Algebraic Solutions to One Variable Inequalities

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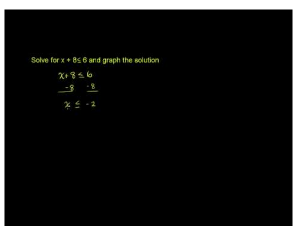
CHAPTER 1 Algebraic Solutions to One Variable Inequalities

Here you'll learn how to solve one variable inequalities.

The Morgan Silver Dollar is a very valuable American dollar minted between 1978 and 1921. When placed on a scale with twenty 1-gram masses, the scale tips toward the Morgan Dollar. Draw a picture to represent this scenario and then write the inequality.

Watch This

Khan Academy One Step Inequalities



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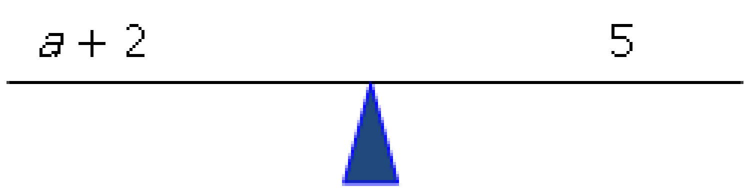
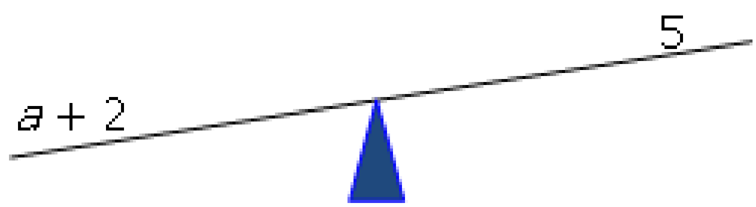
Guidance

Linear equations are of the form $ax+b=0$, where $a \neq 0$. With linear equations there is always an equals sign. Linear inequalities are mathematical statements relating expressions by using one or more inequality symbols $<, >, \leq$, or $\geq$. In other words, the left side no longer equals the right side, it is less than, greater than, less than or equal to, or greater than or equal to.

Recall that one method for solving an equation is to use the balance method. To solve for $a+2=5$, you would draw the following balance:



If you were to use the balance method to solve the linear inequality version, it would look more like this: $a+2>5$



Think about it this way. It is like having someone heavier on the $(a+2)$ side of the balance and someone light on the (5) side of the balance. The $(a+2)$ person has a weight greater than (>) the (5) person and therefore the balance moves down to the ground.

The rules for solving inequalities are basically the same as you used for solving linear equations. If you have parentheses, remove these by using the distributive property. Then you must isolate the variable by moving constants to one side and variables to the other side of the inequality sign. You also have to remember that whatever you do to one side of the inequality, you must do to the other. The same was true when you were working with linear equations. One additional rule is to reverse the sign of the inequality if you are multiplying or dividing both sides by a negative number.

It is important for you to remember what the symbols mean. Always remember that the mouth of the sign opens toward the larger number. So $8 > 5$, the mouth of the >sign opens toward the 8 so 8 is larger than 5. You know that's true. $6b-5 < 300$, the mouth opens toward the 300, so 300 is larger than $6b-5$.

Example A

Solve: $15 < 4+3x$

Solution: Remember that whatever you do to one side of the inequality sign, you do to the other.

$15-4 < 4-4+3x$	Subtract 4 from both sides to isolate the variable
$11 < 3x$	Simplify
$\frac{11}{3} < \frac{3x}{3}$	Divide by 3 to solve for the variable
$x > \frac{11}{3}$	Simplify

Do a quick check to see if this is true. $\frac{11}{3}$ is approximately 3.67. Try substituting 0, 3, and 4 into the equation.

$15 < 4+3x$	$15 < 4+3x$	$15 < 4+3x$
$15 < 4+3(0)$	$15 < 4+3(3)$	$15 < 4+3(4)$
$15 < 4$ False	$15 < 13$ False	$15 < 16$ True

The only value out of 0, 3, and 4 where $x > \frac{11}{3}$ is 4. If you look at the statements above, it was the only inequality that gave a true statement.

Example B

Solve: $2y+3>7$

Solution: Use the same rules as if you were solving any algebraic expression. Remember that whatever you do to one side of the inequality sign, you do to the other.

$$2y+3-3>7-3 \quad \text{Subtract 3 from both sides to isolate the variable}$$
$$2y>4 \quad \text{Simplify}$$
$$\frac{2y}{2}>\frac{4}{2} \quad \text{Divide by 2 to solve for the variable}$$
$$y>2 \quad \text{Simplify}$$

Do a quick check to see if this is true. Try substituting 0, 4, and 8 into the equation.

$$\begin{aligned} 2y+3&>7 \\ 2(0)+3&>7 \\ 3&>7 \text{ False} \end{aligned} \qquad \begin{aligned} 2y+3&>7 \\ 2(4)+3&>7 \\ 11&>7 \text{ True} \end{aligned} \qquad \begin{aligned} 2y+3&>7 \\ 2(8)+3&>7 \\ 19&>7 \text{ True} \end{aligned}$$

The values out of 0, 4, and 8 where $y>2$ are 4 and 8. If you look at the statements above, these values when substituted into the inequality gave true statements.

