Lesson 12: Decimal Expansions of Fractions, Part 2

Student Outcomes

- Students apply the method of rational approximation to determine the decimal expansion of a fraction.
- Students relate the method of rational approximation to the long division algorithm.

Lesson Notes

In this lesson, students use the idea of intervals of tenths, hundredths, thousandths, and so on to determine the decimal expansion of rational numbers. Since there is an explicit value that can be determined, students use what they know about mixed numbers and operations with fractions to pin down specific digits as opposed to the guess and check method used with irrational numbers. The general strategy is for students to compare a fractional value, say $\frac{2}{11}$, to a known decimal digit, that is $\frac{2}{11} = 0.1 +$ "something." Students find the difference between these two values, then work to find the next decimal digit in the expansion. The process continues until students notice a pattern in their work, leading them to recognize that the decimal expansion must be that of an infinite, repeating decimal block.

This lesson includes a fluency activity that will take approximately 10 minutes to complete. The fluency activity is a personal white board exchange with problems on volume that can be found at the end of the exercises for this lesson.

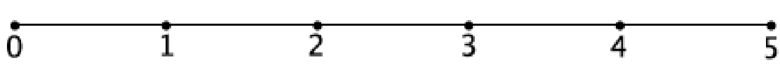
Classwork

Discussion (20 minutes)

Example 1

Write the decimal expansion of $\frac{35}{11}$.

- Our goal is to write the decimal expansion of a fraction, in this case $\frac{35}{11}$. To do so, begin by locating $\frac{35}{11}$ on the number line. What is its approximate location? Explain.



  - *The number* $\frac{35}{11}$ *would lie between* 3 *and* 4 *on the number line because* $\frac{35}{11} = \frac{33}{11} + \frac{2}{11} = 3 + \frac{2}{11}$.
- The goal is to use rational approximation to determine the decimal expansion of a number, instead of having to check a series of intervals as we did with the decimal expansions of irrational numbers. To determine the decimal expansion of $\frac{35}{11}$, focus only on the fraction $\frac{2}{11}$. Then, methodically determine between which interval of tenths $\frac{2}{11}$ would lie. Given that we are looking at an interval of tenths, can you think of a way to do this?

MP.3 Provide time for students to discuss strategies in small groups; then, share their ideas with the class. Encourage students to critique the reasoning of their classmates.

- We know that $\frac{35}{11}$ has a decimal expansion beginning with 3 in the ones place because $\frac{35}{11} = 3 + \frac{2}{11}$. Now we want to determine the tenths digit, the hundredths digit, and then the thousandths digit.

3.

Ones	Tenths	Hundredths	Thousandths

- To figure out the tenths digit, we will use an inequality based on tenths. We are looking for the consecutive integers, m and $m + 1$, that $\frac{2}{11}$ would lie between when m and $m + 1$ are intervals of tenths, i.e.:

$$\frac{m}{10} < \frac{2}{11} < \frac{m+1}{10}$$

3.0 3.1 3.2 3.3 3.4 3.5 3.6 3.7 3.8 3.9

Scaffolding:
An alternative way of asking this question is: "In which interval could we place the fraction $\frac{2}{11}$?" Show students the number line labeled with tenths.

Give students time to make sense of the above inequality. Since the intervals of tenths are represented by $\frac{m}{10}$ and $\frac{m+1}{10}$, consider using concrete numbers, which is clearer than looking at consecutive intervals of tenths on the number line. The chart below may help students make sense of the intervals and the inequality.

