Absolute Value Equations

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Printed: March 27, 2015

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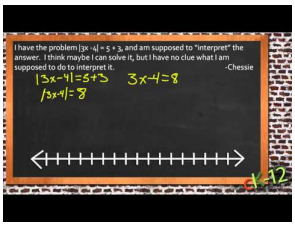
CHAPTER 1 Absolute Value Equations

Here you'll learn how to how to solve more complicated absolute value equations and interpret your answers.

What if you were asked to solve an absolute value equation like $|3x-4|=5$? How could you interpret the solution? After completing this Concept, you'll be able to interpret the solutions to absolute value equations like this one by graphing them on a number line.

Watch This



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Guidance

In the previous concept, we saw how to solve simple absolute value equations. In this concept, you will see how to solve more complicated absolute value equations.

Example A

Solve the equation $|x-4|=5$ *and interpret the answers.*

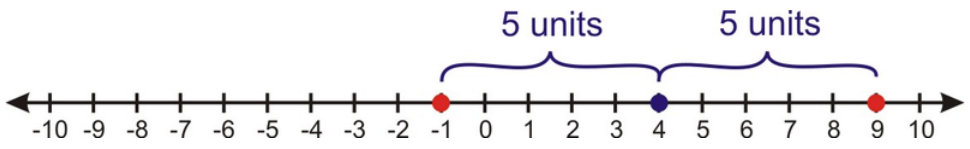
Solution

We consider two possibilities: the expression inside the absolute value sign is non-negative or is negative. Then we solve each equation separately.

$$\begin{aligned} x-4&=5 \quad \text{and} \quad x-4=-5 \\ x&=9 \qquad\qquad\quad x=-1 \end{aligned}$$

$x=9$ and $x=-1$ are the solutions.

The equation $|x-4|=5$ can be interpreted as "what numbers on the number line are 5 units away from the number 4?" If we draw the number line we see that there are two possibilities: 9 and -1.



Example B

Solve the equation $|x+3|=2$ *and interpret the answers.*

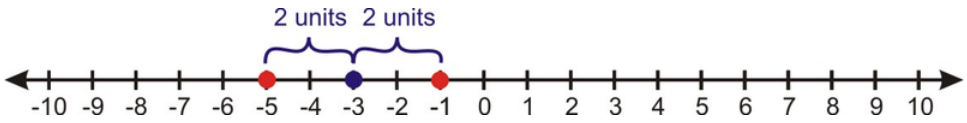
Solution

Solve the two equations:

$$x+3=2 \quad \text{and} \quad x+3=-2$$
$$x=-1 \qquad\qquad x=-5$$

$x=-5$ and $x=-1$ are the answers.

The equation $|x+3|=2$ can be re-written as: $|x-(-3)|=2$. We can interpret this as "what numbers on the number line are 2 units away from -3?" There are two possibilities: -5 and -1.



Solve Real-World Problems Using Absolute Value Equations

Example C

A company packs coffee beans in airtight bags. Each bag should weigh 16 ounces, but it is hard to fill each bag to the exact weight. After being filled, each bag is weighed; if it is more than 0.25 ounces overweight or underweight, it is emptied and repacked. What are the lightest and heaviest acceptable bags?

Solution

The weight of each bag is allowed to be 0.25 ounces away from 16 ounces; in other words, the *difference* between the bag's weight and 16 ounces is allowed to be 0.25 ounces. So if x is the weight of a bag in ounces, then the equation that describes this problem is $|x-16|=0.25$.

Now we must consider the positive and negative options and solve each equation separately:

$$x-16=0.25 \quad \text{and} \quad x-16=-0.25$$
$$x=16.25 \qquad\qquad x=15.75$$

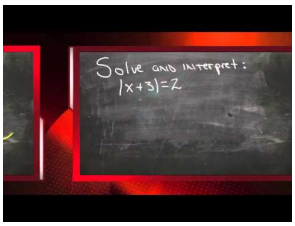
The lightest acceptable bag weighs 15.75 ounces and the heaviest weighs 16.25 ounces.

We see that $16.25-16=0.25$ *ounces* and $16-15.75=0.25$ *ounces*. The answers are 0.25 ounces bigger and smaller than 16 ounces respectively.

The answer checks out.

The answer you just found describes the lightest and heaviest acceptable bags of coffee beans. But how do we describe the total possible range of acceptable weights? That's where inequalities become useful once again.

Watch this video for help with the Examples above.



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CK-12 Foundation: Analyze Solutions to Absolute Value Equations

Vocabulary

- The absolute value of a number is its distance from zero on a number line.
- $|x| = x$ if x is not negative, and $|x| = -x$ if x is negative.
- An equation or inequality with an absolute value in it **splits into two equations,** one where the expression inside the absolute value sign is positive and one where it is negative. When the expression within the absolute value is **positive**, then the absolute value signs do nothing and can be omitted. When the expression within the absolute value is **negative,** then the expression within the absolute value signs must be negated before removing the signs.
- Inequalities of the type $|x| < a$ can be rewritten as "$-a < x < a$."
- Inequalities of the type $|x| > b$ can be rewritten as "$x < -b$ or $x > b$."

Guided Practice

Solve the equation $|2x-7| = 6$ *and interpret the answers.*

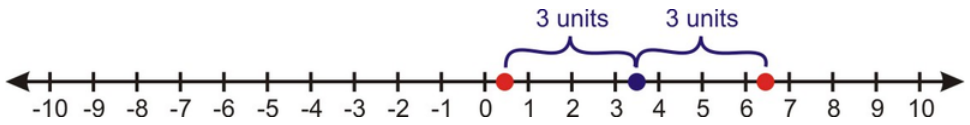
Solution

Solve the two equations:

$$\begin{aligned} 2x-7 &= 6 \\ 2x &= 13 \\ x &= \frac{13}{2} \end{aligned} \quad \text{and} \quad \begin{aligned} 2x-7 &= -6 \\ 2x &= 1 \\ x &= \frac{1}{2} \end{aligned}$$

Answer: $x = \frac{13}{2}$ and $x = \frac{1}{2}$.

The interpretation of this problem is clearer if the equation $|2x-7| = 6$ is divided by 2 on both sides to get $\frac{1}{2}|2x-7| = 3$. Because $\frac{1}{2}$ is nonnegative, we can distribute it over the absolute value sign to get $\left|x - \frac{7}{2}\right| = 3$. The question then becomes "What numbers on the number line are 3 units away from $\frac{7}{2}$?" There are two answers: $\frac{13}{2}$ and $\frac{1}{2}$.



Explore More

Solve the absolute value equations and interpret the results by graphing the solutions on the number line.

1. $|x-5| = 10$

2. $|x+2|=6$
3. $|5x-2|=3$
4. $|x-4|=-3$
5. $\left|2x-\frac{1}{2}\right|=10$
6. $|-x+5|=\frac{1}{5}$
7. $\left|\frac{1}{2}x-5\right|=100$
8. $|10x-5|=15$
9. $|0.1x+3|=0.015$
10. $|27-2x|=3x+2$