Lesson 9: Decimal Expansions of Fractions, Part 1

Classwork

Opening Exercises 1–2

1. a. We know that the fraction $\frac{5}{8}$ can be written as a finite decimal because its denominator is a product of 2's. Which power of 10 will allow us to easily write the fraction as a decimal? Explain.

   b. Write the equivalent fraction using the power of 10.

2. a. We know that the fraction $\frac{17}{125}$ can be written as a finite decimal because its denominator is a product of 5's. Which power of 10 will allow us to easily write the fraction as a decimal? Explain.

   b. Write the equivalent fraction using the power of 10.

Example 1

Write the decimal expansion of the fraction $\frac{5}{8}$.

Example 2

Write the decimal expansion of the fraction $\frac{17}{125}$.

Example 3

Write the decimal expansion of the fraction $\frac{35}{11}$.

Example 4

Write the decimal expansion of the fraction $\frac{6}{7}$.

Exercises 3–5

3. a. Choose a power of ten to use to convert this fraction to a decimal: $\frac{4}{13}$. Explain your choice.

   b. Determine the decimal expansion of $\frac{4}{13}$ and verify you are correct using a calculator.

4. Write the decimal expansion of $\frac{1}{11}$. Verify you are correct using a calculator.

5. Write the decimal expansion of $\frac{19}{21}$. Verify you are correct using a calculator.

Lesson Summary

Multiplying a fraction's numerator and denominator by the same power of 10 to determine its decimal expansion is similar to including extra zeroes to the right of a decimal when using the long division algorithm. The method of multiplying by a power of 10 reduces the work to whole number division.

Example: We know that the fraction $\frac{5}{3}$ has an infinite decimal expansion because the denominator is not a product of 2's and/or 5's. Its decimal expansion is found by the following procedure:

$\frac{5}{3} = \frac{5 \times 10^2}{3} \times \frac{1}{10^2}$ Multiply numerator and denominator by 10^2

$= \frac{166 \times 3 + 2}{3} \times \frac{1}{10^2}$ Rewrite the numerator as a product of a number multiplied by the denominator

$= \left(\frac{166 \times 3}{3} + \frac{2}{3}\right) \times \frac{1}{10^2}$ Rewrite the first term as a sum of fractions with the same denominator

$= \left(166 + \frac{2}{3}\right) \times \frac{1}{10^2}$ Simplify

$= \frac{166}{10^2} + \left(\frac{2}{3} \times \frac{1}{10^2}\right)$ Use the distributive property

$= 1.66 + \left(\frac{2}{3} \times \frac{1}{10^2}\right)$ Simplify

$= 166 \times \frac{1}{10^2} + \frac{2}{3} \times \frac{1}{10^2}$ Simplify the first term using what you know about place value

Notice that the value of the remainder, $\left(\frac{2}{3} \times \frac{1}{10^2}\right) = \frac{2}{300} = 0.006$, is quite small and does not add much value to the number. Therefore, 1.66 is a good estimate of the value of the infinite decimal for the fraction $\frac{5}{3}$.

Problem Set

1. a. Choose a power of ten to convert this fraction to a decimal: $\frac{4}{11}$. Explain your choice.

   b. Determine the decimal expansion of $\frac{4}{11}$ and verify you are correct using a calculator.

2. Write the decimal expansion of $\frac{5}{13}$. Verify you are correct using a calculator.

3. Write the decimal expansion of $\frac{23}{39}$. Verify you are correct using a calculator.

4. Tamer wrote the decimal expansion of $\frac{3}{7}$ as 0.418571, but when he checked it on a calculator it was 0.428571. Identify his error and explain what he did wrong.

$$\frac{3}{7} = \frac{3 \times 10^6}{7} \times \frac{1}{10^6}$$
$$= \frac{3000000}{7} \times \frac{1}{10^6}$$

$$3{,}000{,}000 = 418{,}571 \times 7 + 3$$

$$\frac{3}{7} = \frac{418571 \times 7 + 3}{7} \times \frac{1}{10^6}$$
$$= \left(\frac{418571 \times 7}{7} + \frac{3}{7}\right) \times \frac{1}{10^6}$$
$$= \left(418571 + \frac{3}{7}\right) \times \frac{1}{10^6}$$
$$= 418{,}571 \times \frac{1}{10^6} + \left(\frac{3}{7} \times \frac{1}{10^6}\right)$$
$$= \frac{418571}{10^6} + \left(\frac{3}{7} \times \frac{1}{10^6}\right)$$
$$= 0.418571 + \left(\frac{3}{7} \times \frac{1}{10^6}\right)$$

5. Given that $\frac{6}{7} = 0.857142 + \left(\frac{6}{7} \times \frac{1}{10^6}\right)$. Explain why 0.857142 is a good estimate of $\frac{6}{7}$.