Graphing Basic Absolute Value Functions

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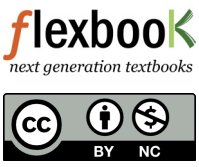

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CHAPTER 1 Graphing Basic Absolute Value Functions

Here you'll learn about the basic properties of absolute value functions.

While on vacation, you go scuba diving. You start at an unknown sea level of zero feet or higher. You then dive to a depth of 90 feet below sea level. What is the vertex of the absolute value function that represents your possible distance from sea level after diving?

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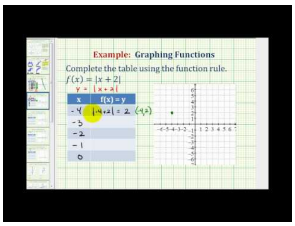

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James Sousa: Ex: Graph an Absolute Value Function Using a Table of Values

Guidance

In the *Solving Absolute Value Equations* concept, we learned how to solve and define absolute value equations. We will now take this idea one step further and graph absolute value equations.

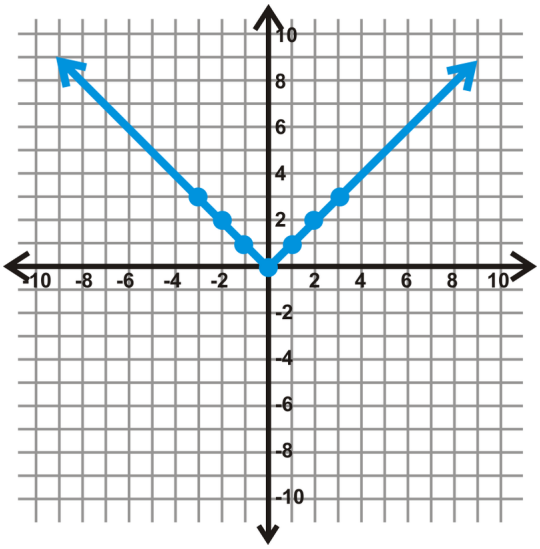
Investigation: Graphing the Parent Graph of an Absolute Value Function

1. We are going to graph $y = |x|$. Draw a table for x and y, with the $x-$values ranging from -3 to 3.

TABLE 1.1:

x	$\|x\|$	y
-3	$\|-3\|$	3
-2	$\|-2\|$	2
-1	$\|-1\|$	1
0	$\|0\|$	0
1	$\|1\|$	1
2	$\|2\|$	2
3	$\|3\|$	3

2. Recall that the absolute value of a number is always positive. Now that you have 7 sets of points, plot each one and graph the function.



3. Notice that this function is very similar to the linear function, $y = x$. Draw this line on the graph in a different color or with a dashed line.

4. Now, fold your graph on the $x-$axis. What do you notice?

In the investigation, you should discover that when you fold your graph on the $x-$axis, the line $y = x$ becomes the absolute value equation, $y = |x|$. That is because the absolute value of a number can never be zero; therefore, the range will always be positive. We call $y = |x|$ the **parent graph** because it is the most basic of all the absolute value functions. We will also compare other absolute value functions to this graph. All linear absolute value functions have this "V" shape.

In general, we can define the graph of $y = |x|$ as $y = \begin{cases} x; & x \geq 0 \\ -x; & x < 0 \end{cases}$. From this, we see that each side, is the mirror image of the other over a vertical line, through the vertex.

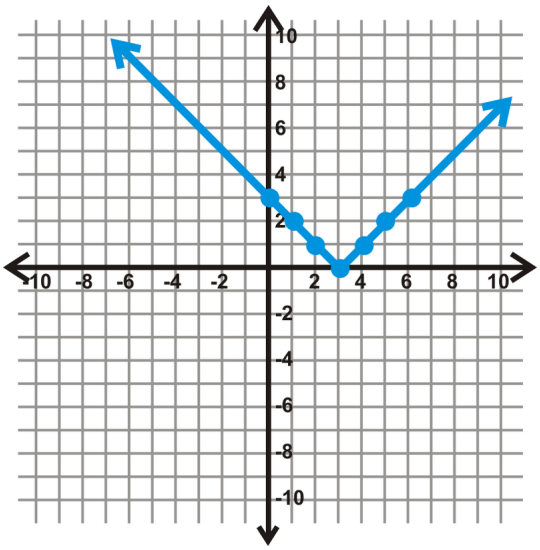
Example A

Use a table to graph $y = |x - 3|$. Determine the domain and range.

