# Solving Absolute Value Equations

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# CHAPTER 1 Solving Absolute Value Equations

Here you'll learn to solve absolute value equations.

To determine the height of skeletal remains, archaeologists use the equation $H = 2.26f + 66.4$, where $H$ is the height in centimeters and $f$ is the length of the skeleton's femur (also in cm). The equation has a margin of error of $\pm 3.42cm$. Dr. Jordan found a skeletal femur that is 46.8 cm. Determine the greatest height and the least height of this person.

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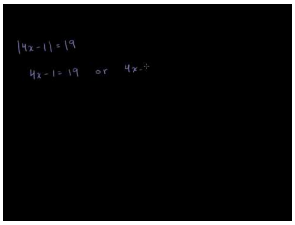
Watch the first part of this video.



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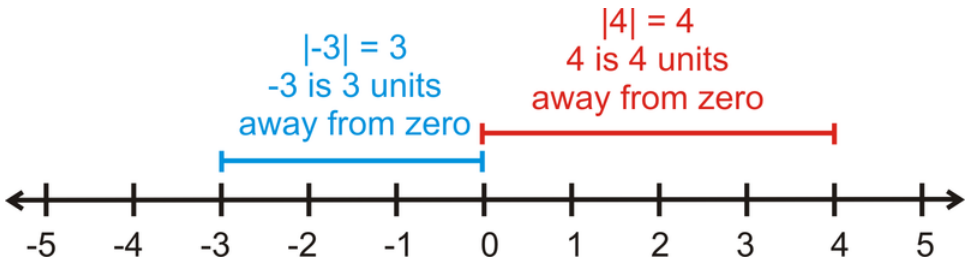
Khan Academy: Absolute Value Equations

## Guidance

**Absolute value** is the distance a number is from zero. Because distance is always positive, the absolute value will always be positive. Absolute value is denoted with two vertical lines around a number, $|x|$.




$|5| = 5$ $|-9| = 9$ $|0| = 0$ $|-1| = 1$

When solving an absolute value equation, the value of $x$ could be two different possibilities; whatever makes the absolute value positive OR whatever makes it negative. Therefore, there will always be TWO answers for an absolute value equation.

If $|x| = 1$, then $x$ can be 1 or -1 because $|1| = 1$ and $|-1| = 1$.

If $|x| = 15$, then $x$ can be 15 or -15 because $|15| = 15$ and $|-15| = 15$.

From these statements we can conclude:

$$|x| = \begin{cases} x \text{ if } x \geq 0 \\ -x \text{ if } x < 0 \end{cases}$$

### Example A

Determine if $x = -12$ is a solution to $|2x - 5| = 29$.

**Solution:** Plug in -12 for $x$ to see if it works.

$$\begin{aligned} |2(-12) - 5| &= 29 \\ |-24 - 5| &= 29 \\ |-29| &= 29 \end{aligned}$$

-12 is a solution to this absolute value equation.

### Example B

Solve $|x + 4| = 11$.

**Solution:** There are going to be two answers for this equation. $x + 4$ can equal 11 or -11.

$$\begin{gathered} |x+4| = 11 \\ \swarrow \quad \searrow \\ x+4 = 11 \quad x+4 = -11 \\ or \\ x = 7 \qquad x = -15 \end{gathered}$$

Test the solutions:

$$\begin{aligned} |7+4| &= 11 \qquad |-15+4| = 11 \\ |11| &= 11 \qquad |-11| = 11 \end{aligned}$$

### Example C

Solve $\left|\frac{2}{3}x - 5\right| = 17$.

