Linear Inequalities

CK-12 Study Guides

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Algebra I

Big Picture

Inequalities tell the relationship between two numbers or expressions, much like an equation. However, unlike equations, inequalities contain a range of solutions. This guide will go over with you some of the most popular methods of solving inequalities as well as some tips for solving the more complex ones. Solving inequalities can be a multi-step process and requires the proper application of inequality properties.

Key Terms

Inequality: Show that an expression is either larger or smaller than another expression. Contains inequality symbols.

Properties of Inequalities

Inequalities can be recognized by their inequality symbols. The four inequality symbols are:

Symbol	Meaning
$<$	less than
$\leq$	less than or equal to
$>$	greater than
$\geq$	greater than or equal to

Properties of equality are useful for understanding how to rearrange and solve inequalities. When solving inequalities, the inequality must be true at every step. A very important difference between equations and inequalities is that changing signs on one side of the equality changes the relationship, so the inequality sign must be switched.

Addition Property of Inequality:

If $a > b$, then $a+c > b+c$

If $a < b$, then $a+c < b+c$

If $a \geq b$, then $a+c \geq b+c$

If $a \leq b$, then $a+c \leq b+c$

Subtraction Property of Inequality:

If $a > b$, then $a-c > b-c$

If $a < b$, then $a-c < b-c$

If $a \geq b$, then $a-c \geq b-c$

If $a \leq b$, then $a-c \leq b-c$

Multiplication Property of Inequality:

When multiplied by a *positive* number c ($c > 0$),

If $a > b$, then $ac > bc$

If $a < b$, then $ac < bc$

If $a \geq b$, then $ac \geq bc$

If $a \leq b$, then $ac \leq bc$

When multiplied by a *negative* number c ($c < 0$),

If $a > b$, then $ac < bc$

If $a < b$, then $ac > bc$

If $a \geq b$, then $ac \leq bc$

If $a \leq b$, then $ac \geq bc$

Division Property of Inequality:

When divided by a *positive* number c ($c > 0$),

If $a > b$, then $\frac{a}{c} > \frac{b}{c}$

If $a < b$, then $\frac{a}{c} < \frac{b}{c}$

If $a \geq b$, then $\frac{a}{c} \geq \frac{b}{c}$

If $a \leq b$, then $\frac{a}{c} \leq \frac{b}{c}$

When multiplied by a *negative* number c ($c < 0$),

If $a > b$, then $\frac{a}{c} < \frac{b}{c}$

If $a < b$, then $\frac{a}{c} > \frac{b}{c}$

If a ≥ b, then $\frac{a}{c} \leq \frac{b}{c}$

If a ≤ b, then $\frac{a}{c} \geq \frac{b}{c}$

Flip the direction of the inequality symbol when you multiply or divide each side of an inequality by a negative number.

Notes

This guide was created by Nicole Crawford, Jane Li, and Jin Yu. To learn more about the student authors, visit http://www.ck12.org/about/ck-12-interns/.

Linear Inequalities cont.

Solving One-Step Inequalities

Using Addition

Isolate the variable by adding the subtracted number from both sides:

$$x-a < b \quad \text{or} \quad x-a > b$$
$$x-a+a < b+a \qquad x-a+a > b+a$$
$$x < b+a \qquad x > b+a$$

This is also true for inequalities $x-a \le b$ and $x-a \ge b$

Using Subtraction

Isolate the variable by subtracting the added number from both sides:

$$x+a < b \quad \text{or} \quad x+a > b$$
$$x+a-a < b-a \qquad x+a-a > b-a$$
$$x < b-a \qquad x > b-a$$

This is also true for inequalities $x+a \le b$ and $x+a \ge b$

Using Multiplication

Isolate the variable by multiplying both sides by the reciprocal of the variable's coefficient:

$$\frac{x}{a} < b \quad \text{or} \quad \frac{x}{a} > b$$
$$\frac{x}{a} \cdot a < b \cdot a \qquad \frac{x}{a} \cdot a > b \cdot a$$
$$x < ab \qquad x > ab$$

- When the inequality is multiplied by a negative number ($a < 0$), the direction of the inequality changes!

This is also true for inequalities $\frac{x}{a} \le b$ *and* $\frac{x}{a} \ge b$

Using Division

Isolate the variable by multiplying both sides by the reciprocal of the variable's coefficient:

$$ax < b \quad \text{or} \quad ax > b$$
$$\frac{ax}{a} < \frac{b}{a} \qquad \frac{ax}{a} > \frac{b}{a}$$
$$x < \frac{b}{a} \qquad x > \frac{b}{a}$$

- When the inequality is divided by a negative number ($a < 0$), the direction of the inequality changes!

This is also true for inequalities $ax \le b$ and $ax \ge b$

Multi-Step Inequalities

The steps for solving multi-step inequalities are similar to solving multi-step equations. Make sure each step obeys the properties of inequality and order of operations. *Don't forget to change the direction of the inequality when multiplying or dividing by a negative number!*

1. Simplify as much as you can using order of operations (PEMDAS) and the distributive property.
2. Add or subtract terms to isolate the variable
3. Multiply and divide by the constants that are attached to the variable. Change the direction of the inequality if it is multiplied or divided by a negative number.

Expressing Solutions

The solution of an inequality can be expressed in four ways:

1. Inequality notation: $x < a$
2. Set notation: $\{x|\ x < a\}$. Brackets are used for a set and the vertical line (|) means "such that," so this expression is "the set of all values of x such that x is a real number less than a."
3. Interval notation: $(-\infty, a)$. Indicates the range of the solution. Means "the interval containing all the numbers from $-\infty$ to a, but not including $-\infty$ or a."
   - Square brackets [and] next to a number means that number is included in the solution set.
   - Round brackets (and) next to a number means that number is not included in the solution set. Always use round brackets with infinity–infinity is not an actual number, so it is never included in the interval.
4. Graph on the real number line. See the *Number Lines* study guide on how to draw inequalities on a number line.

Number of Solutions

Inequalities usually have a range of values for the variable that will make the inequality true. There are two other special cases.

1. If the inequality simplifies to a false inequality (no longer true), the inequality is false and has no solution. This means no value for the variable will ever make the inequality true.

   Example: $-1 > 3$

Check your work to make sure you didn't make a mistake that causes the inequality to be false.

2. If the inequality simplifies to a true inequality, all numbers will make the inequality true, and the inequality has an infinite number of solutions.

   Example: $-1 < 3$