Inequalities with Multiplication and Division

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CHAPTER 1 Inequalities with Multiplication and Division

Here you'll learn how to use multiplication and division to find the solutions to one-step inequalities.

What if the distance in miles of a bowling alley from your house is $\frac{1}{5}$ of the distance of a roller-skating rink from your house? The distance of the roller-skating rink is represented by r, and you know that the bowling alley is less than or equal to 6 miles from your house. How far is the roller-skating rink from your house? After completing this Concept, you'll be able to solve inequalities like the one representing this scenario by using multiplication and division.

Watch This

Multimedia Link: For help with solving inequalities involving multiplication and division, visit Khan Academy's website: http://khanexercises.appspot.com/video?v=PNXozoJWsWc .

Guidance

Equations are mathematical sentences in which the two sides have the same "weight." By adding, subtracting, multiplying, or dividing the same value to both sides of the equation, the balance stays in check. However, inequalities begin off-balance. When you perform inverse operations, the inequality will remain off-balance. This is true with inequalities involving both multiplication and division.

Before we can begin to solve inequalities involving multiplication or division, you need to know two properties: the Multiplication Property of Inequality and the Division Property of Inequality.

Multiplication Property of Inequality: For all real positive numbers a, b, and c:

If $x < a$, *then* $x(c) < a(c)$.

If $x > a$, *then* $x(c) > a(c)$.

Division Property of Inequality: For all real positive numbers a, b, and c:

If $x < a$, *then* $x \div (c) < a \div (c)$.

If $x > a$, *then* $x \div (c) > a \div (c)$.

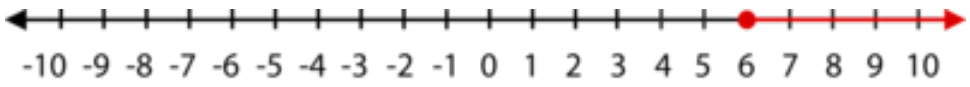
Example A

Consider the inequality $2x \geq 12$. To find the solutions to this inequality, we must isolate the variable x by using the inverse operation of "multiply by 2," which is dividing by 2.

$$2x \geq 12$$
$$\frac{2x}{2} \geq \frac{12}{2}$$
$$x \geq 6$$

This solution can be expressed in four ways. One way is already written: $x \geq 6$. Below are the three remaining ways to express this solution:

- $\{x|x \geq 6\}$
- $[6, \infty)$
- Using a number line:



Example B

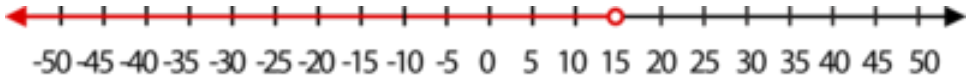
Solve for y : $\frac{y}{5} \leq 3$. *Express the solution using all four methods.*

Solution: The inequality above is read, "y divided by 5 is less than or equal to 3." To isolate the variable y, you must cancel division using its inverse operation, multiplication.

$$\frac{y}{5} \cdot \frac{5}{1} \leq 3 \cdot \frac{5}{1}$$
$$y \leq 15$$

One method of writing the solution is $y \leq 15$.

The other three are:

- $(-\infty, 15]$
- $\{y|y \leq 15\}$
- 

Multiplying and Dividing an Inequality by a Negative Number

Notice that the two properties in this Concept focused on c being only positive. This is because those particular properties of multiplication and division do not apply when the number being multiplied (or divided) is negative.

Think of it this way. When you multiply a value by –1, the number you get is the negative of the original.

$$6(-1) = -6$$

Multiplying each side of a sentence by –1 results in the opposite of both values.

$$5x(-1) = 4(-1)$$
$$-5x = -4$$

When multiplying by a negative, you are doing the "opposite" of everything in the sentence, including the verb.

$$x > 4$$
$$x(-1) > 4(-1)$$
$$-x < -4$$

This concept is summarized below.

