# One Variable Inequalities

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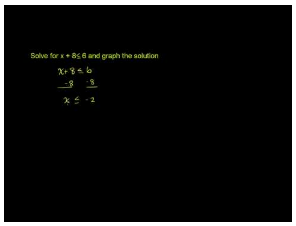
# CHAPTER 1 One Variable Inequalities

Here you are going to learn about one variable linear inequalities.

Janet holds up a card that reads $2x+6=16$. Donna holds up a card that reads $2x+6>16$. Andrew says they are not the same but Donna argues with him. Show, using an example, that Andrew is correct.

## Watch This

Khan Academy One Step Inequalities

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## Guidance

One variable linear inequalities have a different form than one variable linear equations. Linear equations have the general form of $ax+b=c$, where $a \neq 0$. Linear inequalities can have one of four forms: $ax+b>c, ax+b<c, ax+b \geq c$, or $ax+b \leq c$. You should notice the difference is that instead of an equals sign, there is an inequality symbol.

When you solve for a linear inequality, you follow the same rules as you would for a linear equation; however, you must remember one big rule: ***If you divide or multiply by a negative number while solving, you must reverse the sign of the inequality.***

### Example A

In the following table, a linear equation has been solved. Solve for the inequality using the similar steps. Are the steps the same? Is the inequality still true if you substitute 8 in for $p$?

**TABLE 1.1:**

| Equation | Inequality | Is the inequality still true? |
|---|---|---|
| $2p+4=20$<br>$2p+4-4=20-4$<br>$2p=16$<br>$\frac{2p}{2}=\frac{16}{2}$<br>$p=8$ | $2p+4<20$ | ? |

**Solution:**

**TABLE 1.2:**

| Equation | Inequality | Is the inequality still true? |
|---|---|---|
| $2p+4=20$ | $2p+4<20$ | no |

**TABLE 1.2:** (continued)

| Equation | Inequality | Is the inequality still true? |
|---|---|---|
| $2p+4-4=20-4$ | $2p+4-4<20-4$ | |
| $2p=16$ | $2p<16$ | |
| $\frac{2p}{2}=\frac{16}{2}$ | $\frac{2p}{2}<\frac{16}{2}$ | |
| $p=8$ | $p<8$ | |

No, there is no difference in the steps used to find the two solutions.

### Example B

In the following table, a linear equation has been solved. Solve for the inequality using the similar steps. Are the steps the same? Is the inequality still true if you substitute 6 in for $x$?

**TABLE 1.3:**

| Equation | Inequality | Is the inequality still true? |
|---|---|---|
| $3x+5=23$ | $3x+5\geq 23$ | ? |
| $3x+5-5=23-5$ | | |
| $3x=18$ | | |
| $\frac{3x}{3}=\frac{18}{3}$ | | |
| $x=6$ | | |

**Solution:**

**TABLE 1.4:**

| Equation | Inequality | Is the inequality still true? |
|---|---|---|
| $3x+5=23$ | $3x+5\geq 23$ | yes |
| $3x+5-5=23-5$ | $3x+5-5\geq 23-5$ | |
| $3x=18$ | $3x\geq 18$ | |
| $\frac{3x}{3}=\frac{18}{3}$ | $\frac{3x}{3}\geq\frac{18}{3}$ | |
| $x=6$ | $x\geq 6$ | |

No, there is no difference in the steps used to find the two solutions.

### Example C

In the following table, a linear equation has been solved. Solve for the inequality using the similar steps. Are the steps the same? Is the inequality still true if you substitute 3 in for $c$?

**TABLE 1.5:**

| Equation | Inequality | Is the inequality still true? |
|---|---|---|
| $5-3c=-4$ | $5-3c\leq -4$ | ? |
| $5-5-3c=-4-5$ | | |
| $-3c=-9$ | | |
| $\frac{-3c}{-3}=\frac{-9}{-3}$ | | |
| $c=3$ | | |

**Solution:**

**TABLE 1.6:**

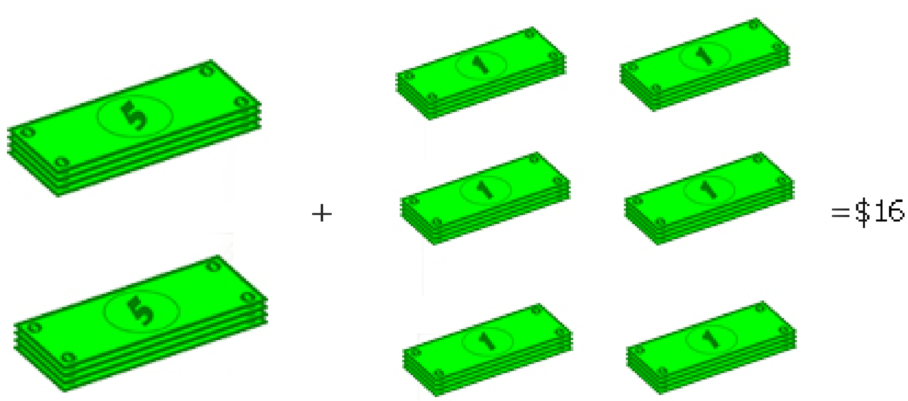
| **Equation** | **Inequality** | **Is the inequality still true?** |
|---|---|---|
| $5-3c=-4$ | $5-3c\leq-4$ | yes |
| $5-5-3c=-4-5$ | $5-5-3c\leq-4-5$ | |
| $-3c=-9$ | $-3c\leq-9$ | |
| $\frac{-3c}{-3}=\frac{-9}{-3}$ | $\frac{-3c}{-3}\geq\frac{-9}{-3}$ | |
| $c=3$ | $c\geq3$ | |

Yes, there was a difference in the steps used for the two solutions. When dividing by –3, the sign of the inequality was reversed.