Integer	Next Integer
1	2
3	4
5	6
12	13
114	115
m	$m + 1$

Tenth	Next Tenth
$0.1 = \frac{1}{10}$	$0.2 = \frac{2}{10}$
$0.3 = \frac{3}{10}$	$0.4 = \frac{4}{10}$
$0.5 = \frac{5}{10}$	$0.6 = \frac{6}{10}$
$1.2 = \frac{12}{10}$	$1.3 = \frac{13}{10}$
$11.4 = \frac{114}{10}$	$11.5 = \frac{115}{10}$
$\frac{m}{10}$	$\frac{m+1}{10}$

- Multiplying through by 10, we get:

$$m < 10\left(\frac{2}{11}\right) < m + 1$$

$$10\left(\frac{2}{11}\right) = \frac{20}{11}$$
$$= \frac{11}{11} + \frac{9}{11}$$
$$= 1 + \frac{9}{11}.$$

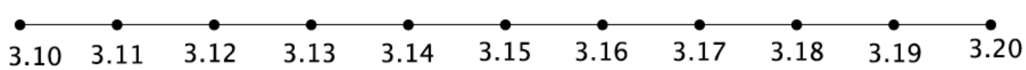
- This implies that $m = 1$. Why does the statement that $10\left(\frac{2}{11}\right) = 1 + \frac{9}{11}$ imply that $m = 1$?
  - *It implies that $m = 1$ because m and $m + 1$ are consecutive integers. Since $10\left(\frac{2}{11}\right) = 1 + \frac{9}{11} = 1\frac{9}{11}$, the number $1\frac{9}{11}$ would be between the two consecutive integers 1 and 2, thus implying that $m = 1$.*
- Now we know that the decimal expansion of $\frac{35}{11}$ has a one in the tenths place:

3.	1		
Ones	**Tenths**	**Hundredths**	**Thousandths**

- Since $\frac{35}{11} = 3 + \frac{2}{11}$ and the decimal expansion of the number is $3.1 = 3 + \frac{1}{10}$, we need to find the difference between these two representations. In other words, we need to find out what is left over after we remove the $\frac{1}{10}$ from the fraction $\frac{2}{11}$:

$$\frac{2}{11} - \frac{1}{10} = \frac{20}{110} - \frac{11}{110} = \frac{9}{110}.$$

- The next step is to find out which interval of hundredths will contain the fraction $\frac{9}{110}$.



Provide time for students to make a prediction and possibly develop a plan for determining the answer.

- The process is the same as looking for the interval of tenths. That is, we are looking for consecutive integers m and $m + 1$ so that

$$\frac{m}{100} < \frac{9}{110} < \frac{m+1}{100}.$$

  By what number should we multiply each term of the inequality to make our work here easier?
  - *Multiplying through by 100 will eliminate the fractions at the beginning and at the end of the inequality.*
- Multiplying through by 100, we get:

$$m < \frac{900}{110} < m + 1.$$

- Between which two integers, m and $m+1$, will we find the fraction $\frac{900}{110}$? Explain.
  - *The fraction* $\frac{900}{110}$ *is between* 8 *and* 9. *The reason is that* $\frac{900}{110} = \frac{880}{110} + \frac{20}{110} = 8 + \frac{2}{11}$.
- Now we know that the decimal expansion of $\frac{35}{11}$ has an 8 in the hundredths place:

Ones	Tenths	Hundredths	Thousandths
3.	1	8	

- Back to our original goal:

$$\frac{35}{11} = 3 + \frac{2}{11}.$$

By substitution we got:

$$\frac{35}{11} = 3 + \frac{1}{10} + \frac{900}{110}$$
$$= 3.1 + \frac{900}{110}$$

We know that $\frac{35}{11} = 3 + \frac{2}{11}$, and our work so far has shown the decimal expansion to be $3.18 = 3 + \frac{1}{10} + \frac{8}{100}$.

