Compound Inequalities

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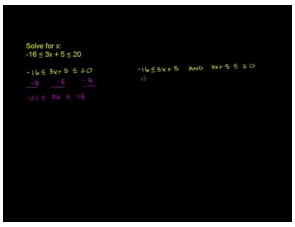
CHAPTER 1 Compound Inequalities

Here you will solve two inequalities that have been joined together by the words "and" and "or."

Mr. Garcia, the Spanish teacher, announces that the students' final grades will consist of 40% projects (0-100 score). The remaining 60% (0-100) will come from the final exam. Going into the final exam, Madison has an 84 in the project score. Within what grade range must she fall on the final exam to raise her score to an A (90-100 overall score)?

Watch This



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Khan Academy: Compound Inequalities 3

Guidance

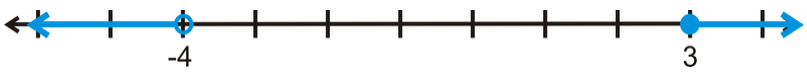
Compound inequalities are inequalities that have been joined by the words "and" or "or." For example:

$-2 < x \leq 5$ Read, "x is greater than -2 *and* less than or equal to 5."

$x \geq 3$ or $x < -4$ Read, "x is greater than or equal to 3 *or* less than -4."

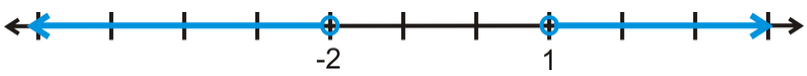
Notice that both of these inequalities have two inequality signs. So, it is like solving or graphing two inequalities at the same time. When graphing, look at the inequality to help you. The first compound inequality above, $-2 < x \leq 5$, has the x in between -2 and 5, so the shading will also be between the two numbers.

And, with the "or" statement, the shading will go in opposite directions.



Example A

Write the inequality statement given by the graph below.



Solution: Because the shading goes in opposite directions, we know this is an "or" statement. Therefore, the statement is $x < -2$ or $x > 1$.

Example B

Solve and graph $-3 < 2x+5 \leq 11$.

Solution: This is like solving two inequalities at the same time. You can split the statement apart to have two inequalities, $-3 < 2x+5$ and $2x+5 \leq 11$ and solve. You can also leave the compound inequality whole to solve.

$$\begin{array}{r} -3 < 2x+5 \leq 11 \\ \underline{-5 \qquad -5 \quad -5} \\ \frac{-8}{2} < \frac{2x}{2} \leq \frac{6}{2} \\ -4 < x \leq 3 \end{array}$$

Test a solution, $x = 0$:

$$\begin{aligned} -3 < 2(0)+5 &\leq 11 \\ -3 < 5 &\leq 11 \end{aligned}$$

Here is the graph:

Example C

Solve and graph $-32 > -5x+3$ or $x-4 \leq 2$.

Solution: When solving an "or" inequality, solve the two inequalities separately, but show the solution on the same number line.

$$\begin{array}{l} -32 > -5x+3 \text{ or } x-4 \leq 2 \\ \underline{-3 \qquad -3} \qquad \underline{+4 \quad +4} \\ \frac{-35}{-5} > \frac{-5x}{-5} \qquad\qquad x \leq 6 \\ 7 < x \end{array}$$

Notice that in the first inequality, we had to flip the inequality sign because we divided by -5. Also, it is a little more complicated to test a solution for these types of inequalities. You still test one point, but it will only work for one of the inequalities. Let's test $x = 10$. First inequality: $-32 > -5(10)+3 \rightarrow -32 > -47$. Second inequality: $10-4 \leq 2 \rightarrow 5 \not\leq 2$. Because $x = 10$ works for the first inequality, is a solution. Here is the graph.

