Solve Inequalities by Using the Distributive Property

Jen Kershaw

AUTHOR
Jen Kershaw

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Printed: March 27, 2015

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CHAPTER 1

Solve Inequalities by Using the Distributive Property

Here you'll solve inequalities by using the distributive property.

Have you ever tried to build a deck? Take a look at this situation.

Ms. Layne wants to build a rectangular deck in her back yard. She wants the length of the deck to be exactly 9 feet. She wants the perimeter of her deck to be, at most, 28 feet. The perimeter of any rectangle can be found by using the expression $P = 2(l + w)$, where l represents the length and w represents the width.

Write an inequality that could be used to represent w, the possible widths, in feet, she could use for her deck. Would a deck with a width of 6 feet result in a deck with the perimeter she wants?

This Concept will teach you how to use the distributive property to solve inequalities. It is exactly what you will need to figure out the deck dilemma.

Guidance

The inequalities that you will see in this Concept involve parentheses. We can simplify an equation with parentheses by using the distributive property. We can do this with inequalities as well. Using the distributive property can help you to simplify an inequality so that it is easier to solve.

Solve for q: $-9(q+3) < 45$

Apply the distributive property to the left side of the inequality. Multiply each of the two numbers inside the parentheses by -9 and then add those products.

$$\begin{aligned} -9(q+3) &< 45 \\ (-9 \times q) + (-9 \times 3) &< 45 \\ -9q + (-27) &< 45 \end{aligned}$$

Now, solve as you would solve any two-step inequality. Since -27 is added to $-9q$, we can get $-9q$ by itself on one side of the inequality by subtracting -27 from both sides. Remember, subtracting -27 from a number is the same as adding its opposite, 27, to that number.

$$\begin{aligned} -9q + (-27) &< 45 \\ -9q + (-27) - (-27) &< 45 - (-27) \\ -9q + (-27 + 27) &< 45 + 27 \\ -9q + 0 &< 72 \\ -9q &< 72 \end{aligned}$$

To get q by itself on one side of the inequality, we need to divide both sides by -9. Since we are dividing both sides by a negative number, you need to reverse the inequality symbol.

$$\begin{aligned} -9q &< 72 \\ \frac{-9q}{-9} &> \frac{72}{-9} \\ 1q &> -8 \\ q &> -8 \end{aligned}$$

The solution is $q > -8$.

$\frac{1}{2}(x+4) \leq 10$

First, we use the distributive property to multiply one-half with both of the terms inside the parentheses.

$\frac{1}{2}x+2 \leq 10$

Next, we subtract two from both sides of the inequality.

$\frac{1}{2}x \leq 8$

Now we can multiply both sides by the reciprocal of one-half which will cancel out the one-half leaving our variable alone. This is an example of the multiplicative inverse property.

$\frac{2}{1}\left(\frac{1}{2}\right)x \leq 8\left(\frac{2}{1}\right)$

The answer is that $x \leq 16$.

Example A

$-5(x+2) > 15$

Solution: $x < -5$

Example B

$6(x-4) \geq 24$

Solution: $x \geq 8$

Example C

$-2(y+3) \leq 12$

Solution: $y \geq -9$

Now let's go back to the dilemma at the beginning of the Concept.

Consider part *a* first.

You know that the length is 9 feet, so substitute 9 for l into the expression $2(l+w)$. This expression represents the actual perimeter of the deck.

actual perimeter $= 2(l+w) = 2(9+w)$

Since she wants the perimeter to be "at most" 28 feet, you should use the "less than or equal to" $(\leq)$ symbol. Translate this problem into an inequality.

$$\begin{array}{c} \textit{She wants the } \underline{\textit{perimeter}} \textit{ of her deck to be, } \underline{\textit{at most}}, \underline{28 \textit{ feet}}. \\ \downarrow \qquad\qquad\qquad\qquad \downarrow \quad \downarrow \\ 2(9+w) \qquad\qquad\qquad\quad \leq \quad 28 \end{array}$$

So, this problem can be represented by the inequality $2(9+w) \leq 28$.