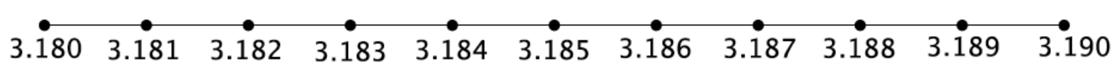
As before, we need to find the difference between $\frac{2}{11}$ and $\left(\frac{1}{10} + \frac{8}{100}\right)$:

$$\frac{2}{11} - \left(\frac{1}{10} + \frac{8}{100}\right) = \frac{2}{11} - \frac{18}{100}$$
$$= \frac{200}{1100} - \frac{198}{1100}$$
$$= \frac{2}{1100}.$$

Then, again, by substitution:

$$\frac{35}{11} = 3 + \frac{1}{10} + \frac{900}{110}$$
$$\frac{35}{11} = 3 + \frac{1}{10} + \frac{8}{100} + \frac{2}{1100}$$
$$= 3.18 + \frac{2}{1100}.$$

- Now, look at the interval of thousandths. Where do you expect $\frac{2}{1100}$ to lie on the number line? Write and explain a plan for determining the interval of thousandths in which the number belongs.



Provide students time to make a prediction and develop a plan for determining the answer. Students should recognize that $\frac{2}{1100} = \frac{2}{11} \times \frac{1}{100}$ and that we've placed the fraction $\frac{2}{11}$ first, but for a different place value.

- Note that $\frac{2}{1100} = \frac{2}{11} \times \frac{1}{100}$. The reappearance of the fraction $\frac{2}{11}$ is meaningful in that we can expect a decimal digit to repeat, but in a different place value since we are now looking for the thousandths digit. We are looking for consecutive integers m and $m+1$ so that
$$\frac{m}{1000} < \frac{2}{1100} < \frac{m+1}{1000}.$$
What should we multiply each term by?
  - *Multiplying through by* $1{,}000$ *will eliminate the fractions at the beginning and at the end of the inequality.*
- Multiplying through by $1{,}000$, we get:
$$m < \frac{20}{11} < m + 1.$$
However, we already know that:
$$\frac{20}{11} = \frac{11}{11} + \frac{9}{11}$$
$$= 1 + \frac{9}{11}$$
- Therefore, the next digit in the decimal expansion of $\frac{35}{11}$ will be 1:

3.	1	8	1
Ones	**Tenths**	**Hundredths**	**Thousandths**

- As before, we have the reappearance of the fraction $\frac{9}{11}$. So, we can expect the next decimal digit to be 8, followed by the reappearance of $\frac{2}{11}$, and so on. Therefore, the decimal expansion of $\frac{35}{11} = 3.1818\ldots$.
- Perform the long division algorithm on the fraction $\frac{35}{11}$, and be prepared to share your observations.

Provide time for students to work. Ask students: How is this method of rational approximation similar to the long division algorithm? Students should notice that the algorithm became repetitive with the appearance of the numbers 2 and 9, alternating with each step. Conclude the discussion by pointing out that the method of rational approximation is similar to the long division algorithm.

Exercises 1–3 (5 minutes)

Students work independently or in pairs to complete Exercises 1–3.

Exercises 1–3

1. **Use rational approximation to determine the decimal expansion of $\frac{5}{3}$.**

$$\frac{5}{3} = \frac{3}{3} + \frac{2}{3}$$
$$= 1 + \frac{2}{3}$$

In the sequence of tenths, we are looking for integers m and $m+1$ so that

$$\frac{m}{10} < \frac{2}{3} < \frac{m+1}{10},$$

which is the same as

$$m < 10\left(\frac{2}{3}\right) < m+1$$

$$\frac{20}{3} = \frac{18}{3} + \frac{2}{3}$$
$$= 6 + \frac{2}{3}.$$

The tenths digit is 6. The difference between $\frac{2}{3}$ and $\frac{6}{10}$ is

$$\frac{2}{3} - \frac{6}{10} = \frac{2}{30}.$$

In the interval of hundredths, we are looking for integers m and $m+1$ so that

$$\frac{m}{100} < \frac{2}{30} < \frac{m+1}{100},$$

which is the same as

$$m < \frac{20}{3} < m+1.$$

But we already know that $\frac{20}{3} = 6 + \frac{2}{3}$; therefore, the hundredths digit is 6. Because we keep getting $\frac{2}{3}$, we can assume the digit of 6 will continue to repeat. Therefore, the decimal expansion of $\frac{5}{3} = 1.666\ldots$.