Intro Problem Revisit Writing the grading as an expression, we get $0.4(84)+0.6x$ where x is the final exam score. Madison wants to get an A, so we will have a compound inequality that ranges between 90 and 100.

$$90 \leq 33.6 + 0.6x \leq 100$$
$$56.4 \leq 0.6x \leq 66.4$$
$$94 \leq x \leq 110.67$$

Unless Mr. Garcia offers extra credit, Madison can't score higher than 100. So, she has to score at least 94 or more, up to 100, to get an A.

Guided Practice

1. Graph $-7 \leq x \leq -1$ on a number line.

Solve the following compound inequalities and graph.

2. $5 \leq -\frac{2}{3}x + 1 \leq 15$
3. $\frac{x}{4} - 7 > 5$ or $\frac{8}{5}x + 2 \leq 18$

Answers

1. This is an "and" inequality, so the shading will be between the two numbers.

2. Solve this just like Example B.

$$5 \leq -\frac{2}{3}x + 1 \leq 15$$
$$\underline{-1 \qquad\quad -1 \quad -1}$$
$$4 \leq -\frac{2}{3}x \leq 14$$
$$-\frac{3}{2}\left(4 \leq -\frac{2}{3}x \leq 14\right)$$
$$-6 \geq x \geq -21$$

Test a solution, $x = -10$:

$$5 \leq -\frac{2}{3}(-12) + 1 \leq 15$$
$$5 \leq 9 \leq 15$$

This solution can also be written $-21 \leq x \leq -6$.

The graph is:

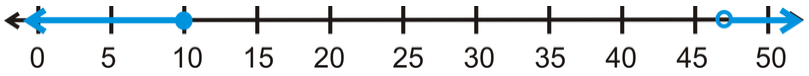
3. This is an "or" compound inequality. Solve the two inequalities separately.

$$\begin{array}{lll} \frac{x}{4}-7>5 & or & \frac{8}{5}x+2\le 18 \\ \underline{\quad +7 \quad +7} & & \underline{\quad -2 \quad -2} \\ 4\cdot\frac{x}{4}>12\cdot 4 & or & \frac{5}{8}\cdot\frac{8}{5}x\le 16\cdot\frac{5}{8} \\ x>48 & or & x\le 10 \end{array}$$

Test a solution, $x=0$:

$$\begin{array}{ll} \frac{0}{4}-7>5 & or \ \frac{8}{5}(0)+2\le 18 \\ -7\not>5 & or \quad 2\le 18 \end{array}$$

Notice that $x=0$ is a solution for the second inequality, which makes it a solution for the entire compound inequality. Here is the graph:



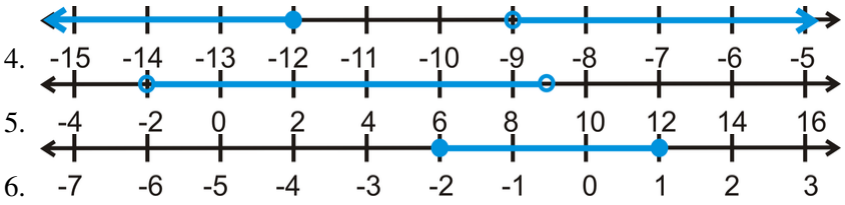
On problems 2 and 3 we changed the scale of the number line to accommodate the solution.

Explore More

Graph the following compound inequalities. Use an appropriate scale.

1. $-1<x<8$
2. $x>5$ or $x\le 3$
3. $-4\le x\le 0$

Write the compound inequality that best fits each graph below.



Solve each compound inequality and graph the solution.

7. $-11<x-9\le 2$
8. $8\le 3-5x<28$
9. $2x-7>-13$ or $\frac{1}{3}x+5\le 1$
10. $0<\frac{x}{5}<4$
11. $-4x+9<35$ or $3x-7\le -16$
12. $\frac{3}{4}x+7\ge -29$ or $16-x>2$
13. $3\le 6x-15<51$
14. $-20<-\frac{3}{2}x+1<16$
15. **Challenge** Write a compound inequality whose solutions are all real numbers. Show why this is true.