Next, consider part *b* .

To find all the possible values of w, solve the inequality. First, apply the distributive property to the right side.

$$\begin{aligned} 2(9+w) &\leq 28 \\ (2\times 9)+(2\times w) &\leq 28 \\ 18+2w &\leq 28 \end{aligned}$$

Now, solve as you would solve any two-step inequality. First, subtract 18 from both sides of the inequality.

$$\begin{aligned} 18+2w &\leq 28 \\ 18-18+2w &\leq 28-18 \\ 0+2w &\leq 10 \\ 2w &\leq 10 \end{aligned}$$

Next, divide both sides of the inequality by 2. Since you are dividing by a positive number, the inequality symbol should stay the same.

$$\begin{aligned} 2w &\leq 10 \\ \frac{2w}{2} &\leq \frac{10}{2} \\ 1w &\leq 5 \\ w &\leq 5 \end{aligned}$$

The value of w must be less than or equal to 5.

Since 6 is greater than, not less than, 5, it is not a possible value of w. So, if she built her deck so it was 6 feet wide, it would have a larger perimeter than she wants.

Guided Practice

Here is one for you to try on your own.

Solve for w: $-2(8+w)+18<28$.

Solution

First, we should apply the distributive property to the left side of the inequality. We can multiply each of the two numbers inside the parentheses by -2 and then add those products.

$$\begin{aligned} -2(8+w)+18 &< 28 \\ (-2\times 8)+(-2\times w)+18 &< 28 \\ -16+(-2w)+18 &< 28 \end{aligned}$$

Next, we can add the like terms (-16 and 18) on the left side of the inequality. Using the commutative and associative properties to reorder the terms on the left side of the equation can make it easier to see how to do this.

$$\begin{aligned} -16+(-2w)+18 &< 28 \\ -16+[(-2w)+18] &< 28 \\ -16+[18+(-2w)] &< 28 \\ (-16+18)+(-2w) &< 28 \\ 2+(-2w) &< 28 \end{aligned}$$

Finally, we solve as you would solve any two-step inequality. Since 2 is added to $-2w$, our first step should be to subtract 2 from both sides of the inequality.

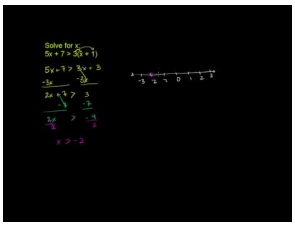
$$\begin{aligned} 2+(-2w) &< 28 \\ 2-2+(-2w) &< 28-2 \\ 0+(-2w) &< 26 \\ -2w &< 26 \end{aligned}$$

Now, we can isolate the variable, w, by dividing both sides of the inequality by -2. Since we are dividing both sides by a negative number, we need to reverse the inequality symbol.

$$\begin{aligned} -2w &< 26 \\ \frac{-2w}{-2} &> \frac{26}{-2} \\ 1w &> -13 \\ w &> -13 \end{aligned}$$

The solution is $w > -13$.

Video Review



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Click image to the left or use the URL below.

URL: http://www.ck12.org/flx/render/embeddedobject/58508

Khan Academy Solving Multi-step Inequalities

Explore More

Directions: Solve each inequality.

1. $3(x+4) > 21$
2. $4(x-1) < 8$
3. $5(y+7) < 70$
4. $-4(x+2) > 8$
5. $3(x-9) \geq 30$
6. $-2(y+4) \geq 16$
7. $5(x+2) \leq 100$
8. $-2(y-3)+12y > 16$
9. $4(x+2)-10x > 38$
10. $3(x-2)+5x \leq 42$
11. $-2(y+4)-2y > 8$
12. $-5(x+2)+6(x-2) \geq 10$
13. $3(x+4)-2(x+1) > 5$
14. $-2(y-4)+8y+2 < 16$
15. $-8(x+2)-9x+2x \geq 14$