Graphs of Absolute Value Equations

Andrew Gloag
Eve Rawley
Anne Gloag

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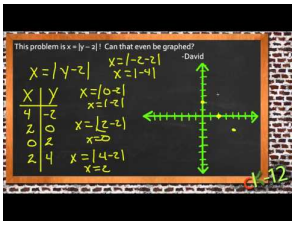
AUTHORS
Andrew Gloag
Eve Rawley
Anne Gloag

CHAPTER 1 Graphs of Absolute Value Equations

Here you'll learn how to make a table of values for absolute value functions so you can graph them.

What if you were given an absolute value function like $y = |x - 8|$? How could you graph this function? After completing this Concept, you'll be able to make a table of values to graph absolute value functions like this one.

Watch This



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Guidance

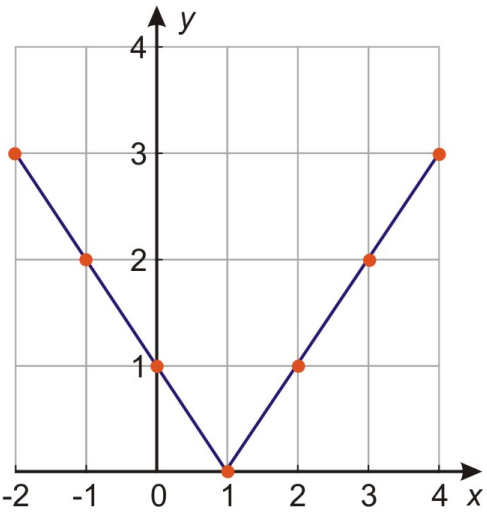
Now let's look at how to graph absolute value functions.

Example A

Consider the function $y = |x - 1|$. We can graph this function by making a table of values:

TABLE 1.1:

x	$y = \lvert x - 1 \rvert$
-2	$y = \lvert -2 - 1 \rvert = \lvert -3 \rvert = 3$
-1	$y = \lvert -1 - 1 \rvert = \lvert -2 \rvert = 2$
0	$y = \lvert 0 - 1 \rvert = \lvert -1 \rvert = 1$
1	$y = \lvert 1 - 1 \rvert = \lvert 0 \rvert = 0$
2	$y = \lvert 2 - 1 \rvert = \lvert 1 \rvert = 1$
3	$y = \lvert 3 - 1 \rvert = \lvert 2 \rvert = 2$
4	$y = \lvert 4 - 1 \rvert = \lvert 3 \rvert = 3$



You can see that the graph of an absolute value function makes a big "V". It consists of two line rays (or line segments), one with positive slope and one with negative slope, joined at the **vertex** or **cusp**.

We've already seen that to solve an absolute value equation we need to consider two options:

1. The expression inside the absolute value is not negative.
2. The expression inside the absolute value is negative.

Combining these two options gives us the two parts of the graph.

For instance, in the above example, the expression inside the absolute value sign is $x-1$. By definition, this expression is nonnegative when $x-1 \geq 0$, which is to say when $x \geq 1$. When the expression inside the absolute value sign is nonnegative, we can just drop the absolute value sign. So for all values of x greater than or equal to 1, the equation is just $y = x-1$.

On the other hand, when $x-1 < 0$ —in other words, when $x < 1$ —the expression inside the absolute value sign is negative. That means we have to drop the absolute value sign but also multiply the expression by -1. So for all values of x less than 1, the equation is $y = -(x-1)$, or $y = -x+1$.

These are both graphs of straight lines, as shown above. They meet at the point where $x-1 = 0$ —that is, at $x = 1$.

We can graph absolute value functions by breaking them down algebraically as we just did, or we can graph them using a table of values. However, when the absolute value equation is linear, the easiest way to graph it is to combine those two techniques, as follows:

1. Find the vertex of the graph by setting the expression inside the absolute value equal to zero and solving for x.
2. Make a table of values that includes the vertex, a value smaller than the vertex, and a value larger than the vertex. Calculate the corresponding values of y using the equation of the function.
3. Plot the points and connect them with two straight lines that meet at the vertex.

Example B

Graph the absolute value function $y = |x+5|$.

Solution

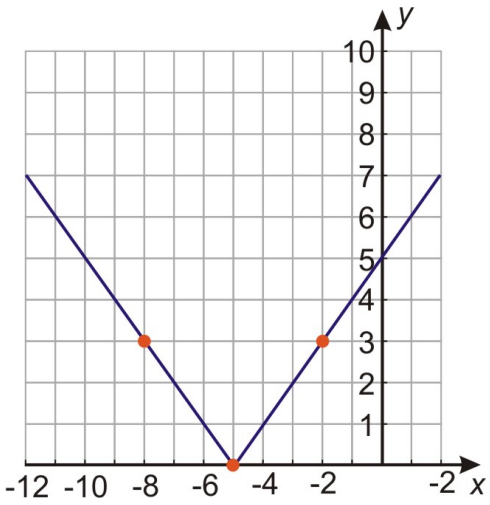
Step 1: Find the vertex by solving $x+5 = 0$. The vertex is at $x = -5$.