2. **Use rational approximation to determine the decimal expansion of $\frac{5}{11}$.**

In the sequence of tenths, we are looking for integers m and $m+1$ so that

$$\frac{m}{10} < \frac{5}{11} < \frac{m+1}{10},$$

which is the same as

$$m < \frac{50}{11} < m+1$$

$$\frac{50}{11} = \frac{44}{11} + \frac{6}{11}$$
$$= 4 + \frac{6}{11}.$$

The tenths digit is 4. The difference between $\frac{5}{11}$ and $\frac{4}{10}$ is

$$\frac{5}{11} - \frac{4}{10} = \frac{6}{110}.$$

In the sequence of hundredths, we are looking for integers m and $m+1$ so that

$$\frac{m}{100} < \frac{6}{110} < \frac{m+1}{100},$$

which is the same as

$$m < \frac{60}{11} < m+1$$

$$\frac{60}{11} = \frac{55}{11} + \frac{5}{11}$$
$$= 5 + \frac{5}{11}.$$

So the hundredths digit is 5. Again, we see the fraction $\frac{5}{11}$, which means the next decimal digit will be 4, as it was in the tenths place. This means we will again see the fraction $\frac{6}{11}$, meaning we will have another digit of 5. Therefore, the decimal expansion of $\frac{5}{11}$ is 0.4545 ….

3. a. **Determine the decimal expansion of the number $\frac{23}{99}$ using rational approximation and long division.**

In the interval of tenths, we are looking for integers m and $m+1$ so that

$$\frac{m}{10} < \frac{23}{99} < \frac{m+1}{10},$$

which is the same as

$$m < \frac{230}{99} < \frac{m+1}{10}$$

$$\frac{230}{99} = \frac{198}{99} + \frac{32}{99}$$
$$= 2 + \frac{32}{99}.$$

The tenths digit is 2. The difference between $\frac{23}{99}$ and $\frac{2}{10}$ is

$$\frac{23}{99} - \frac{2}{10} = \frac{32}{990}.$$

In the interval of hundredths, we are looking for integers m and $m+1$ so that

$$\frac{m}{100} < \frac{32}{99}\left(\frac{1}{10}\right) < \frac{m+1}{100},$$

which is the same as

$$m < \frac{320}{99} < m + 1$$

$$\frac{320}{99} = \frac{297}{99} + \frac{23}{99}$$
$$= 3 + \frac{23}{99}.$$

So, the hundredths digit is 3. Again, we see the fraction $\frac{23}{99}$, which means the next decimal digit will be 2, as it was in the tenths place. This means we will again see the fraction $\frac{32}{99}$, meaning we will have another digit of 3. Therefore, the decimal expansion of $\frac{23}{99}$ is $0.2323\ldots$.

Long division gives us the same decimal expansion of $\frac{23}{99} = 0.2323\ldots$.

b. **When comparing rational approximation to long division, what do you notice?**

The first thing I notice is that the method of rational approximation gives the same decimal expansion as the long division algorithm. This makes sense because when doing long division, I put zeros past the 23, dividing into tenths, hundredths, thousandths, and so on. When I use the method of rational approximation, I do the same thing.

Fluency Exercise (10 minutes)

Please see the White Board Exchange exercise at the end of this lesson. Display the problems one at a time on a whiteboard, document camera, or PowerPoint. Give students about 1 minute to solve each problem, and go over them as a class.

Closing (5 minutes)

Summarize, or ask students to summarize, the main points from the lesson:

- Using rational approximation to write the decimal expansion of a fraction is similar to using the long division algorithm.
- Use the method of rational approximation to write the decimal expansion of a fraction instead of guessing and checking the intervals of tenths, hundredths, thousandths, etc. Determine the interval that a decimal digit is in using computation.

Lesson Summary

The method of rational approximation, used earlier to write the decimal expansion of irrational numbers, can also be used to write the decimal expansion of fractions (rational numbers).

