Solutions to Absolute Value Equations

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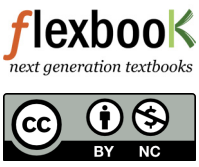

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CHAPTER 1 Solutions to Absolute Value Equations

Here you'll learn how to solve absolute value equations.

Solve for the variable in the equation: $|3x+1|=4$

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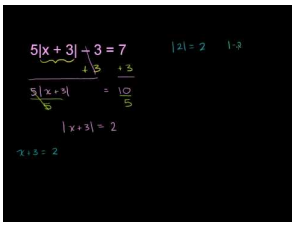

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Guidance

Recall that a linear equation relates mathematical expressions with an equals sign. To solve an absolute linear equation you have to remember the same rules that you have used to solve linear equations with one variable. The difference with absolute value equations is there will often be two solutions instead of just one solution. Consider the following equations:

1.

$$|x|=5$$

This means that x can be 5 or x can be –5. This is because $|5|=|-5|=5$.

2.

$$|x+1|=7$$

This means that $x+1$ can be 7 or $x+1$ can be –7. This is because $|7|=|-7|=7$.

3.

$$|x|=-1$$

The absolute value of x can never be equal to a negative number. Therefore if an absolute value equation is equal to a negative number, there is no solution.

Example A

$|d+3|=2.1$

Solution: Set up two equations to solve. You know that either $d+3=2.1$ OR $d+3=-2.1$. *The quantity inside the absolute value signs could be either the positive or negative of the value on the right side.*

$$
\begin{aligned}
d+3&=2.1 && \\
d+3-3&=2.1-3 && \text{Subtract 3 from both sides to isolate the variable} \\
d&=0.9 && \text{Simplify} \\
&OR && \\
d+3&=-2.1 && \\
d+3-3&=-2.1-3 && \text{Subtract 3 from both sides to isolate the variable} \\
d&=-5.1 && \text{Simplify}
\end{aligned}
$$

Solutions $= 0.9, -5.1$

Example B

$|2(z+4)| = |5|$

Solution: First of all, you know that $|5| = 5$. Now, set up two equations to solve. You know that either $2(z+4) = 5$ OR $2(z+4) = -5$.

$$
\begin{aligned}
2(z+4)&=5 && \\
2z+8&=5 && \text{Remove parentheses} \\
2z+8-8&=5-8 && \text{Subtract 8 from both sides to isolate the variable} \\
2z&=-3 && \text{Simplify} \\
\frac{2z}{2}&=\frac{-3}{2} && \text{Divide by 2 to solve for the variable} \\
z&=\frac{-3}{2} && \text{Simplify} \\
&OR && \\
2(z+4)&=-5 && \\
2z+8&=-5 && \text{Remove parentheses} \\
2z+8-8&=-5-8 && \text{Subtract 8 from both sides to isolate the variable} \\
2z&=-13 && \text{Simplify} \\
\frac{2z}{2}&=\frac{-13}{2} && \text{Divide by 2 to solve for the variable} \\
z&=\frac{-13}{2} && \text{Simplify}
\end{aligned}
$$

Solutions $= \frac{-3}{2}, \frac{-13}{2}$

Example C

$|\frac{1}{2}x+3| = |\frac{4}{5}|$

Solution: First of all, you know that $|\frac{4}{5}| = \frac{4}{5}$. Now, set up two equations to solve. You know that either $\frac{1}{2}x+3 = \frac{4}{5}$ OR $\frac{1}{2}x+3 = -\frac{4}{5}$.

$$\frac{1}{2}x+3=\frac{4}{5}$$

$$\left(\frac{5}{5}\right)\frac{1}{2}x+\left(\frac{10}{10}\right)3=\left(\frac{2}{2}\right)\frac{4}{5}$$ Multiply to get common denominator (LCD = 10)

$$\frac{5}{10}x+\frac{30}{10}=\frac{8}{10}$$ Simplify

$$5x+30=8$$ Simplify

$$5x+30-30=8-30$$ Subtract 30 from both sides to isolate the variable

$$5x=-22$$ Simplify

$$\frac{5x}{5}=\frac{-22}{5}$$ Divide by 5 to solve for the variable

$$x=\frac{-22}{5}$$ Simplify

OR

$$\frac{1}{2}x+3=\frac{-4}{5}$$

$$\left(\frac{5}{5}\right)\frac{1}{2}x+\left(\frac{10}{10}\right)3=\left(\frac{2}{2}\right)\frac{-4}{5}$$ Multiply to get common denominator (LCD = 10)

$$\frac{5}{10}x+\frac{30}{10}=\frac{-8}{10}$$ Simplify

$$5x+30=-8$$ Simplify

$$5x+30-30=-8-30$$ Subtract 30 from both sides to isolate the variable

$$5x=-38$$ Simplify

$$\frac{5x}{5}=\frac{-38}{5}$$ Divide by 5 to solve for the variable

$$x=\frac{-38}{5}$$ Simplify

Solutions $=\frac{-22}{5},\frac{-38}{5}$

Concept Problem Revisited

Solve for the variable in the expression: $|3x+1|=4$

Because $|3x+1|=4$, the expression $3x+1$ is equal to 4 ***or*** –4.

