

Solving Absolute Value Equations

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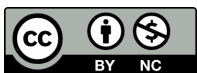
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CHAPTER

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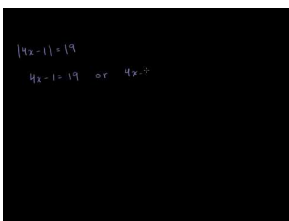
Solving Absolute Value Equations

Here you'll learn to solve absolute value equations.

To determine the height of skeletal remains, archaeologists use the equation $H = 2.26f + 66.4$, where H is the height in centimeters and f is the length of the skeleton's femur (also in cm). The equation has a margin of error of ± 3.42 cm. Dr. Jordan found a skeletal femur that is 46.8 cm. Determine the greatest height and the least height of this person.

Watch This

Watch the first part of this video.

**MEDIA**

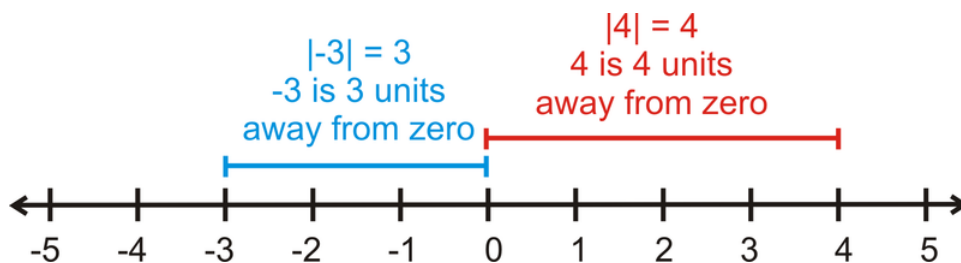
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[Khan Academy: Absolute Value Equations](#)

Guidance

Absolute value is the distance a number is from zero. Because distance is always positive, the absolute value will always be positive. Absolute value is denoted with two vertical lines around a number, $|x|$.



$$|5|=5$$

$$|-9|=9$$

$$|0|=0$$

$$|-1|=1$$

When solving an absolute value equation, the value of x could be two different possibilities; whatever makes the absolute value positive OR whatever makes it negative. Therefore, there will always be TWO answers for an absolute value equation.

If $|x|=1$, then x can be 1 or -1 because $|1|=1$ and $|-1|=1$.

If $|x|=15$, then x can be 15 or -15 because $|15|=15$ and $|-15|=15$.

From these statements we can conclude:

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

Example A

Determine if $x = -12$ is a solution to $|2x - 5| = 29$.

Solution: Plug in -12 for x to see if it works.

$$\begin{aligned} |2(-12) - 5| &= 29 \\ |-24 - 5| &= 29 \\ |-29| &= 29 \end{aligned}$$

-12 is a solution to this absolute value equation.

Example B

Solve $|x + 4| = 11$.

Solution: There are going to be two answers for this equation. $x + 4$ can equal 11 or -11.

$$\begin{aligned} |x + 4| &= 11 \\ \swarrow \quad \searrow \\ x + 4 &= 11 \quad x + 4 = -11 \\ \text{or} \\ x &= 7 \quad x = -15 \end{aligned}$$

Test the solutions:

$$\begin{aligned} |7 + 4| &= 11 & |-15 + 4| &= 11 \\ |11| &= 11 & |-11| &= 11 \end{aligned}$$

Example C

Solve $\left| \frac{2}{3}x - 5 \right| = 17$.

Solution: Here, what is inside the absolute value can be equal to 17 or -17.

$$\left| \frac{2}{3}x - 5 \right| = 17$$

$$\swarrow \quad \searrow$$

$$\frac{2}{3}x - 5 = 17 \quad \frac{2}{3}x - 5 = -17$$

$$\frac{2}{3}x = 22 \quad \text{or} \quad \frac{2}{3}x = -12$$

$$x = 22 \cdot \frac{3}{2} \quad x = -12 \cdot \frac{3}{2}$$

$$x = 33 \quad x = -18$$

Test the solutions:

$$\left| \frac{2}{3}(33) - 5 \right| = 17 \quad \left| \frac{2}{3}(-18) - 5 \right| = 17$$

$$|22 - 5| = 17 \quad |-12 - 5| = 17$$

$$|17| = 17 \quad |-17| = 17$$

Intro Problem Revisit First, we need to find the height of the skeleton using the equation $H = 2.26f + 66.4$, where $f = 46.8$.

