Compound Inequalities

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CHAPTER 1 Compound Inequalities

Here you'll learn how to find the solution to a compound inequality and graph it, as well as how to use a graphing calculator to solve the inequality.

What if you spent 2 more hours doing your homework this week than you spent last week? If the number of hours that you spent last week is represented by h, and you spent between 6 and 9 hours this week, how many hours did you spend doing your homework last week? In this Concept, you'll learn how to solve inequalities such as the one representing this scenario and how to graph the solution. You'll also learn how to use a graphing calculator to find the solution.

Guidance

Inequalities that relate to the same topic can be written as a **compound inequality**. A compound inequality involves the connecting words **"and"** and **"or."**

The word **and** in mathematics means the **intersection** between the sets.

"What the sets have in common."

The word **or** in mathematics means the **union** of the sets.

"Combining both sets into one large set."

Inequalities Involving "And"

Consider, for example, the speed limit situation from the Solving Multi-Step Inequalities Concept. Using interval notation, the solutions to this situation can be written as [0, 65]. As an inequality, what is being said it this:

The speed must be at least 0 mph and at most 65 mph.

Using inequalities to represent "at least" and "at most," the following sentences are written:

$$s \geq 0 \text{ and } s \leq 65$$

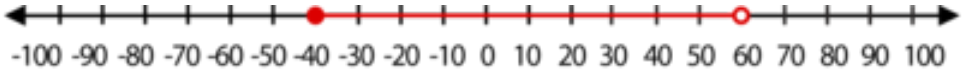
This is an example of a compound inequality. It can be shortened by writing:

$$0 \leq s \leq 65$$

Example A

Graph the solutions to $-40 \leq y < 60$.

Solution: Color in a circle above –40 to represent "less than or equal to." Draw an uncolored circle above 60. The variable is placed between these two values, so the solutions occur between these two numbers.



Inequalities Involving "Or"

A restaurant offers discounts to children 3 years or younger or to adults over 65. Write an inequality for the possible ages eligible to receive the discount.

Begin by writing an inequality to represent each piece. "3 years or younger" means you must be born but must not have celebrated your fourth birthday.

$$0 \leq a < 4$$

"Adults over 65" implies $a > 65$.

The word **or** between the phrases allows you to graph all the possibilities on one number line.

Solving "And" Compound Inequalities

When we solve compound inequalities, we separate the inequalities and solve each of them separately. Then, we combine the solutions at the end.

Example B

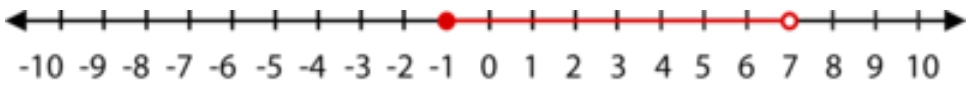
Solve for x and graph your solution: $3x - 5 < x + 9 \leq 5x + 13$.

Solution:

To solve $3x - 5 < x + 9 \leq 5x + 13$, begin by separating the inequalities.

$$\begin{aligned} 3x - 5 &< x + 9 \\ 2x &< 14 \\ x &< 7 \end{aligned} \qquad \text{and} \qquad \begin{aligned} x + 9 &\leq 5x + 13 \\ -4 &\leq 4x \\ -1 &\leq x \text{ or } x \geq -1 \end{aligned}$$

The answers are $x < 7$ and $x \geq -1$ and can be written as $-1 \leq x < 7$. You graph the solutions that satisfy both inequalities.



Solving "Or" Compound Inequalities

To solve an "or" compound inequality, separate the individual inequalities. Solve each separately. Then combine the solutions to finish the problem.

Example C

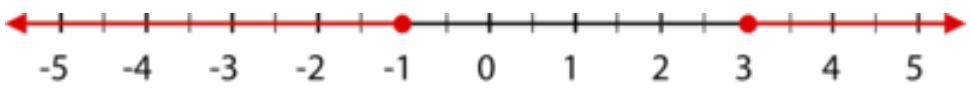
Solve for x and graph the solution: $9 - 2x \leq 3$ *or* $3x + 10 \leq 6 - x$.

Solution:

To solve $9 - 2x \leq 3$ or $3x + 10 \leq 6 - x$, begin by separating the inequalities.

$$\begin{aligned} 9 - 2x &\leq 3 \\ -2x &\leq -6 \\ x &\geq 3 \end{aligned} \qquad \text{or} \qquad \begin{aligned} 3x + 10 &\leq 6 - x \\ 4x &\leq -4 \\ x &\leq -1 \end{aligned}$$

The answers are $x \geq 3$ or $x \leq -1$.



Using a Graphing Calculator to Solve Compound Inequalities

As you have seen in previous Concepts, graphing calculators can be used to solve many complex algebraic sentences.

Example D

Solve $7x - 2 < 10x + 1 < 9x + 5$ using a graphing calculator.

Solution:

This is a compound inequality: $7x - 2 < 10x + 1$ and $10x + 1 < 9x + 5$.

To enter a compound inequality:

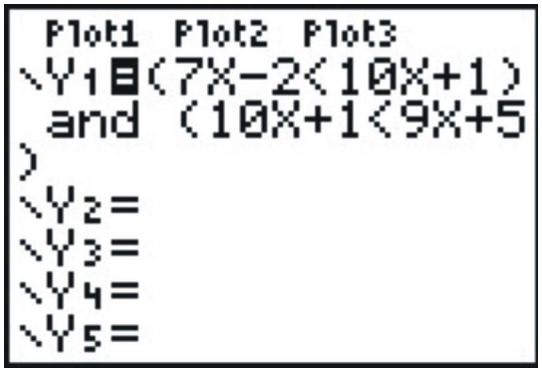
Press the **[Y=]** button.