Multiplication/Division Rule of Inequality: For any real number a, and any **negative** number c,

If $x < a$, *then* $x \cdot c > a \cdot c$

If $x < a$, *then* $\frac{x}{c} > \frac{a}{c}$

As with the other properties of inequalities, these also hold true for $\leq$ *or* $\geq$.

Example C

Solve for r : $-3r < 9$.

Solution: To isolate the variable r, we must cancel "multiply by –3" using its inverse operation, dividing by –3.

$$\frac{-3r}{-3} < \frac{9}{-3}$$

Since you are dividing by –3, everything becomes opposite, including the inequality sign.

$$r > -3$$

Example D

Solve for p : $12p < -30$.

Solution: To isolate the variable p, we must cancel "multiply by 12" using its inverse operation, dividing by 12.

$$\frac{12p}{12} < \frac{-30}{12}$$

Because 12 is **not** negative, you do **not** switch the inequality sign.

$$p < \frac{-5}{2}$$

.

In set notation, the solution would be: $\left(-\infty, \frac{-5}{2}\right)$.

Video Review

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Guided Practice

Solve for m : $-\frac{m}{3} < 2.4$.

Solution:

To isolate the variable m, we must cancel "divide by 3" using its inverse operation, multiplying by 3. We must also cancel out the negative, so we would multiply by -1. Multiplying by 3 and -1 means multiplying by -3.

$$-3 \cdot -\frac{m}{3} < -3 \cdot 2.4$$

Because -3 is negative, you need to switch the inequality sign.

$$m > -7.2$$

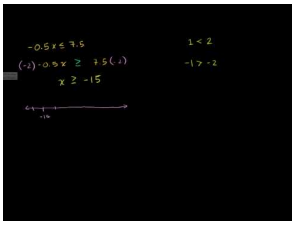
.

In set notation, the solution would be: $(-7.2, \infty)$.

This means that m must be greater than -7.2, but not equal to it.

Explore More

Sample explanations for some of the practice exercises below are available by viewing the following video. Note that there is not always a match between the number of the practice exercise in the video and the number of the practice exercise listed in the following exercise set. However, the practice exercise is the same in both. CK-12 Basic Algebra: Inequalities Using Multiplication and Division (10:27)



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1. In which cases do you change the inequality sign?

Solve each inequality. Give the solution using inequality notation and with a solution graph.

2. $3x \leq 6$

3. $\frac{x}{5} > -\frac{3}{10}$
4. $-10x > 250$
5. $\frac{x}{-7} \geq -5$
6. $9x > -\frac{3}{4}$
7. $\frac{x}{-15} \leq 5$
8. $620x > 2400$
9. $\frac{x}{20} \geq -\frac{7}{40}$
10. $-0.5x \leq 7.5$
11. $75x \geq 125$
12. $\frac{x}{-3} > -\frac{10}{9}$
13. $\frac{k}{-14} \leq 1$
14. $\frac{x}{-15} < 8$
15. $\frac{x}{2} > 40$
16. $\frac{x}{-3} \leq -12$
17. $\frac{x}{25} < \frac{3}{2}$
18. $\frac{x}{-7} \geq 9$
19. $4x < 24$
20. $238 < 14d$
21. $-19m \leq -285$
22. $-9x \geq -\frac{3}{5}$
23. $-5x \leq 21$

Mixed Review

24. After 3 dozen cookies, Anna has fewer than 24 to make.
    a. Write an inequality to represent this situation. Let $c =$ *the number of cookies Anna had to make.*
    b. Write the solutions in set notation.
25. Tracey's checking account balance is $31.85. He needs to deposit enough money to pay his satellite T.V. bill, which is $97.12.
    a. Write an inequality to represent this situation.
    b. Write the solutions as an algebraic sentence.
26. Solve for $v : v = -|2 - (-19) + 6|$.
27. A piggy bank is filled with dimes and quarters. The total amount of money is $26.00.
    a. Graph all the combinations that make this statement true.
    b. If $13.50 is in quarters, how many dimes must be in the piggy bank?