Lesson 12: Decimal Expansions of Fractions, Part 2

Classwork

Example 1

Write the decimal expansion of $\frac{35}{11}$.

Exercises 1–3

1. Use rational approximation to determine the decimal expansion of $\frac{5}{3}$.

2. Use rational approximation to determine the decimal expansion of $\frac{5}{11}$.

3. a. Determine the decimal expansion of the number $\frac{23}{99}$ using rational approximation and long division.

b. When comparing rational approximation to long division, what do you notice?

Lesson Summary

The method of rational approximation, used earlier to write the decimal expansion of irrational numbers, can also be used to write the decimal expansion of fractions (rational numbers).

When used with rational numbers, there is no need to guess and check to determine the interval of tenths, hundredths, thousandths, etc. in which a number will lie. Rather, computation can be used to determine between which two consecutive integers, m and $m+1$, a number would lie for a given place value. For example, to determine where the fraction $\frac{1}{8}$ lies in the interval of tenths, compute using the following inequality:

$\frac{\boldsymbol{m}}{\boldsymbol{10}} < \frac{\boldsymbol{1}}{\boldsymbol{8}} < \frac{\boldsymbol{m+1}}{\boldsymbol{10}}$ Use the denominator of 10 because of our need to find the tenths digit of $\frac{1}{8}$

$\boldsymbol{m < \frac{10}{8} < m+1}$ Multiply through by 10

$\boldsymbol{m < 1\frac{1}{4} < m+1}$ Simplify the fraction $\frac{10}{8}$

The last inequality implies that $m = 1$ and $m + 1 = 2$, because $1 < 1\frac{1}{4} < 2$. Then the tenths digit of the decimal expansion of $\frac{1}{8}$ is 1.

Next, find the difference between the number $\frac{1}{8}$ and the known tenths digit value, $\frac{1}{10}$, i.e., $\frac{1}{8} - \frac{1}{10} = \frac{2}{80} = \frac{1}{40}$.

Use the inequality again, this time with $\frac{1}{40}$, to determine the hundredths digit of the decimal expansion of $\frac{1}{8}$.

$\frac{m}{100} < \frac{1}{40} < \frac{m+1}{100}$ Use the denominator of 100 because of our need to find the hundredths digit of $\frac{1}{8}$

$m < \frac{100}{40} < m+1$ Multiply through by 100

$m < 2\frac{1}{2} < m+1$ Simplify the fraction $\frac{100}{40}$

The last inequality implies that $m = 2$ and $m + 1 = 3$, because $2 < 2\frac{1}{2} < 3$. Then the hundredths digit of the decimal expansion of $\frac{1}{8}$ is 2.

Continue the process until the decimal expansion is complete or you notice a pattern of repeating digits.

Problem Set

1. Explain why the tenths digit of $\frac{3}{11}$ is 2, using rational approximation.
2. Use rational approximation to determine the decimal expansion of $\frac{25}{9}$.
3. Use rational approximation to determine the decimal expansion of $\frac{11}{41}$ to at least 5 digits.
4. Use rational approximation to determine which number is larger, $\sqrt{10}$ or $\frac{28}{9}$.
5. Sam says that $\frac{7}{11} = 0.63$, and Jaylen says that $\frac{7}{11} = 0.636$. Who is correct? Why?