When used with rational numbers, there is no need to guess and check to determine the interval of tenths, hundredths, thousandths, etc. in which a number will lie. Rather, computation can be used to determine between which two consecutive integers, m and $m+1$, a number would lie for a given place value. For example, to determine where the fraction $\frac{1}{8}$ lies in the interval of tenths, compute using the following inequality:

$\frac{m}{10} < \frac{1}{8} < \frac{m+1}{10}$	Use the denominator of 10 because of our need to find the tenths digit of $\frac{1}{8}$
$m < \frac{10}{8} < m+1$	Multiply through by 10
$m < 1\frac{1}{4} < m+1$	Simplify the fraction $\frac{10}{8}$

The last inequality implies that $m = 1$ and $m+1 = 2$, because $1 < 1\frac{1}{4} < 2$. Then the tenths digit of the decimal expansion of $\frac{1}{8}$ is 1.

Next, find the difference between the number $\frac{1}{8}$ and the known tenths digit value, $\frac{1}{10}$, i.e., $\frac{1}{8} - \frac{1}{10} = \frac{2}{80} = \frac{1}{40}$.

Use the inequality again, this time with $\frac{1}{40}$, to determine the hundredths digit of the decimal expansion of $\frac{1}{8}$.

$\frac{m}{100} < \frac{1}{40} < \frac{m+1}{100}$	Use the denominator of 100 because of our need to find the hundredths digit of $\frac{1}{8}$
$m < \frac{100}{40} < m+1$	Multiply through by 100
$m < 2\frac{1}{2} < m+1$	Simplify the fraction $\frac{100}{40}$

The last inequality implies that $m = 2$ and $m+1 = 3$, because $2 < 2\frac{1}{2} < 3$. Then the hundredths digit of the decimal expansion of $\frac{1}{8}$ is 2.

Exit Ticket (5 minutes)

Name ______________________________ Date ________________

Lesson 12: Decimal Expansions of Fractions, Part 2

Exit Ticket

Use rational approximation to determine the decimal expansion of $\frac{41}{6}$.

Exit Ticket Sample Solutions

Use rational approximation to determine the decimal expansion of $\frac{41}{6}$.

$$\frac{41}{6} = \frac{36}{36} + \frac{5}{6}$$
$$= 6 + \frac{5}{6}$$

The ones digit is 6. In the interval of tenths, we are looking for integers m and $m+1$ so that

$$\frac{m}{10} < \frac{5}{6} < \frac{m+1}{10},$$

which is the same as

$$m < \frac{50}{6} < m+1$$

$$\frac{50}{6} = \frac{48}{6} + \frac{2}{6}$$
$$= 8 + \frac{1}{3}.$$

The tenths digit is 8. The difference between $\frac{5}{6}$ and $\frac{8}{10}$ is

$$\frac{5}{6} - \frac{8}{10} = \frac{1}{30}.$$

In the interval of hundredths, we are looking for integers m and $m+1$ so that

$$\frac{m}{100} < \frac{1}{30} < \frac{m+1}{100},$$

which is the same as

$$m < \frac{10}{3} < m+1$$

$$\frac{10}{3} = \frac{9}{3} + \frac{1}{3}$$
$$= 3 + \frac{1}{3}.$$

The hundredths digit is 3. Again, we see the fraction $\frac{1}{3}$, which means the next decimal digit will be 3, as it was in the hundredths place. This means we will again see the fraction $\frac{1}{3}$, meaning we will have another digit of 3. Therefore, the decimal expansion of $\frac{41}{6}$ is $6.8333\ldots$.

Problem Set Sample Solutions

1. Explain why the tenths digit of $\frac{3}{11}$ is 2, using rational approximation.

In the interval of tenths, we are looking for integers m and $m+1$ so that

$$\frac{m}{10} < \frac{3}{11} < \frac{m+1}{10},$$

which is the same as

$$m < \frac{30}{11} < m+1$$
$$\frac{30}{11} = \frac{22}{11} + \frac{8}{11}$$
$$= 2 + \frac{8}{11}.$$

In looking at the interval of tenths, we see that the number $\frac{3}{11}$ must be between $\frac{2}{10}$ and $\frac{3}{10}$ because $\frac{2}{10} < \frac{3}{11} < \frac{3}{10}$. For this reason, the tenths digit of the decimal expansion of $\frac{3}{11}$ must be 2.

