Solutions to Compound Inequalities

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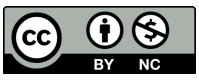

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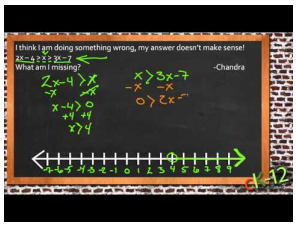
CHAPTER 1 Solutions to Compound Inequalities

Here you'll learn how to separate compound inequalities with *"and"* or *"or"* and solve them separately. You'll then learn how to combine your answers into a single solution and graph the solution set.

What if you had a compound inequality like $0 \leq 2x+6 \leq 6$? How could you solve it and graph its solution set? After completing this Concept, you'll be able to graph the solution set of compound inequalities like this one on a number line.

Watch This



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CK-12 Foundation: 0606S Solving Compound Inequalities (H264)

Guidance

When we solve compound inequalities, we separate the inequalities and solve each of them separately. Then, we combine the solutions at the end.

Example A

Solve the following compound inequalities and graph the solution set.

a) $-2 < 4x-5 \leq 11$

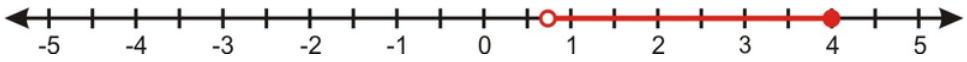
b) $3x-5 < x+9 \leq 5x+13$

Solution

a) First we re-write the compound inequality as two separate inequalities with *and*. Then solve each inequality separately.

$$\begin{aligned} -2 &< 4x-5 \\ 3 &< 4x \\ \frac{3}{4} &< x \end{aligned} \qquad \textit{and} \qquad \begin{aligned} 4x-5 &\leq 11 \\ 4x &\leq 16 \\ x &\leq 4 \end{aligned}$$

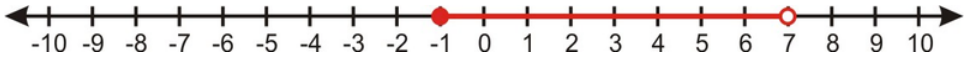
Answer: $\frac{3}{4} < x$ and $x \leq 4$. This can be written as $\frac{3}{4} < x \leq 4$.



b) Re-write the compound inequality as two separate inequalities with *and*. Then solve each inequality separately.

$$\begin{array}{ccc} 3x-5<x+9 & & x+9\le 5x+13 \\ 2x<14 & \textit{and} & -4\le 4x \\ x<7 & & -1\le x \end{array}$$

Answer: $x<7$ and $x\ge -1$. This can be written as: $-1\le x<7$.



Example B

Solve the following compound inequalities and graph the solution set.

a) $9-2x\le 3$ or $3x+10\le 6-x$

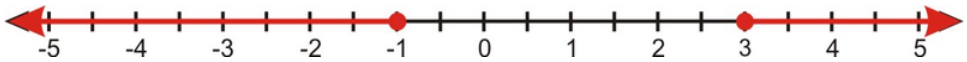
b) $\frac{x-2}{6}\le 2x-4$ or $\frac{x-2}{6}>x+5$

Solution

a) Solve each inequality separately:

$$\begin{array}{ccc} 9-2x\le 3 & & 3x+10\le 6-x \\ -2x\le -6 & \textit{or} & 4x\le -4 \\ x\ge 3 & & x\le -1 \end{array}$$

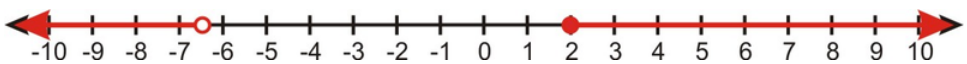
Answer: $x\ge 3$ or $x\le -1$



b) Solve each inequality separately:

$$\begin{array}{ccc} \frac{x-2}{6}\le 2x-4 & & \frac{x-2}{6}>x+5 \\ x-2\le 6(2x-4) & & x-2>6(x+5) \\ x-2\le 12x-24 & \textit{or} & x-2>6x+30 \\ 22\le 11x & & -32>5x \\ 2\le x & & -6.4>x \end{array}$$

Answer: $x\ge 2$ or $x<-6.4$



One thing you may notice in the video for this Concept is that in the second problem, the two solutions joined with "or" overlap, and so the solution ends up being the set of all real numbers, or $(-\infty, \infty)$. This happens sometimes with compound inequalities that involve "or"; for example, if the solution to an inequality ended up being "$x < 5$ or $x > 1$," the solution set would be all real numbers. This makes sense if you think about it: all real numbers are either a) less than 5, or b) greater than or equal to 5, and the ones that are greater than or equal to 5 are also greater than 1—so all real numbers are either a) less than 5 or b) greater than 1.

Compound inequalities with "and," meanwhile, can turn out to have *no* solutions. For example, the inequality "$x < 3$ and $x > 4$" has no solutions: no number is both greater than 4 and less than 3. If we write it as $4 < x < 3$ it's even more obvious that it has no solutions; $4 < x < 3$ implies that $4 < 3$, which is false.

Solve Real-World Problems Using Compound Inequalities

Many application problems require the use of compound inequalities to find the solution.