Example C

Solve: $-2c-5<8$

Solution: Again, to solve this inequality, use the same rules as if you were solving any algebraic expression. Remember that whatever you do to one side of the inequality sign, you do to the other.

$$-2c-5+5<8+5 \quad \text{Add 5 to both sides to isolate the variable}$$
$$-2c<13 \quad \text{Simplify}$$
$$\frac{-2c}{-2}<\frac{13}{-2} \quad \text{Divide by -2 to solve for the variable}$$

Note: When you divide by a negative number, the inequality sign reverses.

$$c>\frac{-13}{2} \quad \text{Simplify}$$

Do a quick check to see if this is true. $\frac{-13}{2}$ is equal to –6.5. Try substituting –8, 0, and 2 into the equation.

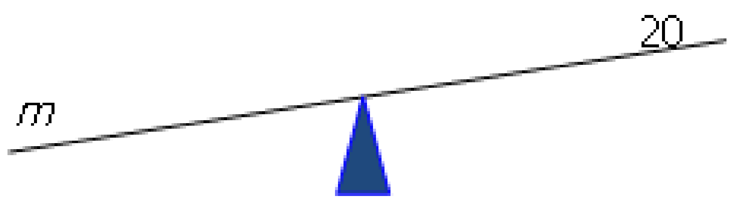
$$\begin{aligned} -2c-5&<8 \\ -2(-8)-5&<8 \\ 11&<8 \text{ False} \end{aligned} \qquad \begin{aligned} -2c-5&<8 \\ -2(0)-5&<8 \\ -5&<8 \text{ True} \end{aligned} \qquad \begin{aligned} -2c-5&<8 \\ -2(2)-5&<8 \\ -9&<8 \text{ True} \end{aligned}$$

The values out of –8, 0, and 2 where $c>\frac{-13}{2}$ are 0 and 2. If you look at the statements above, these values when substituted into the inequality gave true statements.

Concept Problem Revisited

The Morgan Silver Dollar is a very valuable American dollar minted between 1978 and 1921. When placed on a scale with twenty 1-gram masses, the scale tips toward the Morgan Dollar. Draw a picture to represent this scenario and then write the inequality.

Let $m =$ Morgan dollar



Since the weight of the Morgan dollar is greater than 20 g, the mouth of the inequality sign would open towards the variable, m. Therefore the inequality equation would be:

$m > 20$

Guided Practice

Solve each inequality.

1. $4t + 3 > 11$

2. $2z + 7 \leq 5z + 28$

3. $9(j - 2) \geq 6(j + 3) - 9$

Answers:

1. $t > 2$. Here are the steps:

$$\begin{aligned} 4t + 3 &> 11 \\ 4t + 3 - 3 &> 11 - 3 && \text{Subtract 3 from both sides to isolate the variable} \\ 4t &> 8 && \text{Simplify} \\ \frac{4t}{4} &> \frac{8}{4} && \text{Divide by 4 to solve for the variable} \\ t &> 2 \end{aligned}$$

2. $z \geq -7$. Here are the steps:

$$\begin{aligned}
2z+7 &\leq 5z+28 \\
2z-2z+7 &\leq 5z-2z+28 && \text{Subtract } 2z \text{ from both sides to get variables on same side of the inequality sign.} \\
7 &\leq 3z+28 && \text{Simplify} \\
7-28 &\leq 3z+28-28 && \text{Subtract 28 from both sides to isolate the variable} \\
-21 &\leq 3z && \text{Simplify} \\
\frac{3z}{3} &\geq \frac{-21}{3} && \text{Divide by 3 to solve for the variable} \\
z &\geq -7
\end{aligned}$$

3. $j \geq 9$. Here are the steps:

$$\begin{aligned}
9(j-2) &\geq 6(j+3)-9 \\
9j-18 &\geq 6j+18-9 && \text{Remove parentheses} \\
9j-18 &\geq 6j+9 && \text{Combine like terms on each side of inequality sign} \\
9j-6j-18 &\geq 6j-6j+9 && \text{Subtract } 6j \text{ from both sides to get variables on same side of the inequality sign.} \\
3j-18 &\geq 9 && \text{Simplify} \\
3j-18+18 &\geq 9+18 && \text{Add 18 to both sides to isolate the variable} \\
3j &\geq 27 && \text{Simplify} \\
\frac{3j}{3} &\geq \frac{27}{3} && \text{Divide by 3 to solve for the variable} \\
j &\geq 9
\end{aligned}$$

Explore More

Solve for the variable in the following inequalities.

1. $a+8>4$
2. $4c-1>7$
3. $5-3k<6$
4. $3-4t \leq -11$
5. $6 \geq 11-2b$
6. $\frac{e}{5}-3>-1$
7. $\frac{1}{5}(r-3)<-1$
8. $\frac{1}{3}(f+2)<4$
9. $\frac{p+3}{4} \geq -2$
10. $\frac{1}{2}(5-w) \leq -3$
11. $3(2x-5)<2(x-1)+3$
12. $2(y+8)+5(y-1)>6$
13. $2(d-3)<-3(d+3)$
14. $3(g+3) \geq 2(g+1)-2$
15. $2(3s-4)+1 \leq 3(4s+1)$