2. Use rational approximation to determine the decimal expansion of $\frac{25}{9}$.

$$\frac{25}{9} = \frac{18}{9} + \frac{7}{9}$$
$$= 2 + \frac{7}{9}$$

The ones digit is 2. In the interval of tenths, we are looking for integers m and $m+1$ so that

$$\frac{m}{10} < \frac{7}{9} < \frac{m+1}{10},$$

which is the same as

$$m < \frac{70}{9} < m+1$$
$$\frac{70}{9} = \frac{63}{9} + \frac{7}{9}$$
$$= 7 + \frac{7}{9}.$$

The tenths digit is 7. The difference between $\frac{7}{9}$ and $\frac{7}{10}$ is

$$\frac{7}{9} - \frac{7}{10} = \frac{7}{90}.$$

In the interval of hundredths, we are looking for integers m and $m+1$ so that

$$\frac{m}{100} < \frac{7}{90} < \frac{m+1}{100},$$

which is the same as

$$m < \frac{70}{9} < m+1.$$

But we already know that $\frac{70}{9} = 7 + \frac{7}{9}$; therefore, the hundredths digit is 7. Because we keep getting $\frac{7}{9}$, we can assume the digit of 7 will continue to repeat. Therefore, the decimal expansion of $\frac{25}{9} = 2.777\ ....$

3. **Use rational approximation to determine the decimal expansion of $\frac{11}{41}$ to at least 5 digits.**

In the interval of tenths, we are looking for integers m and $m+1$ so that

$$\frac{m}{10} < \frac{11}{41} < \frac{m+1}{10},$$

which is the same as

$$m < \frac{110}{41} < m+1$$

$$\frac{110}{41} = \frac{82}{41} + \frac{28}{41} = 2 + \frac{28}{41}.$$

The tenths digit is 2. The difference between $\frac{11}{41}$ and $\frac{2}{10}$ is

$$\frac{11}{41} - \frac{2}{10} = \frac{28}{410}.$$

In the interval of hundredths, we are looking for integers m and $m+1$ so that

$$\frac{m}{100} < \frac{28}{410} < \frac{m+1}{100},$$

which is the same as

$$m < \frac{280}{41} < m+1$$

$$\frac{280}{41} = \frac{246}{41} + \frac{34}{41} = 6 + \frac{34}{41}.$$

The hundredths digit is 6. The difference between $\frac{11}{41}$ and $\left(\frac{2}{10} + \frac{6}{100}\right)$ is

$$\frac{11}{41} - \left(\frac{2}{10} + \frac{6}{100}\right) = \frac{11}{41} - \frac{26}{100} = \frac{34}{4100}.$$

In the interval of thousandths, we are looking for integers m and $m+1$ so that

$$\frac{m}{1000} < \frac{34}{4100} < \frac{m+1}{1000},$$

which is the same as

$$m < \frac{340}{41} < m+1$$

$$\frac{340}{41} = \frac{328}{41} + \frac{12}{41} = 8 + \frac{12}{41}.$$

The thousandths digit is 8. The difference between $\frac{11}{41}$ and $\left(\frac{2}{10} + \frac{6}{100} + \frac{8}{1000}\right)$ is

$$\frac{11}{41} - \left(\frac{2}{10} + \frac{6}{100} + \frac{8}{1000}\right) = \frac{11}{41} - \frac{268}{1000} = \frac{12}{41000}.$$