### Concept Problem Revisited

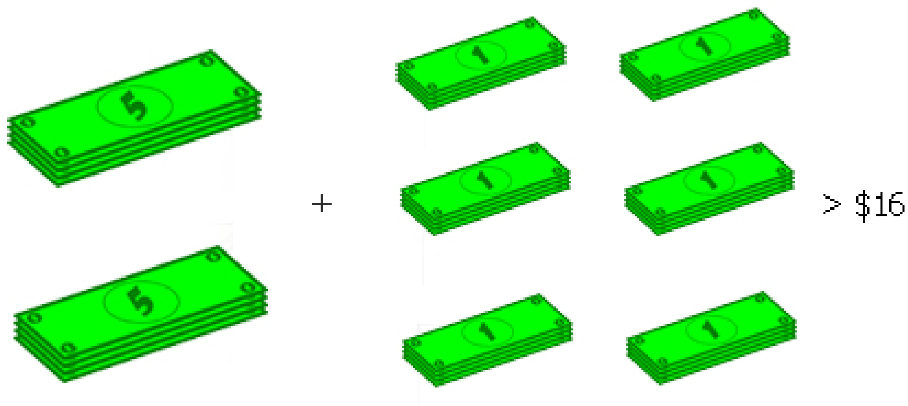
Janet holds up a card that reads $2x+6=16$. Donna holds up a card that reads $2x+6>16$. Andrew says they are not the same but Donna argues with him. Show, using an example, that Andrew is correct.

Andrew could use a real world example. For example, say Andrew held out two \$5 bills and six \$1 bills. Andrew holds Janet's card and says, "Is this true?"




The answer would be yes.

Now let's try it with Donna's inequality.




This amount of money is not greater than $16; it is just equal to $16. The two mathematical statements are not the same.

## Guided Practice

1. In the following table, a linear equation has been solved. Solve for the inequality using similar steps, but remember if you multiply or divide by a negative number you should reverse the inequality sign. Is the inequality still true if you substitute –10 in for *a*?

### TABLE 1.7:

| Equation | Inequality | Is the inequality still true? |
|---|---|---|
| $4.6a+8.2=2.4a-13.8$<br>$4.6a+8.2+13.8=2.4a-13.8+13.8$<br>$4.6a+22=2.4a$<br>$4.6a-4.6a+22=2.4a-4.6a$<br>$22=-2.2a$<br>$\frac{22}{-2.2}=\frac{-2.2a}{-2.2}$<br>$a=-10$ | $4.6a+8.2>2.4a-13.8$ | ? |

2. In the following table, a linear equation has been solved. Solve for the inequality using similar steps, but remember if you multiply or divide by a negative number you should reverse the inequality sign. Is the inequality still true if you substitute 6 in for *w*?

### TABLE 1.8:

| Equation | Inequality | Is the inequality still true? |
|---|---|---|
| $3(w+4)=2(3+2w)$<br>$3w+12=6+4w$<br>$3w+12-12=6-12+4w$<br>$3w=-6+4w$<br>$3w-4w=-6+4w-4w$<br>$-w=-6$<br>$\frac{-w}{-1}=\frac{-6}{-1}$<br>$w=6$ | $3(w+4)<2(3+2w)$ | ? |

3. In the following table, a linear equation has been solved. Solve for the inequality using similar steps, but remember if you multiply or divide by a negative number you should reverse the inequality sign. Is the inequality still true if you substitute –10 in for $h$?

### TABLE 1.9:

| Equation | Inequality | Is the inequality still true? |
|---|---|---|
| $\frac{1}{3}(2-h)=4$ | $\frac{1}{3}(2-h)\geq 4$ | ? |
| $\frac{1}{3}(2-h)=4\left(\frac{3}{3}\right)$ | | |
| $\frac{1}{3}(2-h)=\frac{12}{3}$ | | |
| $2-h=12$ | | |
| $2-2-h=12-2$ | | |
| $-h=10$ | | |
| $\frac{-h}{-1}=\frac{10}{-1}$ | | |
| $h=-10$ | | |

**Answers:**

1.