The inequality symbols are found by pressing **[TEST] [2nd] [MATH]**

Enter the inequality as:

$$Y_1 = (7x - 2 < 10x + 1) \; AND \; (10x + 1 < 9x + 5)$$

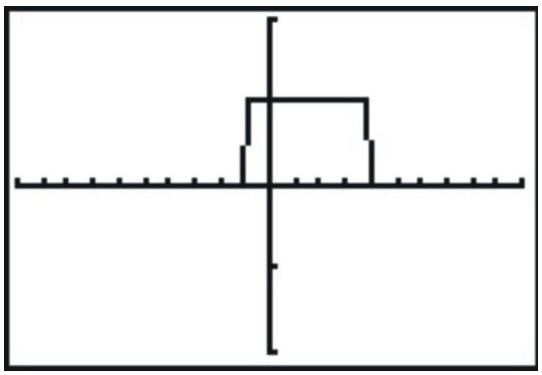
To enter the **[AND]** symbol, press **[TEST]**. Choose **[LOGIC]** on the top row and then select option 1.



The resulting graph is as shown below.

The solutions are the values of x for which $y = 1$.

In this case, $-1 < x < 4$.



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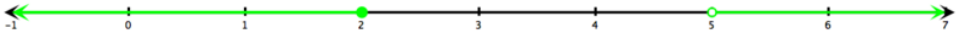
Guided Practice

Graph the solution set for $-1+b>4$ *or* $2b+7\leq 11$.

Solution:

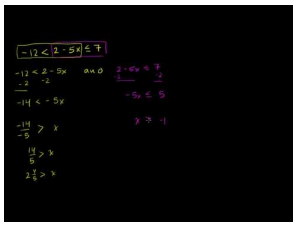
Start by solving for b for each inequality.

Start with the inequalities.	$-1+b>4$	$2b+7\leq 11$
Simplify by adding opposites.	$b>5$	$2b\leq 4$
Divide if necessary.		$b\leq 2$



Explore More

Sample explanations for some of the practice exercises below are available by viewing the following video. Note that there is not always a match between the number of the practice exercise in the video and the number of the practice exercise listed in the following exercise set. However, the practice exercise is the same in both. CK-12 Basic Algebra: Compound Inequalities (11:45)



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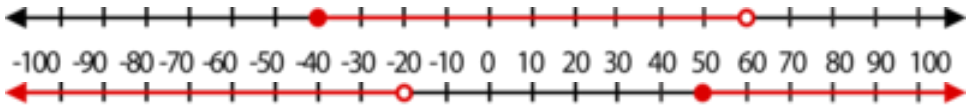
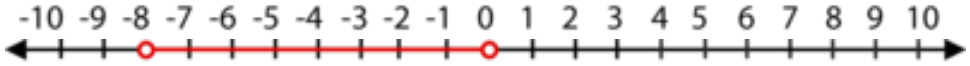
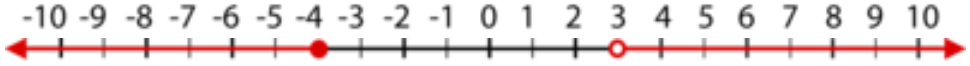
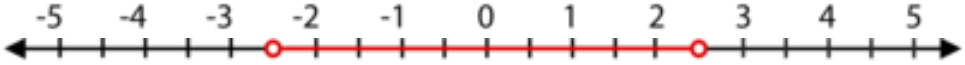
1. Describe the solution set to a compound inequality joined by the word "and."
2. How would your answer to question #1 change if the joining word was "or."
3. Write the process used to solve a compound inequality.

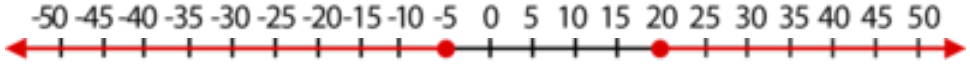
Write the compound inequalities represented by the following graphs.

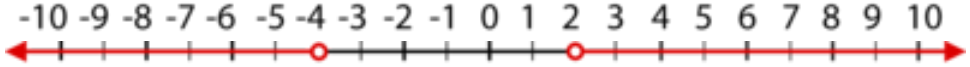
4. 

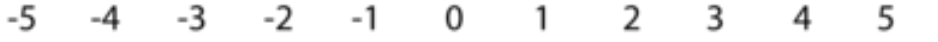
5. 

6. 

7. 

8. 

9. 

10. 

Graph each compound inequality on a number line.

11. $-4 \leq x \leq 6$
12. $x < 0$ or $x > 2$
13. $x \geq -8$ or $x \leq -20$
14. $-15 < x \leq 85$

In 15–30, solve the following compound inequalities and graph the solution on a number line.

15. $-5 \leq x - 4 \leq 13$
16. $-2 < 4x - 5 \leq 11$
17. $\frac{x-2}{6} \leq 2x - 4$ or $\frac{x-2}{6} > x + 5$
18. $1 \leq 3x + 4 \leq 4$
19. $-12 \leq 2 - 5x \leq 7$

20. $\frac{3}{4} \leq 2x+9 \leq \frac{3}{2}$
21. $-2 < \frac{2x-1}{3} < -1$
22. $5x+2(x-3) \geq 2$
23. $3x+2 \leq 10$ or $3x+2 \geq 15$
24. $4x-1 \geq 7$ or $\frac{9x}{2} < 3$
25. $3-x < -4$ or $3-x > 10$
26. $\frac{2x+3}{4} < 2$ or $-\frac{x}{5}+3\frac{2}{5}$
27. $2x-7 \leq -3$ or $2x-3 > 11$
28. $-6d > 48$ or $10+d > 11$
29. $6+b < 8$ or $b+6 \geq 6$
30. $4x+3 \leq 9$ or $-5x+4 \leq -12$