**Solution:** Here, what is inside the absolute value can be equal to 17 or -17.

$$\left|\frac{2}{3}x-5\right|=17$$

$$\swarrow \quad \searrow$$

$$\frac{2}{3}x-5=17 \qquad \frac{2}{3}x-5=-17$$

$$\frac{2}{3}x=22 \quad or \quad \frac{2}{3}x=-12$$

$$x=22\cdot\frac{3}{2} \qquad x=-12\cdot\frac{3}{2}$$

$$x=33 \qquad x=-18$$

Test the solutions:

$$\left|\frac{2}{3}(33)-5\right|=17 \qquad \left|\frac{2}{3}(-18)-5\right|=17$$

$$|22-5|=17 \qquad |-12-5|=17$$

$$|17|=17 \qquad |-17|=17$$

**Intro Problem Revisit** First, we need to find the height of the skeleton using the equation $H=2.26f+66.4$, where $f=46.8$.

$$H=2.26(46.8)+66.4$$
$$H=172.168cm$$

Now, let's use an absolute value equation to determine the margin of error, and thus the greatest and least heights.

$$|x-172.168|=3.42$$

$$\swarrow \quad \searrow$$

$$x-172.168=3.42 \quad x-172.168=-3.42$$

$$or$$

$$x=175.588 \qquad x=168.748$$

So the person could have been a maximum of 175.588 cm or a minimum of 168.748 cm. In inches, this would be 69.13 and 66.44 inches, respectively.

## Guided Practice

1. Is $x=-5$ a solution to $|3x+22|=6$?

Solve the following absolute value equations.

2. $|6x-11|+2=41$

3. $\left|\frac{1}{2}x+3\right|=9$

## Answers

1. Plug in -5 for $x$ to see if it works.

$$\begin{aligned} |3(-5)+22| &= 6 \\ |-15+22| &= 6 \\ |-7| &\neq 6 \end{aligned}$$

-5 is not a solution because $|-7| = 7$, not 6.

2. Find the two solutions. Because there is a 2 being added to the left-side of the equation, we first need to subtract it from both sides so the absolute value is by itself.

$$\begin{aligned} |6x-11|+2 &= 41 \\ |6x-11| &= 39 \\ \swarrow & \quad \searrow \\ 6x-11=39 & \quad 6x-11=-39 \\ 6x=50 & \quad 6x=-28 \\ x=\frac{50}{6} \quad or & \quad x=-\frac{28}{6} \\ =\frac{25}{3} \text{ or } 8\frac{1}{3} & \quad =-\frac{14}{3} \text{ or } -4\frac{2}{3} \end{aligned}$$

Check both solutions. It is easier to check solutions when they are improper fractions.

$$\begin{aligned} \left|6\left(\frac{25}{3}\right)-11\right| &= 39 \\ |50-11| &= 39 \quad and \\ |39| &= 39 \end{aligned} \qquad \begin{aligned} \left|6\left(-\frac{14}{3}\right)-11\right| &= 39 \\ |-28-11| &= 39 \\ |-39| &= 39 \end{aligned}$$

3. What is inside the absolute value is equal to 9 or -9.

$$\begin{aligned} \left|\frac{1}{2}x+3\right| &= 9 \\ \swarrow & \quad \searrow \\ \frac{1}{2}x+3=9 & \quad \frac{1}{2}x+3=-9 \\ \frac{1}{2}x=6 \quad or & \quad \frac{1}{2}x=-12 \\ x=12 & \quad x=-24 \end{aligned}$$

Test solutions:

$$\begin{aligned} \left|\frac{1}{2}(12)+3\right| &= 9 \\ |6+3| &= 9 \\ |9| &= 9 \end{aligned} \qquad \begin{aligned} \left|\frac{1}{2}(-24)+3\right| &= 9 \\ |-12+3| &= 9 \\ |-9| &= 9 \end{aligned}$$

## Explore More

Determine if the following numbers are solutions to the given absolute value equations.

1. $|x-7|=16;9$
2. $|\frac{1}{4}x+1|=4;-8$
3. $|5x-2|=7;-1$

Solve the following absolute value equations.

4. $|x+3|=8$
5. $|2x|=9$
6. $|2x+15|=3$
7. $\left|\frac{1}{3}x-5\right|=2$
8. $\left|\frac{x}{6}+4\right|=5$
9. $|7x-12|=23$
10. $\left|\frac{3}{5}x+2\right|=11$
11. $|4x-15|+1=18$
12. $|-3x+20|=35$
13. $|12x-18|=0$
14. What happened in #13? Why do you think that is?
15. **Challenge** When would an absolute value equation have no solution? Give an example.