Example C

The speed of a golf ball in the air is given by the formula $v = -32t + 80$. When is the ball traveling between 20 ft/sec and 30 ft/sec?

Solution

First we set up the inequality $20 \leq v \leq 30$, and then replace v with the formula $v = -32t + 80$ to get $20 \leq -32t + 80 \leq 30$.

Then we separate the compound inequality and solve each separate inequality:

$$\begin{aligned} 20 &\leq -32t + 80 \\ 32t &\leq 60 \\ t &\leq 1.875 \end{aligned} \quad \text{and} \quad \begin{aligned} -32t + 80 &\leq 30 \\ 50 &\leq 32t \\ 1.56 &\leq t \end{aligned}$$

Answer: $1.56 \leq t \leq 1.875$

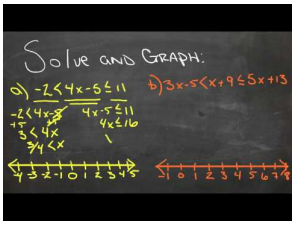
To check the answer, we plug in the minimum and maximum values of t into the formula for the speed.

For $t = 1.56$, $v = -32t + 80 = -32(1.56) + 80 = 30\ ft/sec$

For $t = 1.875$, $v = -32t + 80 = -32(1.875) + 80 = 20\ ft/sec$

So the speed is between 20 and 30 ft/sec. **The answer checks out.**

Watch this video for help with the Examples above.



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CK-12 Foundation: Solving Compound Inequalities

Vocabulary

- **Compound inequalities** combine two or more inequalities with **"and"** or **"or."**

- **"And"** combinations mean that only solutions for *both* inequalities will be solutions to the compound inequality.
- **"Or"** combinations mean solutions to *either* inequality will also be solutions to the compound inequality.

Guided Practice

William's pick-up truck gets between 18 to 22 miles per gallon of gasoline. His gas tank can hold 15 gallons of gasoline. If he drives at an average speed of 40 miles per hour, how much driving time does he get on a full tank of gas?

Solution

Let $t =$ driving time. We can use dimensional analysis to get from time per tank to miles per gallon:

$$\frac{t \text{ hours}}{1 \text{ tank}} \times \frac{1 \text{ tank}}{15 \text{ gallons}} \times \frac{40 \text{ miles}}{1 \text{ hour}} \times \frac{40t}{15} \frac{\text{miles}}{\text{gallon}}$$

Since the truck gets between 18 and 22 miles/gallon, we set up the compound inequality $18 \leq \frac{40t}{15} \leq 22$. Then we separate the compound inequality and solve each inequality separately:

$$\begin{aligned} 18 &\leq \frac{40t}{15} \\ 270 &\leq 40t \\ 6.75 &\leq t \end{aligned} \qquad \text{and} \qquad \begin{aligned} \frac{40t}{15} &\leq 22 \\ 40t &\leq 330 \\ t &\leq 8.25 \end{aligned}$$

Answer: $6.75 \leq t \leq 8.25$.

Andrew can drive between 6.75 and 8.25 hours on a full tank of gas.

If we plug in $t = 6.75$ we get $\frac{40t}{15} = \frac{40(6.75)}{15} = 18$ *miles per gallon.*

If we plug in $t = 8.25$ we get $\frac{40t}{15} = \frac{40(8.25)}{15} = 22$ *miles per gallon.*

The answer checks out.

Explore More

Solve the following compound inequalities and graph the solution on a number line.

1. $-5 \leq x - 4 \leq 13$
2. $1 \leq 3x + 5 \leq 4$
3. $-12 \leq 2 - 5x \leq 7$
4. $\frac{3}{4} \leq 2x + 9 \leq \frac{3}{2}$
5. $-2 \leq \frac{2x-1}{3} < -1$
6. $4x - 1 \geq 7$ or $\frac{9x}{2} < 3$
7. $3 - x < -4$ or $3 - x > 10$
8. $\frac{2x+3}{4} < 2$ or $-\frac{x}{5} + 3 < \frac{2}{5}$
9. $2x - 7 \leq -3$ or $2x - 3 > 11$
10. $4x + 3 < 9$ or $-5x + 4 \leq -12$
11. How would you express the answer to problem 1 as a set?
12. How would you express the answer to problem 1 as an interval?

13. How would you express the answer to problem 6 as a set?
14. Could you express the answer to problem 6 as a single interval? Why or why not?
    a. How would you express the first part of the solution in interval form?
    b. How would you express the second part of the solution in interval form?
15. Express the answers to problems 2 through 5 in interval notation.
16. Solve the inequality "$x \geq -3$ or $x < 1$" and express the answer in interval notation.
17. How many solutions does the inequality "$x \geq 2$ and $x \leq 2$" have?
18. To get a grade of B in her Algebra class, Stacey must have an average grade greater than or equal to 80 and less than 90. She received the grades of 92, 78, 85 on her first three tests.
    a. Between which scores must her grade on the final test fall if she is to receive a grade of B for the class? (Assume all four tests are weighted the same.)
    b. What range of scores on the final test would give her an overall grade of C, if a C grade requires an average score greater than or equal to 70 and less than 80?
    c. If an A grade requires a score of at least 90, and the maximum score on a single test is 100, is it possible for her to get an A in this class? (Hint: look again at your answer to part a.)