Absolute Value Equations

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CHAPTER 1 Absolute Value Equations

Here you'll learn how to find the solutions to absolute value equations.

Suppose that a movie director has been told by a film studio that the length of the movie he's making must not differ from 120 minutes by more than 10 minutes. What would be the longest and shortest acceptable movies he could make? What absolute value equation could you set up to find out? In this Concept, you'll learn to solve problems like this one by setting up and finding the solutions to absolute value equations.

Guidance

Absolute value situations can also involve unknown variables. For example, suppose the distance from zero is 16. What two points can this represent?

Begin by writing an absolute value sentence to represent this situation.

$$16 = |n|, \text{ where } n = \text{the missing value}$$

Which two numbers are 16 units from zero?

$$n = 16 \text{ or } n = -16$$

Absolute value situations can also involve distances from points other than zero. We treat such cases as compound inequalities, separating the two independent equations and solving separately.

Example A

Solve for x : $|x-4|=5$.

Solution: This equation looks like the distance definition:

$$\text{distance} = |x-y| \text{ or } |y-x|$$

The distance is 5, and the value of y is 4. We are looking for two values that are five units away from four on a number line.

Visually, we can see the answers are –1 and 9.

Algebraically, we separate the two absolute value equations and solve.

$$x-4=5 \text{ and } x-4=-(5)$$

By solving each, the solutions become:

$$x = 9 \text{ and } x = -1$$

Example B

Solve $|2x-7|=6$.

Solution:

Begin by separating this into its separate equations.

$$2x-7=6 \text{ and } 2x-7=-6$$

Solve each equation independently.

$$\begin{aligned} 2x-7&=6 \\ 2x-7+7&=6+7 \\ 2x&=13 \\ x&=\frac{13}{2} \end{aligned} \qquad \begin{aligned} 2x-7&=-6 \\ 2x-7+7&=-6+7 \\ 2x&=1 \\ x&=\frac{1}{2} \end{aligned}$$

Example C

A company packs coffee beans in airtight bags. Each bag should weigh 16 ounces but it is hard to fill each bag to the exact weight. After being filled, each bag is weighed and if it is more than 0.25 ounces overweight or underweight, it is emptied and repacked. What are the lightest and heaviest acceptable bags?

Solution:

The varying quantity is the weight of the bag of coffee beans. Choosing a letter to represent this quantity and writing an absolute value equation yields:

$$|w-16|=0.25$$

Separate and solve.

$$\begin{aligned} w-16&=0.25 \\ w&=16.25 \end{aligned} \qquad \begin{aligned} w-16&=-0.25 \\ w&=15.75 \end{aligned}$$

The lightest bag acceptable is 15.75 ounces and the heaviest bag accepted is 16.25 ounces.

Video Review

MEDIA

Click image to the left or use the URL below.

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Guided Practice

Solve for n: $|-3n+5|=17$.

Solution:

$|-3n+5|=17$ means that $-3n+5=17$ or $-3n+5=-17$.

Solving each one separately, we get:

$$\begin{aligned} -3n+5&=17 \\ -3n&=12 \\ n&=-4 \end{aligned} \qquad \begin{aligned} -3n+5&=-17 \\ -3n&=-22 \\ n&=\frac{22}{3} \end{aligned}$$

Explore More

In 1–12, solve the absolute value equations and interpret the results by graphing the solutions on a number line.

1. $|7u|=77$
2. $|x-5|=10$
3. $|5r-6|=9$
4. $1=\frac{|6+5z|}{5}$
5. $|8x|=32$
6. $|\frac{m}{8}|=1$
7. $|x+2|=6$
8. $|5x-2|=3$
9. $51=|1-5b|$
10. $8=3+|10y+5|$
11. $|4x-1|=19$
12. $8|x+6|=-48$

13. A company manufactures rulers. Their 12-inch rulers pass quality control if they're within $\frac{1}{32}$ inches of the ideal length. What is the longest and shortest ruler that can leave the factory?

Mixed Review

14. A map has a scale of 2 *inch* = 125 *miles*. How far apart would two cities be on the map if the actual distance is 945 miles?

15. Determine the domain and range: $\{(-9,0),(-6,0),(-4,0),(0,0),(3,0),(5,0)\}$.
16. Is the relation in question #15 a function? Explain your reasoning.
17. Consider the problem $3(2x-7)=100$. Lei says the first step to solving this equation is to use the Distributive Property to cancel the parentheses. Hough says the first step to solving this equation is to divide by 3. Who is right? Explain your answer.
18. Graph $4x+y=6$ using its intercepts.
19. Write $\frac{3}{30}$ as a percent. Round to the nearest hundredth.
20. Simplify $-5\frac{2}{3} \div \frac{71}{8}$.