Solution: In general, when you use a table to graph a function, pick some positive and negative numbers, as well as zero. Use the equation to help you determine which $x-$values to pick. Setting what is inside the absolute value equal to zero, we get that $x = 3$. Pick three values on either side of $x = 3$ and then graph.

TABLE 1.2:

x	$\|x-3\|$	y
0	$\|-3\|$	3
1	$\|-2\|$	2
2	$\|-1\|$	1
3	$\|0\|$	0
4	$\|1\|$	1
5	$\|2\|$	2
6	$\|3\|$	3



Notice that this graph shifts to the right 3 when compared to the parent graph. The domain will be all real numbers, $x \in \mathbb{R}$, and the range will be all positive real numbers, including zero, $y \in [0, \infty)$.

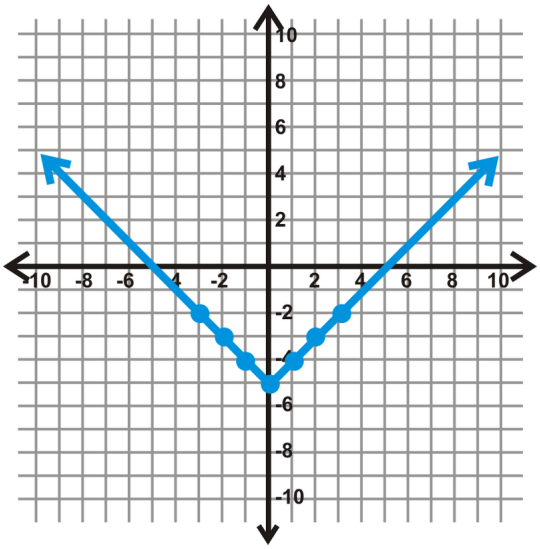
Example B

Use a table to graph $y = |x| - 5$. Determine the domain and range.

Solution: Be careful! Here, the minus 5 is not inside the absolute value. So, first take the absolute value of the $x-$value and then subtract 5. In cases like these, the range can include negative numbers.

TABLE 1.3:

x	$\|x\|-5$	y
-3	$\|-3\|-5$	-2
-2	$\|-2\|-5$	-3
-1	$\|-1\|-5$	-4
0	$\|0\|-5$	-5
1	$\|1\|-5$	-4
2	$\|2\|-5$	-3
3	$\|3\|-5$	-2



Here, the graph shifts down 5 when compared to the parent graph. The domain will be all real numbers, $x \in \mathbb{R}$, and the range will be all real numbers greater than or equal to -5, $y \in [-5, \infty)$.

In these three absolute value graphs, you may have noticed that there is a **minimum** point. This point is called the **vertex.** For example, in Example B, the vertex is (0, -5). The vertex can also be a **maximum.** See the next example.

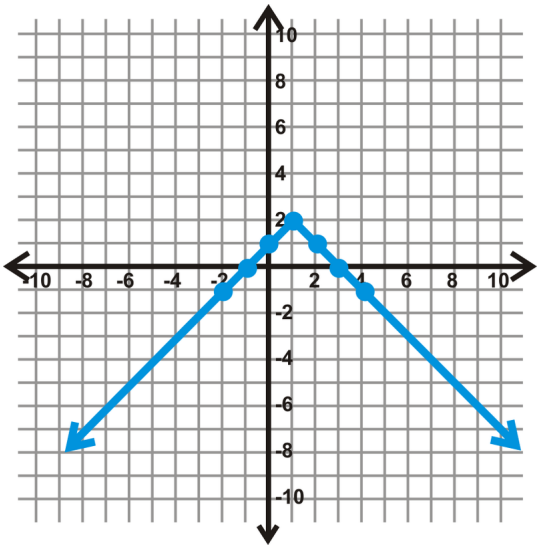
Example C

Use a table to graph $y = -|x-1|+2$. Determine the vertex, domain, and range.

Solution: Determine what makes the inside of the absolute value equation zero, $x = 1$. Then, to make your table of values, pick a couple values on either side of $x = 1$.

TABLE 1.4:

x	$-\|x-1\|+2$	y
-2	$-\|-2-1\|+2$	-1
-1	$-\|-1-1\|+2$	0
0	$-\|0-1\|+2$	1
1	$-\|1-1\|+2$	2
2	$-\|2-1\|+2$	1
3	$-\|3-1\|+2$	0
4	$-\|4-1\|+2$	-1



The vertex is (1, 2) and in this case, it is the maximum value. The domain is $x \in \mathbb{R}$, and the range is $y \in (-\infty, 2]$.