In the interval of ten-thousandths, we are looking for integers m and $m+1$ so that

$$\frac{m}{10000} < \frac{12}{41000} < \frac{m+1}{10000},$$

which is the same as

$$m < \frac{120}{41} < m+1$$

$$\frac{120}{41} = \frac{82}{41} + \frac{38}{41} = 2 + \frac{38}{41}.$$

The ten-thousandths digit is 2*. The difference between* $\frac{11}{41}$ *and* $\left(\frac{2}{10}+\frac{6}{100}+\frac{8}{1000}+\frac{2}{10000}\right)$ *is*

$$\frac{11}{41}-\left(\frac{2}{10}+\frac{6}{100}+\frac{8}{1000}+\frac{2}{10000}\right)=\frac{11}{41}-\frac{2682}{10000}=\frac{38}{410000}.$$

In the interval of hundred-thousandths, we are looking for integers m *and* $m+1$ *so that*

$$\frac{m}{100000}<\frac{38}{410000}<\frac{m+1}{100000},$$

which is the same as

$$m<\frac{380}{41}<m+1$$

$$\frac{380}{41}=\frac{369}{41}+\frac{11}{41}=9+\frac{11}{41}.$$

The hundred-thousandths digit is 9*. We see again the fraction* $\frac{11}{41}$*, so we can expect the decimal digits to repeat at this point. Therefore, the decimal approximation of* $\frac{11}{41}=0.2682926829\ldots$

4. Use rational approximation to determine which number is larger, $\sqrt{10}$ or $\frac{28}{9}$.

The number $\sqrt{10}$ *is between* 3 *and* 4*. In the sequence of tenths,* $\sqrt{10}$ *is between* 3.1 *and* 3.2 *because* $3.1^2<\left(\sqrt{10}\right)^2<3.2^2$*. In the sequence of hundredths,* $\sqrt{10}$ *is between* 3.16 *and* 3.17 *because* $3.16^2<\left(\sqrt{10}\right)^2<3.17^2$*. In the sequence of hundredths,* $\sqrt{10}$ *is between* 3.162 *and* 3.163 *because* $3.162^2<\left(\sqrt{10}\right)^2<3.163^2$*. The decimal expansion of* $\sqrt{10}$ *is approximately* $3.162\ldots$

$$\frac{28}{9}=\frac{27}{9}+\frac{1}{9}$$
$$=3+\frac{1}{9}$$

In the interval of tenths, we are looking for the integers m *and* $m+1$ *so that*

$$\frac{m}{10}<\frac{1}{9}<\frac{m+1}{10},$$

which is the same as

$$m<\frac{10}{9}<m+1$$

$$\frac{10}{9}=\frac{9}{9}+\frac{1}{9}$$
$$=1+\frac{1}{9}.$$

The tenths digit is 1*. Since the fraction* $\frac{1}{9}$ *has reappeared, then we can assume that the next digit is also* 1*, and the work will continue to repeat. Therefore, the decimal expansion of* $\frac{28}{9}=3.1111\ldots$

Therefore, $\frac{28}{9}<\sqrt{10}$*.*

5. Sam says that $\frac{7}{11} = 0.63$, and Jaylen says that $\frac{7}{11} = 0.636$. Who is correct? Why?

In the interval of tenths, we are looking for integers m and $m+1$ so that

$$\frac{m}{10} < \frac{7}{11} < \frac{m+1}{10},$$

which is the same as

$$m < \frac{70}{11} < \frac{m+1}{10}$$

$$\frac{70}{11} = \frac{66}{11} + \frac{4}{11}$$
$$= 6 + \frac{4}{11}.$$

The tenths digit is 6. The difference between $\frac{7}{11}$ and $\frac{6}{10}$ is

$$\frac{7}{11} - \frac{6}{10} = \frac{4}{110}.$$

In the interval of hundredths, we are looking for integers m and $m+1$ so that

$$\frac{m}{100} < \frac{4}{110} < \frac{m+1}{100},$$

which is the same as

$$m < \frac{40}{11} < m+1$$
$$\frac{40}{11} = \frac{33}{11} + \frac{7}{11}$$
$$= 3 + \frac{7}{11}.$$

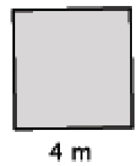
The hundredths digit is 3. Again, we see the fraction $\frac{7}{11}$, which means the next decimal digit will be 6, as it was in the tenths place. This means we will again see the fraction $\frac{4}{11}$, meaning we will have another digit of 3. Therefore, the decimal expansion of $\frac{7}{11}$ is $0.6363\ldots$.

