



## Lesson 16: The Converse of the Pythagorean Theorem

### Student Outcomes

- Students explain a proof of the converse of the Pythagorean Theorem.
- Students apply the theorem and its converse to solve problems.

### Lesson Notes

Students had their first experience with the converse of the Pythagorean Theorem in Module 3, Lesson 14. In that lesson, students learned the proof of the converse by contradiction. That is, they were given a “right” triangle and asked to show that it was not a right triangle by assuming the angle was greater than  $90^\circ$ . The computations using the Pythagorean Theorem led students to an expression that was not possible, i.e., twice a length was equal to zero. This contradiction meant that the angle of the “right” triangle was in fact  $90^\circ$ . In this lesson, students are given two triangles with base and height dimensions of  $a$  and  $b$ . They are told that one of the triangles is a right triangle and has lengths that satisfy the Pythagorean Theorem. Students must use computation and their understanding of the basic rigid motions to show that the triangle with an unmarked angle is also a right triangle. The proof is subtle, so it is important from the beginning that students understand the differences between the triangles used in the discussion of the proof of the converse.

### Classwork

#### Discussion (20 minutes)

- So far you have seen three different proofs of the Pythagorean Theorem:

*If the lengths of the legs of a right triangle are  $a$  and  $b$ , and the length of the hypotenuse is  $c$ , then  $a^2 + b^2 = c^2$ .*

Provide students time to explain to a partner a proof of the Pythagorean Theorem. Allow them to choose any one of the three proofs they have seen. Remind them of the proof from Module 2 that was based on congruent triangles, knowledge about angle sum of a triangle, and angles on a line. Also remind them of the proof from Module 3 that was based on their knowledge of similar triangles and corresponding sides being equal in ratio. Select students to share their proof with the class. Encourage other students to critique the reasoning of the student providing the proof.

- What do you recall about the meaning of the word converse?

Consider pointing out the hypothesis and conclusion of the Pythagorean Theorem and then asking students to describe the converse in those terms.

- *The converse is when the hypothesis and conclusion of a theorem are reversed.*
- You have also seen one proof of the converse:
  - *If the lengths of three sides of a triangle,  $a$ ,  $b$ , and  $c$  satisfy  $c^2 = a^2 + b^2$ , then the triangle is a right triangle, and furthermore, the side of length  $c$  is opposite the right angle.*

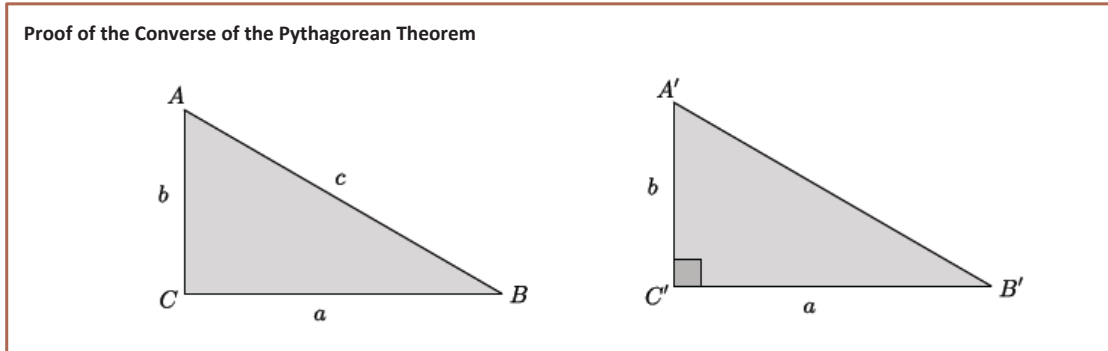
#### Scaffolding:

Provide students samples of converses (and note that converses are not always true):

- If it is a right angle, then the angle measure is  $90^\circ$ .  
Converse: If the angle measure is  $90^\circ$ , then it is a right angle.
- If it is raining, I will study inside the house.  
Converse: If I study inside the house, it is raining.