$$
\begin{aligned}
3x+1 &= 4 \\
3x+1-1 &= 4-1 && \text{Subtract 1 from both sides to isolate the variable} \\
3x &= 3 && \text{Simplify} \\
\frac{3x}{3} &= \frac{3}{3} && \text{Divide by 3 to solve for the variable} \\
x &= 1 && \text{Simplify}
\end{aligned}
$$

OR

$$
\begin{aligned}
3x+1 &= -4 \\
3x+1-1 &= -4-1 && \text{Subtract 1 from both sides to isolate the variable} \\
3x &= -5 && \text{Simplify} \\
\frac{3x}{3} &= \frac{-5}{3} && \text{Divide by 3 to solve for the variable} \\
x &= \frac{-5}{3} && \text{Simplify}
\end{aligned}
$$

Just like with regular linear equations, you can check both answers.

$$
\begin{aligned}
|3x+1| &= 4 \\
\left|3\left(\frac{-5}{3}\right)+1\right| &= 4 \\
|-5+1| &= |-4| = 4
\end{aligned}
\qquad\qquad
\begin{aligned}
|3x+1| &= 4 \\
|3(1)+1| &= 4 \\
|4| &= 4
\end{aligned}
$$

Guided Practice

Solve each equation.

1. $|4a-2|=3$
2. $|2b-8|-3=4$
3. $|\frac{1}{2}c-5|=3$

Answers:

1. The solutions are $\frac{5}{4}, \frac{-1}{4}$. Here are the steps:

$$
\begin{aligned}
4a-2&=3 && \\
4a-2+2&=3+2 && \text{Add 2 to both sides to isolate the variable} \\
4a&=5 && \text{Simplify} \\
\frac{4a}{4}&=\frac{5}{4} && \text{Divide by 4 to solve for the variable} \\
a&=\frac{5}{4} && \\
&OR && \\
4a-2&=-3 && \\
4a-2+2&=-3+2 && \text{Add 2 to both sides to isolate the variable} \\
4a&=-1 && \text{Simplify} \\
\frac{4a}{4}&=\frac{-1}{4} && \text{Divide by 4 to solve for the variable} \\
a&=\frac{-1}{4} &&
\end{aligned}
$$

2. The solutions are $\frac{15}{2}, \frac{1}{2}$. First, isolate the part of the equation with the absolute value sign by adding 3 to both sides. The new equation is $|2b-8|=7$. Then, set up two equations and solve.

$$
\begin{aligned}
2b-8&=7 && \\
2b&=15 && \text{Add 8 to both sides and simplify} \\
\frac{2b}{2}&=\frac{15}{2} && \text{Divide by 2 to solve for the variable} \\
b&=\frac{15}{2} && \\
&OR && \\
2b-8&=-7 && \\
2b-8+8&=-7+8 && \text{Add 8 to both sides to isolate the variable} \\
2b&=1 && \text{Simplify} \\
\frac{2b}{2}&=\frac{1}{2} && \text{Divide by 2 to solve for the variable} \\
b&=\frac{1}{2} &&
\end{aligned}
$$

3. The solutions are $16, 4$. Here are the steps to solve:

$$\frac{1}{2}c-5=3$$

$\frac{1}{2}c-\left(\frac{2}{2}\right)5=\left(\frac{2}{2}\right)3$ Multiply to get common denominator. (LCD $=2$)

$\frac{c}{2}-\frac{10}{2}=\frac{6}{2}$ Simplify

$c-10=6$ Simplify

$c-10+10=6+10$ Add 10 to both sides to isolate the variable

$$c=16$$

OR

$$\frac{1}{2}c-5=-3$$

$\frac{1}{2}c-\left(\frac{2}{2}\right)5=\left(\frac{2}{2}\right)-3$ Multiply to get common denominator. (LCD $=2$)

$\frac{c}{2}-\frac{10}{2}=\frac{-6}{2}$ Simplify

$c-10=-6$ Simplify

$c-10+10=-6+10$ Add 10 to both sides to isolate the variable

$$c=4$$

Explore More

Solve each of the following absolute value linear equations.

1. $|t+2|=4$
2. $|r-2|=7$
3. $|5-k|=6$
4. $|6-y|=12$
5. $-6=|1-b|$
6. $|\frac{1}{5}x-3|=1$
7. $|\frac{1}{2}(r-3)|=2$
8. $|\frac{1}{3}(f+1)|=5$
9. $|3d-11|=-2$
10. $|5w+9|-6=68$
11. $|5(2t+5)+3(t-1)|=-3$
12. $|2.24x-24.63|=2.25$
13. $|6(5j-3)+2|=14$
14. $|7g-8(g+3)|=1$
15. $|e+4(e+3)|=17$