\_\_\_}{11}$$

$$\frac{142}{11} = \_\_\_\_\_\_\_\_\_\_$$

b. Does the number $\frac{142}{11}$ have a finite or infinite decimal expansion? Explain how you know.

5. Which fractions produced an infinite decimal expansion? Why do you think that is?

Exercises 6–10

6. Does the number $\frac{65}{13}$ have a finite or infinite decimal expansion? Based on our definition of rational numbers having a decimal expansion that repeats eventually, is the number rational? Explain.

7. Does the number $\frac{17}{11}$ have a finite or infinite decimal expansion? Based on our definition of rational numbers having a decimal expansion that repeats eventually, is the number rational? Explain.

8. Does the number $\pi = 3.1415926535897\ldots$ have a finite or infinite decimal expansion? Based on our definition of rational numbers having a decimal expansion that repeats eventually, is the number rational? Explain.

9. Does the number $\frac{860}{999} = 0.860860860\ldots$ have a finite or infinite decimal expansion? Based on our definition of rational numbers having a decimal expansion that repeats eventually, is the number rational? Explain.

10. Does the number $\sqrt{2} = 1.41421356237\ldots$ have a finite or infinite decimal expansion? Based on our definition of rational numbers having a decimal expansion that repeats eventually, is the number rational? Explain.

Lesson Summary

The long division algorithm is a procedure that can be used to determine the decimal expansion of infinite decimals.

Every rational number has a decimal expansion that repeats eventually. For example, the number 32 is rational because it has a repeat block of the digit 0 in its decimal expansion, $32.\bar{0}$. The number $\frac{1}{3}$ is rational because it has a repeat block of the digit 3 in its decimal expansion, $0.\bar{3}$. The number $0.454545\ldots$ is rational because it has a repeat block of the digits 45 in its decimal expansion, $0.\overline{45}$.

Problem Set

1. Write the decimal expansion of $\frac{7000}{9}$. Based on our definition of rational numbers having a decimal expansion that repeats eventually, is the number rational? Explain.

2. Write the decimal expansion of $\frac{6555555}{3}$. Based on our definition of rational numbers having a decimal expansion that repeats eventually, is the number rational? Explain.

3. Write the decimal expansion of $\frac{350000}{11}$. Based on our definition of rational numbers having a decimal expansion that repeats eventually, is the number rational? Explain.

4. Write the decimal expansion of $\frac{12000000}{37}$. Based on our definition of rational numbers having a decimal expansion that repeats eventually, is the number rational? Explain.

5. Someone notices that the long division of 2,222,222 by 6 has a quotient of 370,370 and remainder 2 and wonders why there is a repeating block of digits in the quotient, namely 370. Explain to the person why this happens.

6. Is the number $\frac{9}{11} = 0.81818181\ldots$ rational? Explain.

7. Is the number $\sqrt{3} = 1.73205080\ldots$ rational? Explain.

8. Is the number $\frac{41}{333} = 0.1231231231\ldots$ rational? Explain.