MP.3

- The following is another proof of the converse. Assume we are given a triangle  $ABC$  so that the sides,  $a$ ,  $b$ , and  $c$  satisfy  $c^2 = a^2 + b^2$ . We want to show that  $\angle ACB$  is a right angle. To do so, we construct a right triangle  $A'B'C'$  with leg lengths of  $a$  and  $b$  and right angle  $\angle A'C'B'$ .



- What do we know or not know about each of these triangles?
  - In the first triangle,  $ABC$ , we know that  $a^2 + b^2 = c^2$ . We do not know if angle  $C$  is a right angle. In the second triangle,  $A'B'C'$ , we know that it is a right triangle.
- What conclusions can we draw from this?
  - By applying the Pythagorean Theorem to  $\triangle A'B'C'$ , we get  $|A'B'|^2 = a^2 + b^2$ . Since we are given  $c^2 = a^2 + b^2$ , then by substitution,  $|A'B'|^2 = c^2$ , and then  $|A'B'| = c$ . Since  $c$  is also  $|AB|$ , then  $|A'C'| = |AC|$ . That means that both triangles have sides,  $a$ ,  $b$ , and  $c$ , that are the exact same lengths. Therefore, if we translated one triangle along a vector (or applied any required rigid motion(s)), it would map onto the other triangle showing a congruence. Congruence is degree preserving, which means that  $\angle ACB$  is a right angle, i.e.,  $90^\circ = \angle A'C'B' = \angle ACB$ .

MP.3

Provide students time to explain to a partner a proof of the converse of the Pythagorean Theorem. Allow them to choose either proof that they have seen. Remind them of the proof from Module 3 that was a proof by contradiction, where we assumed that the triangle was not a right triangle and then showed that the assumption was wrong. Select students to share their proof with the class. Encourage other students to critique the reasoning of the student providing the proof.

**Exercises 1–7 (15 minutes)**

Students complete Exercises 1–7 independently. Remind students that since each of the exercises references the side length of a triangle we need only consider the positive square root of each number, because we cannot have a negative length.

**Exercises**

1. Is the triangle with leg lengths of 3 mi., 8 mi., and hypotenuse of length  $\sqrt{73}$  mi. a right triangle? Show your work, and answer in a complete sentence.

$$\begin{aligned}
 3^2 + 8^2 &= (\sqrt{73})^2 \\
 9 + 64 &= 73 \\
 73 &= 73
 \end{aligned}$$

*Yes, the triangle with leg lengths of 3 mi., 8 mi., and hypotenuse of length  $\sqrt{73}$  mi. is a right triangle because it satisfies the Pythagorean Theorem.*

2. What is the length of the unknown side of the right triangle shown below? Show your work, and answer in a complete sentence. Provide an exact answer and an approximate answer rounded to the tenths place.

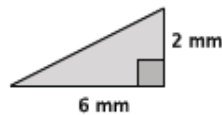


Let  $c$  represent the hypotenuse of the triangle.

$$\begin{aligned} 1^2 + 4^2 &= c^2 \\ 1 + 16 &= c^2 \\ 17 &= c^2 \\ \sqrt{17} &= c \\ 4.1 &\approx c \end{aligned}$$

The length of the hypotenuse of the right triangle is exactly  $\sqrt{17}$  inches and approximately 4.1 inches.

3. What is the length of the unknown side of the right triangle shown below? Show your work, and answer in a complete sentence. Provide an exact answer and an approximate answer rounded to the tenths place.



Let  $c$  represent the hypotenuse of the triangle.

$$\begin{aligned} 2^2 + 6^2 &= c^2 \\ 4 + 36 &= c^2 \\ 40 &= c^2 \\ \sqrt{40} &= c \\ \sqrt{2^3} \times \sqrt{5} &= c \\ \sqrt{2^2} \times \sqrt{2} \times \sqrt{5} &= c \\ 2\sqrt{10} &= c \end{aligned}$$

The length of the hypotenuse of the right triangle is exactly  $2\sqrt{10}$  mm and approximately 6.3 mm.