Then, technically, both Sam and Jaylen are incorrect because the fraction $\frac{7}{11}$ is an infinite decimal. However, Sam is correct to the first two decimal digits of the number, and Jaylen is correct to the first three decimal digits of the number.

Fluency Exercise: White Board Exchange [Key]

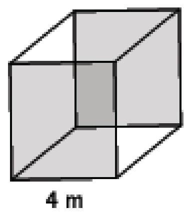
1. Find the area of the square shown below.

   $A = 4^2$
   $= 16\ m^2$

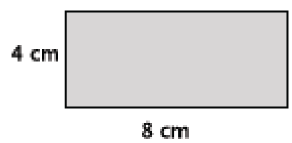


2. Find the volume of the cube shown below.

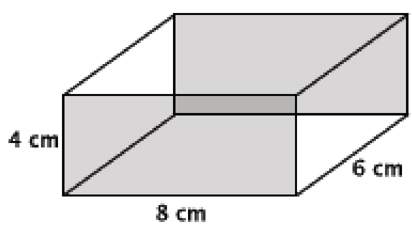
   $V = 4^3$
   $= 64\ m^3$



3. Find the area of the rectangle shown below.

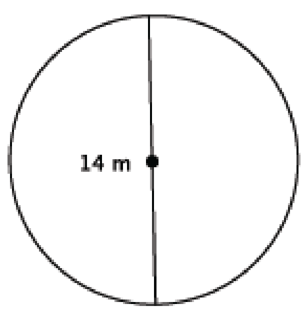


4. Find the volume of the rectangular prism show below.



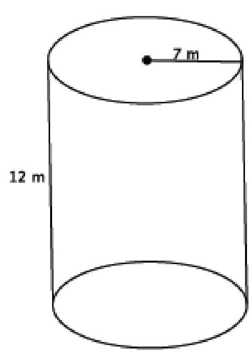
5. Find the area of the circle shown below.

   $A = 7^2\pi$
   $= 49\pi\ m^2$

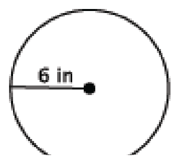


6. Find the volume of the cylinder show below.

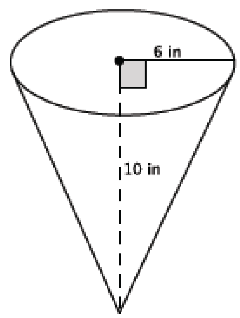
   $V = 49\pi(12)$
   $= 588\pi\ m^3$



7. Find the area of the circle shown below.



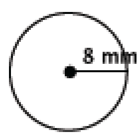
8. Find the volume of the cone show below.



$V = \left(\frac{1}{3}\right) 36\pi(10)$

$= 120\pi \text{ in.}^3$

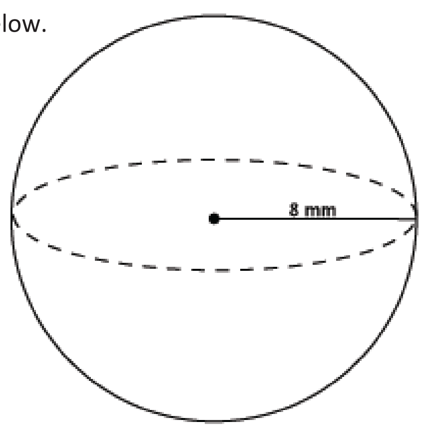
9. Find the area of the circle shown below.



$A = 8^2\pi$

$= 64\pi \text{ mm}^2$

10. Find the volume of the sphere show below.



$V = \left(\frac{4}{3}\right)\pi(64)(8)$

$= \frac{2048}{3}\pi \text{ mm}^3$