Multi-Step Inequalities

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CHAPTER 1 Multi-Step Inequalities

Here you'll learn how to use the distributive, addition, subtraction, multiplication, and division properties to find the solutions to multi-step inequalities.

Suppose that you know that 10 less than 3 times the number of coins in your piggy bank is greater than 200. If the number of coins in your piggy bank is represented by c, how would you go about finding the value of this variable? In this Concept, you'll learn how to solve multi-step inequalities such as the one representing this scenario by using the distributive, addition, subtraction, multiplication, and division properties.

Guidance

Previously we worked on one-step inequalities. Inequalities, like equations, can require several steps to isolate the variable. These inequalities are called **multi-step inequalities.** With the exception of the Multiplication/Division Property of Inequality, the process of solving multi-step inequalities is identical to solving multi-step equations.

Procedure to Solve an Inequality:

1. Remove any parentheses by using the Distributive Property.
2. Simplify each side of the inequality by combining like terms.
3. Isolate the ax term. Use the Addition/Subtraction Property of Inequality to get the variable on one side of the inequality sign and the numerical values on the other.
4. Isolate the variable. Use the Multiplication/Division Property of Inequality to get the variable alone on one side of the inequality.
    a. Remember to reverse the inequality sign if you are multiplying or dividing by a negative number.
5. Check your solution.

Example A

Solve for w : $6x - 5 < 10$.

Solution: Begin by using the checklist above.

1. Parentheses? No

2. Like terms on the same side of inequality? No

3. Isolate the ax term using the Addition Property.

$$6x - 5 + 5 < 10 + 5$$

Simplify.

$$6x < 15$$

4. Isolate the variable using the Multiplication or Division Property.

$$\frac{6x}{6} < \frac{15}{6} = x < \frac{5}{2}$$

5. Check your solution. Choose a number less than 2.5, say 0, and check using the original inequality.

$$6(0) - 5 < 10?$$
$$-5 < 10$$

Yes, the answer checks. $x < 2.5$

Example B

Solve for x: $-9x < -5x - 15$.

Solution: Begin by using the checklist above.

1. Parentheses? No

2. Like terms on the same side of inequality? No

3. Isolate the ax term using the Addition Property.

$$-9x + 5x < -5x + 5x - 15$$

Simplify.

$$-4x < -15$$

4. Isolate the variable using the Multiplication or Division Property.

$$\frac{-4x}{-4} < \frac{-15}{-4}$$

Because the number you are dividing by is negative, you must reverse the inequality sign.

$$x > \frac{15}{4} \rightarrow x > 3\frac{3}{4}$$

5. Check your solution by choosing a number larger than 3.75, such as 10.

$$-9(10) < -5(10) - 15?$$
$$\checkmark \ -90 < -65$$

Identifying the Number of Solutions to an Inequality

Inequalities can have infinitely many solutions, no solutions, or a finite set of solutions. Most of the inequalities you have solved to this point have an infinite number of solutions. By solving inequalities and using the context of a problem, you can determine the number of solutions an inequality may have.

Example C

Find the solutions to $x-5>x+6$.

Solution: Begin by isolating the variable using the Addition Property of Inequality.

$$x-x-5>x-x+6$$

Simplify.

$$-5>6$$

This is an untrue inequality. Negative five is never greater than six. Therefore, the inequality $x-5>x+6$ has no solutions.

Example D

Previously we looked at the following sentence: "The speed limit is 65 miles per hour." Use inequalities and set notation to describe the set of possible speeds at which a car could drive under the speed limit.

Solution:

The speed at which you drive cannot be negative, which means $0 \leq s$, and it must be less than 65 miles per hour, so $s \leq 65$.. Combining these we get $0 \leq s \leq 65$. Therefore, the set of possibilities using interval notation is [0, 65].

This solution set has infinitely many solutions, since there are infinitely many real numbers between 0 and 65.

Video Review

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Guided Practice

Solve for x: $4x-2(3x-9) \leq -4(2x-9)$.

Solution: Begin by using the previous checklist.

1. Parentheses? Yes. Use the Distributive Property to clear the parentheses.

$$4x+(-2)(3x)+(-2)(-9) \leq -4(2x)+(-4)(-9)$$

Simplify.

$$4x - 6x + 18 \leq -8x + 36$$

2. Like terms on the same side of inequality? Yes. Combine these.

$$-2x + 18 \leq -8x + 36$$

3. Isolate the *ax* term using the Addition Property.

$$-2x + 8x + 18 \leq -8x + 8x + 36$$

Simplify.

$$\begin{aligned} 6x + 18 &\leq 36 \\ 6x + 18 - 18 &\leq 36 - 18 \\ 6x &\leq 18 \end{aligned}$$

4. Isolate the variable using the Multiplication or Division Property.

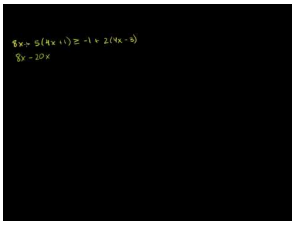
$$\frac{6x}{6} \leq \frac{18}{6} \rightarrow x \leq 3$$

5. Check your solution by choosing a number less than 3, such as –5.

$$\begin{aligned} 4(-5) - 2(3 \cdot -5 - 9) &\leq -4(2 \cdot -5 - 9) \\ \checkmark \; 28 &< 76 \end{aligned}$$

Explore More

Sample explanations for some of the practice exercises below are available by viewing the following video. Note that there is not always a match between the number of the practice exercise in the video and the number of the practice exercise listed in the following exercise set. However, the practice exercise is the same in both. CK-12 Basic Algebra: Multi-Step Inequalities (8:02)



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In 1–15, solve each of the inequalities and graph the solution set.

1. $6x-5<10$
2. $-9x<-5x-15$
3. $-\frac{9x}{5}\leq 24$
4. $\frac{9x}{5}-7\geq -3x+12$
5. $\frac{5x-1}{4}>-2(x+5)$
6. $4x+3<-1$
7. $2x<7x-36$
8. $5x>8x+27$
9. $5-x<9+x$
10. $4-6x\leq 2(2x+3)$
11. $5(4x+3)\geq 9(x-2)-x$
12. $2(2x-1)+3<5(x+3)-2x$
13. $8x-5(4x+1)\geq -1+2(4x-3)$
14. $2(7x-2)-3(x+2)<4x-(3x+4)$
15. $\frac{2}{3}x-\frac{1}{2}(4x-1)\geq x+2(x-3)$

Mixed Review

16. Solve: $10\geq -5f$.
17. Graph $y=-7$ on a coordinate plane.
18. Classify $\sqrt{5}$ using the real number hierarchy.
19. What are some problem-solving methods you have learned so far in this textbook? List one example for each method.
20. A circle has an area of $A=\pi r^2$. What is the radius of a circle with area of $196\pi\ in^2$?
21. Solve for a : $\frac{6}{a}=\frac{-22}{a+4}$.