4. Is the triangle with leg lengths of 9 in., 9 in., and hypotenuse of length  $\sqrt{175}$  in. a right triangle? Show your work, and answer in a complete sentence.

$$\begin{aligned} 9^2 + 9^2 &= (\sqrt{175})^2 \\ 81 + 81 &= 175 \\ 162 &\neq 175 \end{aligned}$$

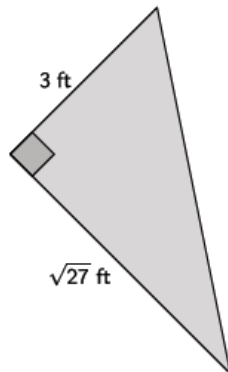
No, the triangle with leg lengths of 9 in., 9 in., and hypotenuse of length  $\sqrt{175}$  in. is not a right triangle because the lengths do not satisfy the Pythagorean Theorem.

5. Is the triangle with leg lengths of  $\sqrt{28}$  cm, 6 cm, and hypotenuse of length 8 cm a right triangle? Show your work, and answer in a complete sentence.

$$\begin{aligned} (\sqrt{28})^2 + 6^2 &= 8^2 \\ 28 + 36 &= 64 \\ 64 &= 64 \end{aligned}$$

Yes, the triangle with leg lengths of  $\sqrt{28}$  cm, 6 cm, and hypotenuse of length 8 cm is a right triangle because the lengths satisfy the Pythagorean Theorem.

6. What is the length of the unknown side of the right triangle shown below? Show your work, and answer in a complete sentence.

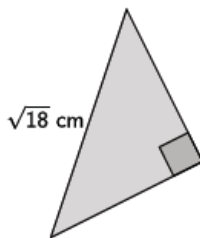


Let  $c$  represent the hypotenuse of the triangle.

$$\begin{aligned} 3^2 + (\sqrt{27})^2 &= c^2 \\ 9 + 27 &= c^2 \\ 36 &= c^2 \\ \sqrt{36} &= \sqrt{c^2} \\ 6 &= c \end{aligned}$$

The length of the hypotenuse of the right triangle is 6 ft.

7. The triangle shown below is an isosceles right triangle. Determine the length of the legs of the triangle. Show your work, and answer in a complete sentence.



Let  $x$  represent the length of the side of the isosceles triangle.

$$\begin{aligned} x^2 + x^2 &= (\sqrt{18})^2 \\ 2x^2 &= 18 \\ \frac{2x^2}{2} &= \frac{18}{2} \\ x^2 &= 9 \\ \sqrt{x^2} &= \sqrt{9} \\ x &= 3 \end{aligned}$$

The leg lengths of the isosceles triangle are 3 cm.

### Closing (5 minutes)

Summarize, or ask students to summarize, the main points from the lesson:

- The converse of the Pythagorean Theorem states that if side lengths of a triangle,  $a$ ,  $b$ ,  $c$ , satisfy  $a^2 + b^2 = c^2$ , then the triangle is a right triangle.
- If the side lengths of a triangle,  $b$ ,  $c$ , do not satisfy  $a^2 + b^2 = c^2$ , then the triangle is not a right triangle.
- We know how to explain a proof of the Pythagorean Theorem and its converse.

#### Lesson Summary

The converse of the Pythagorean Theorem states that if a triangle with side lengths  $a$ ,  $b$ , and  $c$  satisfies  $a^2 + b^2 = c^2$ , then the triangle is a right triangle.

The converse can be proven using concepts related to congruence.

### Exit Ticket (5 minutes)

Name \_\_\_\_\_

Date \_\_\_\_\_

## Lesson 16: The Converse of the Pythagorean Theorem

### Exit Ticket

1. Is the triangle with leg lengths of 7 mm and 7 mm and a hypotenuse of length 10 mm a right triangle? Show your work, and answer in a complete sentence.
  
  
  
  
  
  
  
  
  
  
2. What would the hypotenuse need to be so that the triangle in Problem 1 would be a right triangle? Show work that leads to your answer.
  
  
  
  
  
  
  
  
  
  
3. What would one of the leg lengths need to be so that the triangle in Problem 1 would be a right triangle? Show work that leads to your answer.