Intro Problem Revisit The absolute value function that represents this situation is $y = |x - 90|$, where x is your sea level before diving. By graphing this function, you can see that the vertex occurs at the point (90, 0)

Guided Practice

Graph the following functions using a table. Determine the vertex, domain, and range of each function.

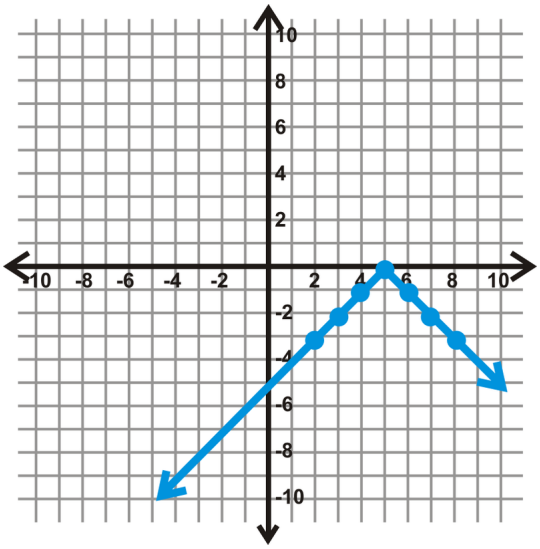
1. $y = -|x - 5|$
2. $y = |x + 4| - 2$

Answers

1. Determine what makes the inside of the absolute value equation zero, $x = 5$. Then, to make your table of values, pick a couple values on either side of $x = 5$.

TABLE 1.5:

x	$-\|x-5\|$	y
2	$-\|2-5\|$	-3
3	$-\|3-5\|$	-2
-4	$-\|4-5\|$	-1
5	$-\|5-5\|$	0
6	$-\|6-5\|$	-1
7	$-\|7-5\|$	-2
8	$-\|8-5\|$	-3

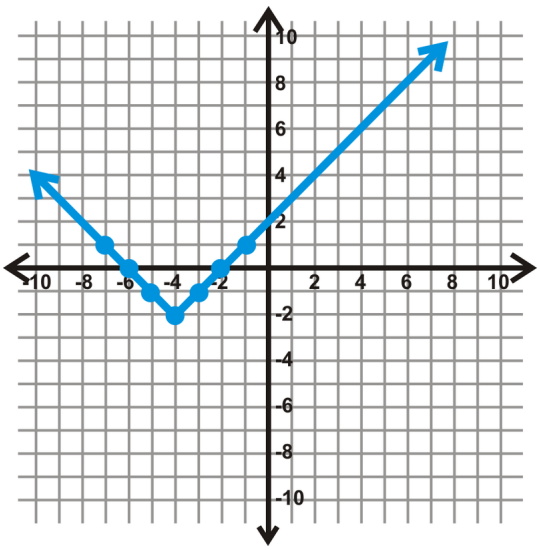


The vertex is (5, 0) and in this case, it is the maximum value. The domain is $x \in \mathbb{R}$, and the range is $y \in (-\infty, 0]$.

2. Determine what makes the inside of the absolute value equation zero, $x = -4$. Then, to make your table of values, pick a couple values on either side of $x = -4$.

TABLE 1.6:

x	$\lvert x+4\rvert-2$	y
-1	$\lvert -1+4\rvert-2$	1
-2	$\lvert -2+4\rvert-2$	0
-3	$\lvert -3+4\rvert-2$	-1
-4	$\lvert -4+4\rvert-2$	-2
-5	$\lvert -5+4\rvert-2$	-1
-6	$\lvert -6+4\rvert-2$	0
-7	$\lvert -7+4\rvert-2$	1



The vertex is (-4, -2) and in this case, it is the minimum value. The domain is $x \in \mathbb{R}$, and the range is $y \in [-2, \infty)$.

Explore More

Graph the following functions using a table. Determine the vertex, domain, and range of each function.

1. $y = |x+6|$
2. $y = |x-4|$
3. $y = -|x|+3$
4. $y = |x|-2$
5. $y = -|x+3|+7$
6. $y = |x-1|-6$
7. $y = 2|x|$
8. $y = -3|x|$
9. $y = \frac{1}{3}|x|$

Use problems 1-9 to answer fill in the blanks.

10. If there is a negative sign in front of the absolute value, the graph is _____ (when compared to the parent graph).
11. If the equation is $y = |x-h|+k$, the vertex will be _____.
12. The domain of an absolute value function is always _____.
13. For $y = a|x|$, if $a > 1$, then the graph will be _____ than the parent graph.
14. For $y = a|x|$, if $0 < a < 1$, then the graph will be _____ than the parent graph.
15. Without making a table, what is the vertex of $y = |x-9|+7$?