### TABLE 1.10:

| Equation | Inequality | Is the inequality still true? |
|---|---|---|
| $4.6a+8.2=2.4a-13.8$ | $4.6a+8.2>2.4a-13.8$ | no |
| $4.6a+8.2+13.8=2.4a-13.8+13.8$ | $4.6a+8.2+13.8>2.4a-13.8+13.8$ | |
| $4.6a+22=2.4a$ | $4.6a+22>2.4a$ | |
| $4.6a-4.6a+22=2.4a-4.6a$ | $4.6a-4.6a+22>2.4a-4.6a$ | |
| $22=-2.2a$ | $22>-2.2a$ | |
| $\frac{22}{-2.2}=\frac{-2.2a}{-2.2}$ | $\frac{22}{-2.2}<\frac{-2.2a}{-2.2}$ | |
| $a=-10$ | $a>-10$ | |

2.

### TABLE 1.11:

| Equation | Inequality | Is the inequality still true? |
|---|---|---|
| $3(w+4)=2(3+2w)$ | $3(w+4)<2(3+2w)$ | no |
| $3w+12=6+4w$ | $3w+12=6<4w$ | |
| $3w+12-12=6-12+4w$ | $3w+12-12<6-12+4w$ | |
| $3w=-6+4w$ | $3w<-6+4w$ | |
| $3w-4w=-6+4w-4w$ | $3w-4w<-6+4w-4w$ | |
| $-w=-6$ | $-w<-6$ | |
| $\frac{-w}{-1}=\frac{-6}{-1}$ | $\frac{-w}{-1}>\frac{-6}{-1}$ | |
| $w=6$ | $w>6$ | |

3.

### TABLE 1.12:

| Equation | Inequality | Is the inequality still true? |
|---|---|---|
| $\frac{1}{3}(2-h)=4$ | $\frac{1}{3}(2-h)\geq 4$ | yes |
| $\frac{1}{3}(2-h)=4\left(\frac{3}{3}\right)$ | $\frac{1}{3}(2-h)=4\left(\frac{3}{3}\right)$ | |

**TABLE 1.12:** (continued)

| Equation | Inequality | Is the inequality still true? |
|---|---|---|
| $\frac{1}{3}(2-h)=\frac{12}{3}$ | $\frac{1}{3}(2-h)\geq\frac{12}{3}$ | |
| $2-h=12$ | $2-h\geq 12$ | |
| $2-2-h=12-2$ | $2-2-h\geq 12-2$ | |
| $-h=10$ | $-h\geq 10$ | |
| $\frac{-h}{-1}=\frac{10}{-1}$ | $\frac{-h}{-1}\leq\frac{10}{-1}$ | |
| $h=-10$ | $h\leq -10$ | |

## Explore More

In the following table, a linear equation has been solved.

**TABLE 1.13:**

| Equation | Inequality | Is the inequality still true? |
|---|---|---|
| $5.2+x+3.6=4.3$ | $5.2+x+3.6\geq 4.3$ | ? |
| $8.8+x=4.3$ | | |
| $8.8-8.8+x=4.3-8.8$ | | |
| $x=-4.5$ | | |

1. Solve for the inequality using similar steps.
2. Were all of the steps the same? Why or why not?
3. Is the inequality still true if you substitute –4.5 in for $x$?

In the following table, a linear equation has been solved.

**TABLE 1.14:**

| Equation | Inequality | Is the inequality still true? |
|---|---|---|
| $\frac{n}{4}-5=-3$ | $\frac{n}{4}-5<-3$ | ? |
| $\frac{n}{4}-5\left(\frac{4}{4}\right)=-3\left(\frac{4}{4}\right)$ | | |
| $\frac{n}{4}-\frac{20}{4}=\frac{-12}{4}$ | | |
| $n-20=-12$ | | |
| $n-20+20=-12+20$ | | |
| $n=8$ | | |

4. Solve for the inequality using similar steps.
5. Were all of the steps the same? Why or why not?
6. Is the inequality still true if you substitute 8 in for $n$?

In the following table, a linear equation has been solved.

**TABLE 1.15:**

| Equation | Inequality | Is the inequality still true? |
|---|---|---|
| $1-z=5(3+2z)+8$ | $1-z<5(3+2z)+8$ | ? |
| $1-z=15+10z+8$ | | |
| $1-z=23+10z$ | | |
| $1-z+z=23+10z+z$ | | |

**TABLE 1.15:** (continued)

| Equation | Inequality | Is the inequality still true? |
|---|---|---|
| $1 = 23 + 11z$ | | |
| $1 - 23 = 23 - 23 + 11z$ | | |
| $-22 = 11z$ | | |
| $\frac{-22}{11} = \frac{11z}{11}$ | | |
| $z = -2$ | | |

7. Solve for the inequality using similar steps.
8. Were all of the steps the same? Why or why not?
9. Is the inequality still true if you substitute –2 in for $z$?

10. The sum of two numbers is greater than 764. If one of the numbers is 416, what could the other number be?
11. 205 less a number is greater than or equal to 112. What could that number be?
12. Five more than twice a number is less than 20. If the number is a whole number, what could the number be?
13. The product of 7 and a number is greater than 42. If the number is a whole number less than 10, what could the number be?
14. Three less than 5 times a number is less than or equal to 12. If the number is a whole number, what could the number be?
15. Double a number and add 12 and the result will be greater than 20. The number is less than 6. What is the number?