Inequalities that Describe Patterns

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CHAPTER 1 Inequalities that Describe Patterns

Here you will learn how to read about a real-life situation and write an inequality that represents this situation. You will then solve the inequality and plug the answer back into the inequality to check your work.

What if you were driving a car at 45 miles per hour and you knew that your destination was less than 150 miles away? What inequality could you set up to solve for the number of hours that you have left to travel? After you've solved the inequality, how could you check to make sure that your answer is correct? Once you've completed this Concept, you'll be able to find and verify solutions to inequalities representing scenarios like these.

Guidance

Sometimes Things Are Not Equal

In some cases there are multiple answers to a problem or the situation requires something that is not exactly equal to another value. When a mathematical sentence involves something other than an equal sign, an **inequality** is formed.

Definition: An **algebraic inequality** is a mathematical sentence connecting an expression to a value, a variable, or another expression with an inequality sign.

Listed below are the most common inequality signs.

$>$ "greater than"

$\geq$ "greater than or equal to"

$\leq$ "less than or equal to"

$<$ "less than"

$\neq$ "not equal to"

Below are several examples of inequalities.

$$3x < 5 \qquad x^2 + 2x - 1 > 0 \qquad \frac{3x}{4} \geq \frac{x}{2} - 3 \qquad 4 - x \leq 2x$$

Example A

Translate the following into an inequality: Avocados cost \$1.59 per pound. How many pounds of avocados can be purchased for less than \$7.00?

Solution: Choose a variable to represent the number of pounds of avocados purchased, say a.

$$1.59(a) < 7$$

You will be asked to solve this inequality in the exercises

Checking a Solution to an Inequality

Unlike equations, inequalities have more than one solution. However, you can check whether a value, such as $x = 6$, is *a solution* to an inequality the same way as you would check if it is *the solution* to an equation–by substituting it in and seeing if you get a true algebraic statement.

The following two examples show you how this works.

Example B

Check whether $m = 11$ is a solution set to $4m + 30 \leq 70$.

Solution:

Plug in $m = 11$, to see if we get a true statement.

$$4(11) + 30 \leq 70$$
$$44 + 30 \leq 70$$
$$74 \leq 70$$

Since $m = 11$ gives us a false statement, it is not a solution to the inequality.

Example C

Check whether $m = 10$ is a solution to $4m + 30 \leq 70$.

Solution:

Substitute in $m = 10$:

$$4(10) + 30 \leq 70$$
$$40 + 30 \leq 70$$
$$70 \leq 70$$

For $70 \leq 70$ to be a true statement, we need $70 < 70$ **or** $70 = 70$. Since $70 = 70$, this is a true statement, so $m = 10$ is a solution.

Video Review

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Guided Practice

1. Check whether $x = 3$ is a solution to $2x - 5 < 7$.

2. Check whether $x = 6$ is a solution to $2x - 5 < 7$.

Solutions:

1. Substitute in $x = 3$, to see if it is a solution to $2x - 5 < 7$.

$$\begin{aligned} 2(3) - 5 &< 7 \\ 6 - 5 &< 7 \\ 1 &< 7 \end{aligned}$$

Since 1 is less than 7, we have a true statement, so $x = 3$ is a solution to $2x - 5 < 7$.

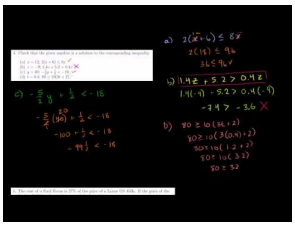
2. Check if $x = 6$ is a solution to $2x - 5 < 7$.

$$\begin{aligned} 2(6) - 5 &< 7 \\ 12 - 5 &< 7 \\ 7 &< 7 \end{aligned}$$

Since 7 is not less than 7, this is a false statement. Thus $x = 6$ is not a solution to $2x - 5 < 7$.

Explore More

Sample explanations for some of the practice exercises below are available by viewing the following video. Note that there is not always a match between the number of the practice exercise in the video and the number of the practice exercise listed in the following exercise set. However, the practice exercise is the same in both. CK-12 Basic Algebra: Equations and Inequalities (16:11)



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1. Define *solution*.
2. What is the difference between an algebraic equation and an algebraic inequality? Give an example of each.
3. What are the five most common inequality symbols?

In 4–7, define the variables and translate the following statements into algebraic equations.

4. A bus can seat 65 passengers or fewer.
5. The sum of two consecutive integers is less than 54.
6. An amount of money is invested at 5% annual interest. The interest earned at the end of the year is greater than or equal to $250.
7. You buy hamburgers at a fast food restaurant. A hamburger costs $0.49. You have at most $3 to spend. Write an inequality for the number of hamburgers you can buy.

For exercises 8–11, check whether the given solution set is the solution set to the corresponding inequality.

8. $x = 12;\ 2(x+6) \leq 8x$
9. $z = -9;\ 1.4z + 5.2 > 0.4z$
10. $y = 40;\ -\frac{5}{2}y + \frac{1}{2} < -18$
11. $t = 0.4;\ 80 \geq 10(3t+2)$

In 12-14, find the solution set.

12. Using the burger and French fries situation from the previous Concept, give three combinations of burgers and fries your family can buy without spending more than $25.00.
13. Solve the avocado inequality from Example A and check your solution.
14. On your new job you can be paid in one of two ways. You can either be paid $1000 per month plus 6% commission on total sales or be paid $1200 per month plus 5% commission on sales over $2000. For what amount of sales is the first option better than the second option? Assume there are always sales over $2000.

Mixed Review

15. Translate into an algebraic equation: 17 less than a number is 65.
16. Simplify the expression: $3^4 \div (9 \times 3) + 6 - 2$.
17. Rewrite the following without the multiplication sign: $A = \frac{1}{2} \cdot b \cdot h$.
18. The volume of a box without a lid is given by the formula $V = 4x(10-x)^2$, where x is a length in inches and V is the volume in cubic inches. What is the volume of the box when $x = 2$?