## Exit Ticket Sample Solutions

1. Is the triangle with leg lengths of 7 mm and 7 mm and a hypotenuse of length 10 mm a right triangle? Show your work, and answer in a complete sentence.

$$\begin{aligned}7^2 + 7^2 &= 10^2 \\49 + 49 &= 100 \\98 &\neq 100\end{aligned}$$

*No, the triangle with leg lengths of 7 mm, 7 mm, and hypotenuse of length 10 mm is not a right triangle because the lengths do not satisfy the Pythagorean Theorem.*

2. What would the hypotenuse need to be so that the triangle in Problem 1 would be a right triangle? Show work that leads to your answer.

*Let  $c$  represent the length of the hypotenuse.*

*Then,*

$$\begin{aligned}7^2 + 7^2 &= c^2 \\49 + 49 &= c^2 \\98 &= c^2 \\\sqrt{98} &= c\end{aligned}$$

*The hypotenuse would need to be  $\sqrt{98}$  mm for the triangle with sides of 7 mm and 7 mm to be a right triangle.*

3. What would one of the leg lengths need to be so that the triangle in Problem 1 would be a right triangle? Show work that leads to your answer.

*Let  $a$  represent the length of one leg.*

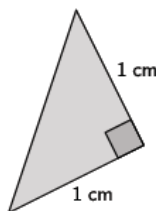
*Then,*

$$\begin{aligned}a^2 + 7^2 &= 10^2 \\a^2 + 49 &= 100 \\a^2 + 49 - 49 &= 100 - 49 \\a^2 &= 51 \\a &= \sqrt{51}\end{aligned}$$

*The leg length would need to be  $\sqrt{51}$  mm so that the triangle with one leg length of 7 mm and the hypotenuse of 10 mm is a right triangle.*

## Problem Set Sample Solutions

1. What is the length of the unknown side of the right triangle shown below? Show your work, and answer in a complete sentence. Provide an exact answer and an approximate answer rounded to the tenths place.

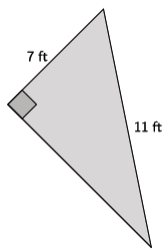


*Let  $c$  represent the hypotenuse of the triangle.*

$$\begin{aligned}1^2 + 1^2 &= c^2 \\1 + 1 &= c^2 \\2 &= c^2 \\\sqrt{2} &= \sqrt{c^2} \\1.4 &\approx c\end{aligned}$$

*The length of the hypotenuse is exactly  $\sqrt{2}$  cm and approximately 1.4 cm.*

2. What is the length of the unknown side of the right triangle shown below? Show your work, and answer in a complete sentence. Provide an exact answer and an approximate answer rounded to the tenths place.



Let  $x$  represent the unknown length of the triangle.

$$\begin{aligned} 7^2 + x^2 &= 11^2 \\ 49 + x^2 &= 121 \\ 49 - 49 + x^2 &= 121 - 49 \\ x^2 &= 72 \\ \sqrt{x^2} &= \sqrt{72} \\ x &= \sqrt{2^2} \times \sqrt{2} \times \sqrt{3^2} \\ x &= 6\sqrt{2} \\ x &\approx 8.5 \end{aligned}$$

The length of the unknown side of the triangle is exactly  $6\sqrt{2}$  ft. and approximately 8.5 ft.

3. Is the triangle with leg lengths of  $\sqrt{3}$  cm, 9 cm, and hypotenuse of length  $\sqrt{84}$  cm a right triangle? Show your work, and answer in a complete sentence.

$$\begin{aligned} (\sqrt{3})^2 + 9^2 &= (\sqrt{84})^2 \\ 3 + 81 &= 84 \\ 84 &= 84 \end{aligned}$$

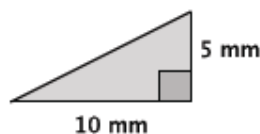
Yes, the triangle with leg lengths of  $\sqrt{3}$  cm, 9 cm, and hypotenuse of length  $\sqrt{84}$  cm is a right triangle because the lengths satisfy the Pythagorean Theorem.