Step 2: Make a table of values:

TABLE 1.2:

x	$y = \lvert x+5 \rvert$
-8	$y = \lvert -8+5 \rvert = \lvert -3 \rvert = 3$
-5	$y = \lvert -5+5 \rvert = \lvert 0 \rvert = 0$
-2	$y = \lvert -2+5 \rvert = \lvert 3 \rvert = 3$

Step 3: Plot the points and draw two straight lines that meet at the vertex:



Example C

Graph the absolute value function: $y = \lvert 3x - 12 \rvert$

Solution

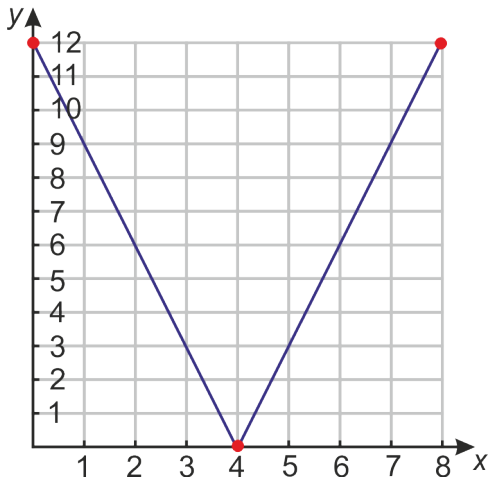
Step 1: Find the vertex by solving $3x - 12 = 0$. The vertex is at $x = 4$.

Step 2: Make a table of values:

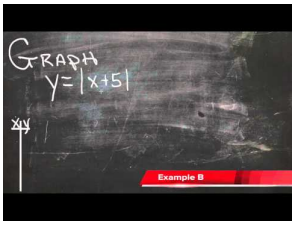
TABLE 1.3:

x	$y = \lvert 3x - 12 \rvert$
0	$y = \lvert 3(0) - 12 \rvert = \lvert -12 \rvert = 12$
4	$y = \lvert 3(4) - 12 \rvert = \lvert 0 \rvert = 0$
8	$y = \lvert 3(8) - 12 \rvert = \lvert 12 \rvert = 12$

Step 3: Plot the points and draw two straight lines that meet at the vertex.



Watch this video for help with the Examples above.



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Vocabulary

- The absolute value of a number is its distance from zero on a number line.
- $|x| = x$ if x is not negative, and $|x| = -x$ if x is negative.
- An equation or inequality with an absolute value in it **splits into two equations,** one where the expression inside the absolute value sign is positive and one where it is negative. When the expression within the absolute value is **positive**, then the absolute value signs do nothing and can be omitted. When the expression within the absolute value is **negative,** then the expression within the absolute value signs must be negated before removing the signs.
- Inequalities of the type $|x| < a$ can be rewritten as "$-a < x < a$."
- Inequalities of the type $|x| > b$ can be rewritten as "$x < -b$ or $x > b$."

Guided Practice

Graph the absolute value function: $y = 3|x-4|$

Solution

Step 1: Find the vertex by solving $x - 4 = 0$. The vertex is at $x = 4$.

Step 2: Make a table of values:

TABLE 1.4:

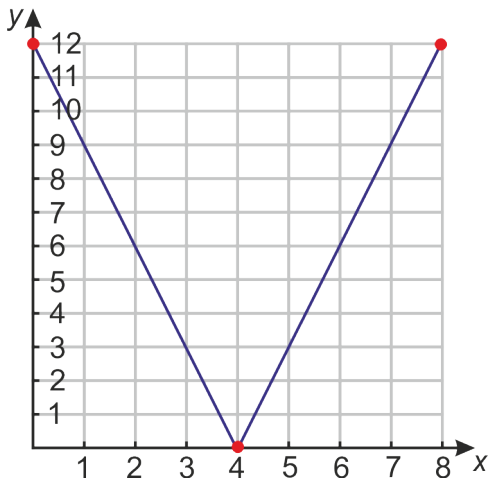
x	$y = 3\|x-4\|$
0	$y = 3\|0-4\| = 3\|-4\| = 3 \cdot 4 = 12$

TABLE 1.4: (continued)

x	$y = 3\|x-4\|$
4	$y = 3\|4-4\| = 3\|0\| = 3 \cdot 0 = 0$
8	$y = 3\|8-4\| = 3\|4\| = 3 \cdot 4 = 12$

Notice this is the same table as Example C. The function $y = 3|x-4|$ is equivalent to the function $y = |3x-12|$. This is because positive numbers can be factored out, or distributed into the absolute value function.

Step 3: Plot the points and draw two straight lines that meet at the vertex.



Explore More

Graph the absolute value functions.

1. $y = |x+3|$
2. $y = |x-6|$
3. $y = |4x+2|$
4. $y = |5-6x|$
5. $y = |2x-1|$
6. $y = 3|2x-7|$
7. $y = 0.05|x-1.25|$
8. $y = \frac{1}{2}|x+10|$
9. $y = \left|\frac{x}{3}-4\right|$
10. $y = -2\left|\frac{x}{2}-5\right|$