$$H = 2.26(46.8) + 66.4$$

$$H = 172.168 \text{ cm}$$

Now, let's use an absolute value equation to determine the margin of error, and thus the greatest and least heights.

$$|x - 172.168| = 3.42$$

$$\swarrow \quad \searrow$$

$$x - 172.168 = 3.42 \quad x - 172.168 = -3.42$$

$$\text{or}$$

$$x = 175.588 \quad x = 168.748$$

So the person could have been a maximum of 175.588 cm or a minimum of 168.748 cm. In inches, this would be 69.13 and 66.44 inches, respectively.

Guided Practice

1. Is $x = -5$ a solution to $|3x + 22| = 6$?

Solve the following absolute value equations.

2. $|6x - 11| + 2 = 41$

3. $\left| \frac{1}{2}x + 3 \right| = 9$

Answers

1. Plug in -5 for x to see if it works.

$$|3(-5) + 22| = 6$$

$$|-15 + 22| = 6$$

$$|-7| \neq 6$$

-5 is not a solution because $|-7| = 7$, not 6.

2. Find the two solutions. Because there is a 2 being added to the left-side of the equation, we first need to subtract it from both sides so the absolute value is by itself.

$$\begin{aligned} |6x - 11| + 2 &= 41 \\ |6x - 11| &= 39 \\ \swarrow \quad \searrow \\ 6x - 11 &= 39 & 6x - 11 &= -39 \\ 6x &= 50 & 6x &= -28 \\ x &= \frac{50}{6} & \text{or} & x = -\frac{28}{6} \\ &= \frac{25}{3} & \text{or} & 8\frac{1}{3} & = -\frac{14}{3} & \text{or} & -4\frac{2}{3} \end{aligned}$$

Check both solutions. It is easier to check solutions when they are improper fractions.

$$\begin{aligned} \left| 6\left(\frac{25}{3}\right) - 11 \right| &= 39 & \left| 6\left(-\frac{14}{3}\right) - 11 \right| &= 39 \\ |50 - 11| &= 39 & \text{and} & |-28 - 11| &= 39 \\ |39| &= 39 & & & |-39| &= 39 \end{aligned}$$

3. What is inside the absolute value is equal to 9 or -9.

$$\begin{aligned} \left| \frac{1}{2}x + 3 \right| &= 9 \\ \swarrow \quad \searrow \\ \frac{1}{2}x + 3 &= 9 & \frac{1}{2}x + 3 &= -9 \\ \frac{1}{2}x &= 6 & \text{or} & \frac{1}{2}x &= -12 \\ x &= 12 & & & x &= -24 \end{aligned}$$

Test solutions:

$$\begin{aligned} \left| \frac{1}{2}(12) + 3 \right| &= 9 & \left| \frac{1}{2}(-24) + 3 \right| &= 9 \\ |6 + 3| &= 9 & |-12 + 3| &= 9 \\ |9| &= 9 & & & |-9| &= 9 \end{aligned}$$

Explore More

Determine if the following numbers are solutions to the given absolute value equations.

1. $|x - 7| = 16; 9$
2. $|\frac{1}{4}x + 1| = 4; -8$
3. $|5x - 2| = 7; -1$

Solve the following absolute value equations.

4. $|x + 3| = 8$
5. $|2x| = 9$
6. $|2x + 15| = 3$
7. $|\frac{1}{3}x - 5| = 2$
8. $|\frac{x}{6} + 4| = 5$
9. $|7x - 12| = 23$
10. $|\frac{3}{5}x + 2| = 11$
11. $|4x - 15| + 1 = 18$
12. $|-3x + 20| = 35$
13. $|12x - 18| = 0$
14. What happened in #13? Why do you think that is?
15. **Challenge** When would an absolute value equation have no solution? Give an example.