4. Is the triangle with leg lengths of  $\sqrt{7}$  km, 5 km, and hypotenuse of length  $\sqrt{48}$  km a right triangle? Show your work, and answer in a complete sentence.

$$\begin{aligned} (\sqrt{7})^2 + 5^2 &= (\sqrt{48})^2 \\ 7 + 25 &= 48 \\ 32 &\neq 48 \end{aligned}$$

No, the triangle with leg lengths of  $\sqrt{7}$  km, 5 km, and hypotenuse of length  $\sqrt{48}$  km is not a right triangle because the lengths do not satisfy the Pythagorean Theorem.

5. What is the length of the unknown side of the right triangle shown below? Show your work, and answer in a complete sentence. Provide an exact answer and an approximate answer rounded to the tenths place.



Let  $c$  represent the hypotenuse of the triangle.

$$\begin{aligned} 5^2 + 10^2 &= c^2 \\ 25 + 100 &= c^2 \\ 125 &= c^2 \\ \sqrt{125} &= \sqrt{c^2} \\ \sqrt{5^3} &= c \\ \sqrt{5^2} \times \sqrt{5} &= c \\ 5\sqrt{5} &= c \\ 11.2 &\approx c \end{aligned}$$

The length of the hypotenuse is exactly  $5\sqrt{5}$  mm and approximately 11.2 mm.

6. Is the triangle with leg lengths of 3, 6, and hypotenuse of length  $\sqrt{45}$  a right triangle? Show your work, and answer in a complete sentence.

$$\begin{aligned} 3^2 + 6^2 &= (\sqrt{45})^2 \\ 9 + 36 &= 45 \\ 45 &= 45 \end{aligned}$$

Yes, the triangle with leg lengths of 3, 6 and hypotenuse of length  $\sqrt{45}$  is a right triangle because the lengths satisfy the Pythagorean Theorem.

7. What is the length of the unknown side of the right triangle shown below? Show your work, and answer in a complete sentence. Provide an exact answer and an approximate answer rounded to the tenths place.



Let  $x$  represent the unknown length of the triangle.

$$\begin{aligned} 2^2 + x^2 &= 8^2 \\ 4 + x^2 &= 64 \\ 4 - 4 + x^2 &= 64 - 4 \\ x^2 &= 60 \\ \sqrt{x^2} &= \sqrt{60} \\ x &= \sqrt{2^2 \times 3 \times 5} \\ x &= 2\sqrt{15} \\ x &\approx 7.7 \end{aligned}$$

The length of the unknown side of the triangle is exactly  $2\sqrt{15}$  inches and approximately 7.7 inches.

8. Is the triangle with leg lengths of  $1, \sqrt{3}$ , and hypotenuse of length 2 a right triangle? Show your work, and answer in a complete sentence.

$$\begin{aligned} 1^2 + (\sqrt{3})^2 &= 2^2 \\ 1 + 3 &= 4 \\ 4 &= 4 \end{aligned}$$

Yes, the triangle with leg lengths of  $1, \sqrt{3}$ , and hypotenuse of length 2 is a right triangle because the lengths satisfy the Pythagorean Theorem.

9. Corey found the hypotenuse of a right triangle with leg lengths of 2 and 3 to be  $\sqrt{13}$ . Corey claims that since  $\sqrt{13} = 3.61$  when estimating to two decimal digits, that a triangle with leg lengths of 2, 3, and a hypotenuse of 3.61 is a right triangle. Is he correct? Explain.

No, Corey is not correct.

$$\begin{aligned} 2^2 + 3^2 &= (3.61)^2 \\ 4 + 9 &= 13.0321 \\ 13 &\neq 13.0321 \end{aligned}$$

No, the triangle with leg lengths of 2, 3, and hypotenuse of length 3.61 is not a right triangle because the lengths do not satisfy the Pythagorean Theorem.

10. Explain a proof of the Pythagorean Theorem.

Consider having students share their proof with a partner while their partner critiques their reasoning. Accept any of the three proofs that the students have seen.

11. Explain a proof of the converse of the Pythagorean Theorem.

Consider having students share their proof with a partner while their partner critiques their reasoning. Accept either of the proofs that the students have seen.